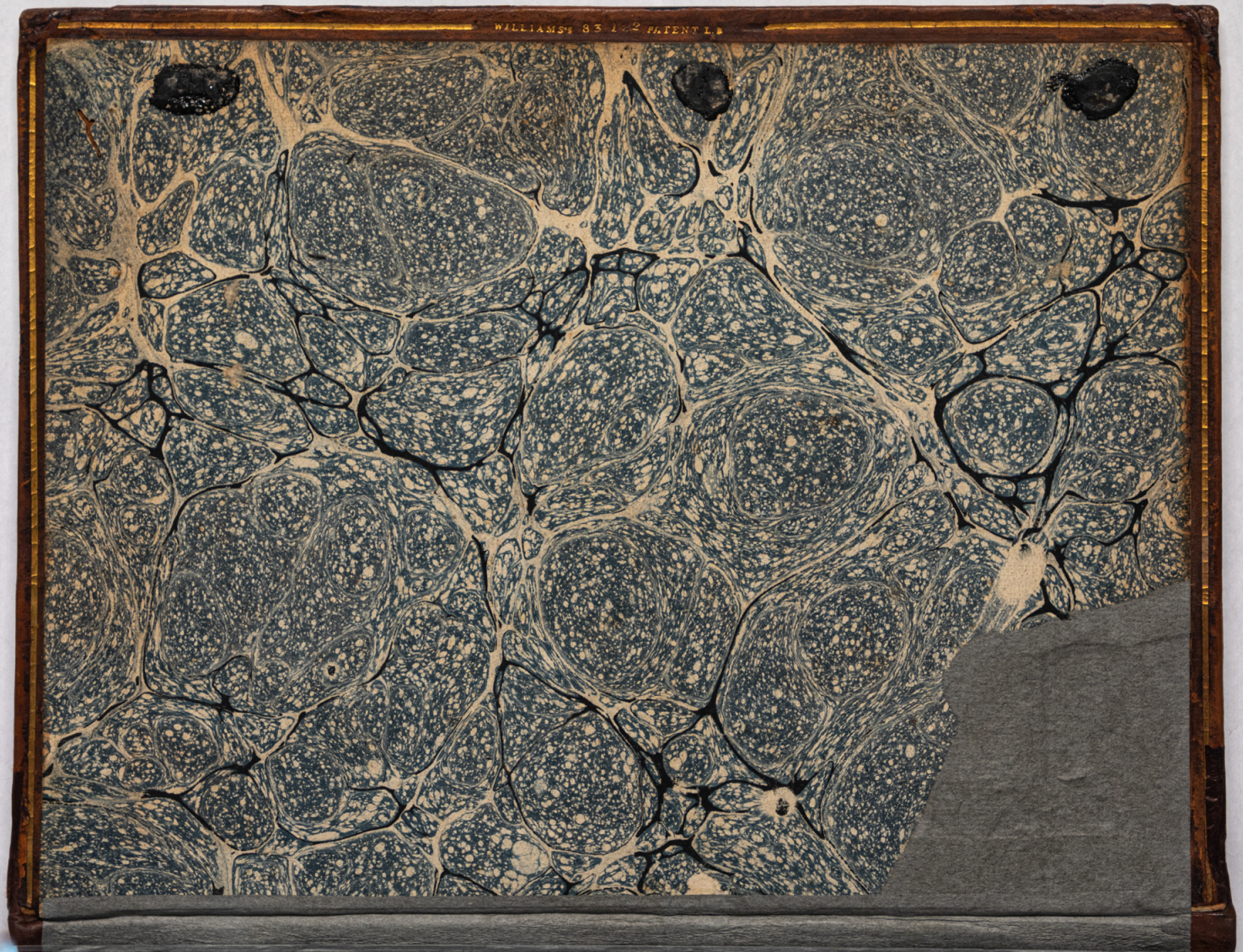



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WILLIAMS' 33 1/2 PATENT L.B

James Barnett
12,178.

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THE

Elements

of

NAVIGATION

Performed by

Patrick Crichton

Educated upon the

Royal Math.l Found.n

By

Lawrence Gwynne

A.M Master

1812

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Arithmetic.

Definitions & Principles.

Arithmetic is a science which teaches the properties of numbers & how to compute or estimate the value of things. An Unit or Unity is any thing considered as one.

Number in general is many units.

Digits or Figures are the marks by which numbers are denoted or expressed & are the names follg.

Digits	1	2	3	4	5	6	7	8	9
Names	One	Two	Three	Four	Five	Six	Seven	Eight	Nine

And with these is used the mark 0 called

cypher, which of itself stands for nothing; but being annexed to a digit, alters its value.

Thus 10 signifies forty; & 400 stands for four hundred.

Integer, or Whole Numbers, are such as express a number of things each of which is considered as an unit. Thus, four pounds, twelve miles, thirty-four gallons, one hundred days, &c are, in each case called an integer or whole number.

Fractional Numbers are those which express the value of some part or parts of an unit.

Thus one half one quarter three quarters &c are each the fractional part of an unit

Notation is the expressing by digits or figures any number proposed in words & the reading of any number that is expressed by figures is called numeration.

Decimal Notation is that kind of numbering in which ten units of an inferior name are equal in value to one unit of the next superior.

Every number is said to consist of as many places as it contains figures.

The value of every digit in any number is changed according to the place it stands in & the reading of any number consists in giving to each figure its right name & value. The right hand place of an integer number is called the place of

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units & from this place all numbers begin whether whole or fractional the integers encreasing in order from the units place towards the left & the fractions decreasing in order from the units place towards the right & to distinguish decimal fractions from integers ..... there is always a point or comma ( ) set on the left hand side of the fractional number so that the integers stand on the left hand side of the mark & the fractions on the right hand.

For the more convenient reading of numbers they are divided into periods of six places each beginning each period into two degrees of three places each the names & order of which are as follows where H stands for the word ten O for hundreds & TH for thousands.

Integers						Decimal Fractions					
Second Period			First Period			First Period			Second Period		
Degree	Degree	Degree	Degree	Degree	Degree	Degree	Degree	Degree	Degree	Degree	Degree
Billions	Billions	Billions	Millions	Millions	Millions	Units	Hundredths	Thousandths	Millionths	Millionths	Millionths
CIII.	XIII.	IIIth.	CII.	XII.	IIth.	C.	X.	C.	X.	C.	IIth.
3	2	1	2	3	4	5	6	7	8	9	0

Decimal Fractions are also thus named	
Tenths	1
Hundredths	2
Thousandths	3
Tenths of Thousandths	4
Hundredths of Thousandths	5
&c.	

The name of the first period is Units, of the second, Millions, of the third, billions, of the fourth, Trillions, &c

In the above order it may be observed that each degree contains the names of Units, Tens Hundreds, the first degree of a period contains the units of that period, & the second contains the thousands thereof; so that from hence it will be easy to read a number consisting of ever so many places by the following directions

Rule 1st Suppose a number parted into as many sets or degrees of three places each, beginning at the units places, as it will admit of & if one or two places remain, they will be the units and tens of the next degree.

2nd Beginning at the left hand and read in each degree, as many hundreds tens & units as the figures in those places of the degree express adding the name thousands if in the second degree of a period & adding the name of the period after reading the hundreds tens and units of its first degree.

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Examples.

Write in words 764000006480.

Ans^r Seventy six thousand four hundred Billion six hundred & forty eight

Write in figures nine hundred & seventy eight thousand seven hundred & fifty six Billion one thousand & one.

Ans^r 978756001001.

Write in words 980065003691.

Ans^r Nine hundred & eighty thousand sixty five Million three thousand six hundred & ninety four

Write in figures two Billion three hundred & ninety four thousand six hundred & fifty four Billion six hundred & ninety eight thousand five hundred & sixty four.

Ans^r 2394654698564.

Write in words 765427986532.

Ans^r Seven Billion six hundred & fifty four thousand two hundred & seventy nine Billion eight hundred & sixty five thousand four hundred & thirty two.

Write in figures Sixty five thousand four hundred & thirty two Billion nine hundred & sixty eight Thousand five hundred & forty two Billion six hundred & eighty four thousand three hundred twenty one

Ans^r 65432968426832l.

All fractional numbers consist of two parts which are usually written one above the other with a line drawn betⁿ them the number below the line called the Denominator shews how many equal parts the unit is divided into: the number above the line, called the numerator, shews by how many of these equal parts the value of that fraction is expressed.

Thus 9 pence is 9 parts in twelve of a Shilling & may be written thus $\frac{9}{12}$ when a Shilling is the unit.

These fractions the denominators of which are 10, or 100, or 1000, or 10000, or 100000, &c are called decimal fractions but fractions with any other denominators are called vulgar fractions.

As decimal fractions are parts of an unit divided into 10, 100, 1000, 10000, &c parts according to the places in the fractional number therefore they are read like whole numbers only calling them so many parts of 10, or of 100, or of 1000 &c.

Thus a decimal fraction of { One / Two / Three / Four / &c } places will be so many parts of { 10, Ten / 100, Hundred / 1000, Thousand / 10000, Ten Thousand / &c }

Cyphers on the right hand of integers increase their value, on the left hand of a decimal fraction diminish its value but on the

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left hand of integers, & on the right hand of fractions do not alter their value.

Thus { 8 is 8 Units / 00...8 Tens / 000...8 hundreds } And { ,8... is 8 parts in 10 / ,08...8 parts in 100 / ,008... 8 parts in 1000 } Of a unit so divided

When a fraction has no integer number prefixed, it is convenient to put 0 in the place of units. A Mixed Number is when a fraction is annexed to a whole number.

Examples.

Write in words 765,079008,0.

Ans^r Seventy six thousand five hundred and forty: And seven hundred ninety thousand & forty Millionth parts

Write in figures two Billion six hundred thousand Million & twenty one And three Million six hundred & twenty thousand two hundred & twenty five ten Millionth parts.

Ans^r 2600000000,02436202025.

Write in words 9876519,078967 3.

Ans^r Nine Million eight hundreds & seventy six thousand five hundred & forty nine And seven hundred and eighty nine thousand six hundred & seventy three hundred Millionth parts.

Like names in different numbers are such figures as stand equally distant from the place of units & have the same denomination annexed to them.

Thus all numbers of pounds sterling are like names & so are all numbers of shillings the like of any number of miles &c.

Besides the decimal notation explained as above there are other kinds in common use, such as the duodecimal in which every superior name contains 12 units of its next inferior name, the sexagenary or sexagesimal in which sixty of an inferior name makes one of its next superior.

The former is used by workmen in the measuring of surfaces & works in building & the latter is used in the division of a Circle & of Time.

The following characters or marks are frequently in arithmetical computations briefly to express the manner of operation

The mark + (more) belongs to addition & shews that the numbers it stands between are to be added together.

Thus 12 + 3 expresses the sum of 12 & 3 or that 3 is to be added to 12 & is thus read 12 more 3.

The mark − (less) is for subtraction & shews that the number following it or on the right hand side of it is to be taken from the number preceding it

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or on the left hand.

Thus 12-3, expresses the diff^{ce} betw^{n} 12 & 3, so that 3 is to be subtracted from 12, & is thus read, 12 less 3 or 12 lessened by 3.

The mark × into for multiplication shews that the numbers on each side of it are to be multiplied the one by the other.

Thus 12×3 denotes the product of 12 into 3 or that 12 is to be multiplied by 3.

Division is expressed by setting the divisor under the dividend with a line drawn betw^{n} them like a fraction.

Thus $\frac{12}{3}$ expresses the quotient of 12 by 3 or that 12 is to be divided by 3

The sign = (equal) shews the results of the operation by the numbers or quantities on the one side of it, is equal either to the numbers or quantities on the other side, or to the result of the operation by those numbers or quantities.

Thus 12+3=15 & 12-3=9 & 12×3=36 & $\frac{12}{3}$=4 severally shews the value of the preceding expressions.

Tables of English Money, Weights & Measures.

Money.

Farth^{gs}	Pence	Shill:	Pounds
960 =	240 =	20 =	1 £
48 =	12 =	1 s	
4 =	1 d		

Note 1, 2, 3 farthings, are thus written, $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$.

Troy Weight.

Grains	Dwts	Ounces	Pounds
5760 =	240 =	12 =	1 lb
480 =	20 =	1 Oz^{e}	
24 =	1 dwt		

Note, Gold & Silver are weighed by Troy Weight.

Avoird^{e} Weight

Drams	Ozs	Pounds	Cwt	Ton
573440 =	35840 =	2240 =	20 =	1
28672 =	1792 =	112 =	1 C	
256 =	16 =	1 lb		
16 =	1 oz			

Note, Provisions, Stores, &c are weigh^{d} by Avoird^{e} or great weight

Wine Measure

Solid inch	Gall^{s}	Hhd	Ton
58212 =	252 =	4 =	2 = 1
29106 =	126 =	2 =	1 P^{e}
14553 =	63 =	1 Hhd	
231 =	1 Gall^{n}		
42 =	1 Tierce		
84 =	1 Puncheon		

Dry Measure.

Pint	Gall:	Pecks	Bush:	Q^{r}
512 =	64 =	32 =	8 =	1
64 =	8 =	4 =	1 bush	
16 =	2 =	1 peck		
8 =	1 gall			

Note, 1 bush = 1 comb 10 qrs = 1 Wey 12 Weys = 1 Last of Corn 36 bush = 1 chald^{r} of Coals

Cloth Measure

4 Nails	=	1 Q^{r} of a Yard
4 Quarters	=	1 Yard
5 Q^{rs}	=	1 English ell
3 Q^{rs}	=	1 Flemish ell
6 Q^{rs}	=	1 French ell
1 Span	=	9 Inches
1 Hand	=	4 Inches.

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Long Measure

Barley corns Inches Feet Yards Poles Furl Mile

1 90 080 – 6336 – 5280 – 1760 – 320 – 8 – 1

23760 – 7920 – 660 – 220 – 40 – 1 furl

594 – 198 – 16½ – 5½ – 1 pole

108 – 36 – 3 – 1 yard

36 – 12 – 1 foot

3 – 1 inch

Also 3 miles make 1 League

And 20 leagues or 60 sea miles make a Degree

But a degree contains about 69½ miles of statute measure. A Fathom 6 feet 2 yards

Time

Seconds Minutes Hours days Year

31556937 – 525910 – 8766 – 365¼ – 1

86400 – 1440 – 24 – 1 day

3600 – 60 – 1 hour

60 – 1 min

Land Measure

Links Poles Roods Acres Mile

625 = 1 pole

2500 = 40 = 1 Rood

100000 = 100 = 10 = 1 Acre

6,400000 = 10240 = 2560 = 640 = 1 Mile

Beer Measure

Gal Inches Pints Galls Firk Kilder Barrell

282 = 8 = 1 gall^{n}

2538 = 72 = 9 = 1 Firk:

5076 = 144 = 18 = 2 = 1 Kilder

10152 = 288 = 36 = 4 = 2 = 1 Barrel

Pence Table Exemplified

Pence	s d	Pence	s d	s d		d			
20 .. is ..	1 8	70 .. is	5 10	10 0 is ½	1 p^{r} ℔ Sterling	0 is ½	1 p^{r} s	0 d .. ½	1 p^{r} £
30	2 6	80	6 8	0 8 ..... ⅓		1 ..... ⅓		60	
40	3 4	90	7 6	5 0 ..... ¼		3 ..... ¼		80	
60	5 0	100	8 4	1 0 0 ..... ⅕		2 ..... ⅙		120	
00	5 0	110	9 2	3 4 ..... ⅙		1½ ..... ⅛			
				2 0 ..... ⅛		3/4 ..... 1/10			
				2 0 ..... 1/10					

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Multiplication Table.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

Shilling Table.

s	£ s
20	1 0
30	1 10
40	2 0
50	2 10
60	3 0
70	3 10
80	4 0
90	4 10

s	£ s
100	5 0
110	5 10
120	6 0
130	6 10
140	7 0
150	7 10
160	8 0
170	8 10

s	£ s
180	9 0
190	9 10
200	10 0
210	10 10
220	11 0
230	11 10
240	12 0

Addition of whole numbers decimal fractions & mixt numbers

Addition is the method of collecting several numbers into one sum. Rule 1st Write the given numbers under each other, so that like names stand under ..... like names that is units under units, tens under tens & under those draw a line.

Rule 2nd. Add up the first or right hand upright row, under which write the overplus of even tens contained in that sum.

Rule 3rd. To the sum of the second row add as many units as there were tens in the sum of the 1st row, writing down as before the excess above even tens contained in the sum & proceed thus until all the rows are added together. But if the numbers to be added together are decimal fractions or mixed numbers place them so that the fractional points may stand under each other, then add them together as above directed for whole numbers & put the fractional point in the sum under the fractional points in the given numbers.

Add ..... together the following numbers 476 + 5723 + 87 + 54632

476
5723
87
54632

60918

• Add together the foll^g numbers 6794306797706791
+907657981760171234712345671679107907867?

6 7 9 4 3 0 6 7 9 7 7 0 6 7 9 1
9 0 7 6 5 7 9 8 1 7 6 8
7 1 2 3 4 7 1 2 3 4 5 6 7
0 7 9 7 0 7 9 0 7 8 6 7
0 8 8 1 4 7 0 3 4 9 1 0 9 9 3

• Add together the foll^g numbers 7647964771+
7697496787176270764327+765432791278017690+70127
07073

7 6 4 7 9 6 4 7 7
7 6 9 7 4 9 6 7 8 7
7 6 2 7 8 7 6 4 3 2 7
7 6 8 4 3 2 7 9 1 2 7 8
7 6 9 8 7 6 4 2 7 9 7 8 7
8 5 4 8 9 3 8 1 2 8 6 6

• Add together the foll^g numbers 7691708967+76327910+
7688972110+767987910707+76791234+76917170791076
079127?

7 6 9 1 7 8 9 6 7
7 6 4 3 2 7 9 1 8 6
7 6 8 9 7 2 1 1 0 7
7 6 7 9 0 7 9 4 0 7 8 7
7 6 7 9 1 2 3 4 7 6 9 1 7
7 6 7 9 1 0 7 6 0 7 9 1 2 7
8 5 3 2 3 4 9 7 1 3 5 9 9 1

Add the following numbers together 674,2767267 + 674,2737 + 769547,927654 + 67807,77787 + 7910,78,673 + 73294,67887?

674,2767267
674,2737
769547,927654
67807,77787
7910,78,673
73294,67887

Ans. 14629008,85671976

Add the following numbers together 7943791279,043795 + 7,07,79806,7980123 + 7910081,27021 + 7043791279,06791001 + 9171237123456789 + 1112796,47910078 + 679806746,79212378 + 764791237,9879 + 79063791279,1 + 7910796,79,706787967?

7943791279,043795
764798004,7980123
7910081,27021
7643791279,06791001
9471237123456789
1112796,47910078
679806746,79212378
764791237,9879
79063791279,1
7910796,79,706787967

Ans. 1681831054490602695 07

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Subtraction of whole numbers, decimal fractions & mixt numbers

Subtraction is the method of taking one number from another & shewing the gen.l or diff.ce or excess. The Subtrahend is the number to be subtracted or taken away. The Minuend is the number from which the Subtrahend is to be taken.

Rule 1st Under the minuend write the subtrahend so that units may stand under units & tens under tens & under them draw a line.

2nd Beginning at y.e right hand take each figure in the lower line from the figure standing over it & write the rem.r or what is left beneath the line under that figure. 3rd But if the figure below is greater than that above it increase the upper figure by 10 which is equal to an unit of the next greater name from this sum take the figure in the lower line & write the rem.r under it.

4th To the next name in the lower line carry the unit borrowed & thus proceed to the highest denomination or name.

N.B. The same directions must be attended to for arranging fractional numbers as are given in the rule for addition.

From 8 6 4 5 3 9 6 2 7 1 0 3 7
Take 9 0 4 6 3 2 9 0 7 3 0 9
Rem. 7 7 4 0 7 6 3 3 0 3 7 2 8

From 7 6 4 3 1 2 6 7 3 1 7 6
Take 9 6 4 3 9 0 5 7 8 6
Rem. 7 5 4 0 6 8 7 0 7 3 9 0

From 6 4 3 2 1 6 4 9 0 7 3 2 9
Take 8 7 6 5 3 4 9 0 3 2 0 7
Rem. 5 5 5 3 6 3 0 0 0 4 1 2 2

From 3 2 1 5 4 3 2 1 9 7 6 4 3
Take 8 6 4 9 6 2 3 0 6 4 2
Rem. 3 1 2 8 9 3 5 9 5 9 0 0 1

From 9 0 8 7 0 3 7 8 3 9 2 8 5 7
Take 8 9 0 4 3 7 2 0 5 9 6 4 8
Rem. 8 7 9 1 2 0 0 5 5 3 3 2 0 9

From 9 8 7 2 0 5 3 7 8 7 6 5 4 2
Take 6 5 7 4 3 9 2 8 7 6 5 9
Rem. 9 8 0 6 9 0 9 8 5 0 8 8 8 3

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From 7 9 0 3 8 5 7 2 , 9 8 5 4 0 2
Take 9 7 6 8 5 3 , 7 2 0 6 3 9
Rem^{r} 7 8 0 0 1 7 1 9 2 5 0 8 1 3

From 8 7 2 9 0 , 1 3 5 2 7 1
Take 6 9 5 4 2 , 8 7 9 3 5 2
Rem^{r} 1 7 7 1 7 , 2 5 5 9 1 9

From 6 3 2 9 5 4 , 2 6 2 8 7 5
Take 2 7 6 8 1 , 6 3 9 5 8 2
Rem^{r} 6 0 5 2 7 , 6 2 3 2 9 3

From 8 7 3 7 4 , 8 3 2 7 6 7 4
Take 5 3 6 7 2 , 9 8 5 4 6 7 6
Rem^{r} 3 3 7 0 4 , 8 4 1 7 2 9 8

From 1 , 6 3 4 , 9 2 4 , 0 4 9 3 5 7
Take 3 2 7 0 8 5 4 , 3 8 7 9 6 4
Rem^{r} 1 3 5 8 0 0 , 7 2 6 1 5 3

From 6 3 8 2 9 0 , 2 7 3 2 7 4
Take 3 6 9 8 2 7 , 6 4 9 8 7 6
Rem^{r} 2 6 8 4 6 3 , 6 2 3 3 9 8

Multiplication of whole numbers decimal fractions & mixt numbers

Multiplication is the method of finding what a given number will amount to when added any given number of times to itself. The number which is to be multiplied is called the Multiplicand. The number which denotes how often the multiplicand is to be taken is called the Multiplier. The number which arises from the operation is called the product. But Multiplicand & Multiplier are called factors

Rule 1st When the multiplier is a single figure beginning at the right hand multiply the first figure of the multiplicand by the multiplier & set the excess of the product above even tens under the first figure of the Multiplicand. Multiply the second figure of the Multiplicand by the multiplier add one to the product for every ten which there was in the first product set the excess of the same above even tens under the 2nd figure of

the Multiplicand & carry one to the product of the third figure for every ten which there was in the sum. Proceed in this manner to the last figure of the Multiplicand when the whole product of that figure is to be set down. 2^nd^ When the multiplier consists of several figures with the factors so that the right hand (place) of the multiplier may stand under the right hand place of the Multiplicand & draw a line under them.

Begin at the right hand side & multiply the whole Multiplicand by each figure of the Multiplier severally setting the respective products one under another & the first (figure) of each under that of the multiplier which produced it. Add all these products together & the sum will be the whole product. If decimal fractions are concerned point off as many figures from the right hand side of the product for fractions as the number of fractional places in both factors amount to. If there be not so many places in the product as this rule requires the Deficiency must be made up by writing cyphers on the left hand side of it.

Multiply 7906799869647 } Factors
By 9 }

Product 71081108826825

Multiply 98764790076291210 } Factors
By 10 }

Product 987647900762912000

Multiply 9376408976432167011 } Factors
By 11 }

Product 103141387407538458

Multiply 1764576321511 } Factors
By 3462547 }

123522034250511
70583052860
6822881607511
3529152643011
1058745792911
705830528611
5293728964511

Product 6099294182860511

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Multiply 964574624753 } Factors
By 42657413 }

209372387425
3858298499000
675202237326
482287312375
57874774650
19291492950
3858298499000

41146274535049392 5 Product

Multiply 9468457964 } Factors
By 3172540 }

85216121676
5681074770 4
37073831856
473522898 20
189369159 28
6677920571 8
378738318 56
28405373892

32879633516851160 Product

Multiplyd 74263245,70794 } Factors
By 406924077883 }

22278973712302
5941059666352
371316228539 70
51984271995558
2970529828317600
148526491415880
6683692113714600
445579474247640
29705298283176000

6 0/3 6

30219502627839095930902 Product

Multiplyd 0980799079 } Factors
By 908764 0235 }

353190391903950
21209123694237
1413961379615600
28279234592316000
424188473884740
49188655286530
5655846318163200
636281770827110

6247870830399608565

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Division of whole numbers decimal fractions & mixt numbers.

Division is the method of finding how often one number is contained in another. The number which is to divided is called the Dividend. The number which is to be divided by is called the Divisor. The quotient is the number arising from the division & shews how often the divisor is contained in the dividend.

Rule 1st Write the divisor on the left hand side of the dividend with a line drawn to separate them & draw another line on the right hand side of the dividend. If the divisor be not greater than an equal number of the left hand figures of the dividend, count as many figures from the left hand side of the dividend as there are in the divisor & put a point under the last of them but if the divisor be greater than an equal number of the left hand figures of the dividend point off one figure more than there are in the divisor. Seek how often the divisor is contained in the fi

gures thus pointed off & put the number of times it is contained in them beyond the line which was drawn on the right hand side of the dividend for the first figure of the quotient. Multiply the divisor by this figure set the product under that part of the dividend which was used in finding it subtract the lower line from the upper one & bring down the next figure of the dividend to the right hand side of the rem^{r} seek how often the divisor is contained in the number set the result in the quotient & proceed in this manner until all the figures of the dividend are used. When any figure of the dividend is taken down & annexed to the rem^{r} as directed above if the result be less than the divisor put a cypher in the quotient & bring down the next figure. When the divisor does not exceed 12 the division may be performed in one line by making the multiplication & subtraction in the mind & carrying the rem^{r} as so many tens to the next figure. There must be as many figures pointed off from the right hand of the quotient for fractions as the number of fractional places in the dividend exceed those

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in the divisor but if there are not so many fractional places in the dividend as in the divisor cyphers must be annexed to the right hand of the dividend until there are as many or more. When the division is finished count the number of fractional places which there are in the dividend & also the number of those which there are in the divisor take the latter from the former & point off as many figures as the rem amount to from the right hand side of the Quotient for fractions. If there arise not so many places as this rule requires supply the places wanting by writing cyphers on the left hand side of it & prefix the fractional point. When it is required to divide any number by another which has cyphers on its right hand side reject those cyphers remove the fractional point as many figures towards the left hand as there were cyphers & let the dividend thus pointed off be divided by the remaining figures of the divisor. If the divisor by unity with

ciphers annex the fractional point as many figures towards the left hand as there were ciphers & the division will be made.

After all the figures in the dividend are used & the fractional point placed properly in the quotient the division may be continued to any length which may be thought necessary by annexing ciphers to the remainder.

Examples

Divide 7647356 by 6

6) 7647356 (4
1274559

Divide 4708325987 by 10

10) 4708325997 (7
.4708325997

Divide 704791280679 by 11

11) 704791280679 (8
.64052843061

Divide 78679127800679 by 12

12) 78679127800679 (11
.6556593983389

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Divide 7918647127964 by 987654

987654)7918647127964(8017632

7901232

1741512

987654

7538587

6913578

6250099

5925924

3241756

2962962

2707914

1975308

812636 Rem^r.

8017632 Factors
987654

32070528
40088160
48105792
56123424
64141056
72158688
7918064631552 Product
812636 Rem^r
7918647127964 Proof

Divide 7684791278,6491 by 9876798?

9876798)7684791278,6491(778064

6913723923

771032048

691372923

79056125 6

79014392

641734449

592607934

49126515 1

39507195 6

9619319 5 Remr

778064 Factors

9876798)

7002576

6224512

7002576

5446448

4668384

5446448

6224512

7002576

7684781659,3296 Product

9619,3195 Remr

7684791278,6491 Proof

WILLIAMS'S 33 1-2 PATENT L.S

Divide 7960427 by 2436294.

2436294) 7960427000 (187

4236 04294

3724384060

3388834352

335549708 0

296523005 8

39026702 2 Rem.

4236 04294 } Factors
187 }

296523005 8

3388834352.

4236042 94..

7921400297 8 Product

39026702 2 Rem.

7960427000 Proof

Reduction

Reduction is the method of reducing a number from one name or denomination to another retaining the same value.

Case 1st

To reduce a number consisting of several names to their least name. Rule 1st Multiply the first or greatest name by the parts which an unit of that name contains of the next less name adding to the product the parts of the second name in the given number.

2nd Multiply this sum by the number of times that an unit of the next less name is contained in one of the second name adding to the product the parts of the third name in the given number.

And thus proceed, until the least name in the given name is arrived at.

Examples

£ s d

Reduce 7988 7 0½ 17 6¾ into Farthings.

20

159 7753 7 Shillings

12

1917 3045 0 Pence

4

7669 2180 3 Farthings

lbs Oz Dwt Gs

Reduce 7 8 26 7 " 8 " 16 " 16 into Grains

12

93 9212 Ozs

20

1878 4256 Dwts

24

7513 7038

3756 8513.

Ans^r 45082 2160 Grains

Tons Cwt 2^rs Qrs Lbs Ozs Dwts

Reduce 2 82 72 " 19 " 2 " 20 " 15 " 15 into Drams

20

5654 39 Cwt

4

22618 38 2^rs

28

180917 10

45230 78.

6333149 0 lbs

16

37998944

63331491

10133038 54 Ozs

10

607982312 9

101330385 5.

Ans 1621286167 9 Drams

Tuns Tcs Hhd Galls Pints

Reduce 2 7 0 2 0 1 1 61 2 into Pints.

2

55257 Pipes

2

110515 Hhds

63

3 3 1 5 4 6

6 0 3 0 9 6

6 9 6 2 5 0 6 Galls

8

Ans. 2 5 5 7 0 0 5 0 Pints

Reduce 7 2 6 2 7 Weys 2s Bush Tchs Gall Pints

7 2 6 2 7 2s 4 3 1 6 into Pints

10

72627 2s

8

581020 Bushells

4

2324083 Tchs

2

16481677 Galls

8

Ans. 37105342 Pints

WILLIAMS' 33 1-2 PATENT L.B

Miles Fur. Poles Yds. Feet In. B.C.

Reduce 790 7 6 „ 7 „ 38 „ 0 „ 2 „ 9 „ 2 into B. Corns

8

6390215 Fur.

40

255608638 Poles

55

1278043190

1278043190

14058475090 Yds

3

42175425269 Feet

12

506105103567 In.

12

Ans 6075183153107 3 B. Corns

Years Days Hours

Reduce 7984 „ 324 „ 17 „ 18 into minutes

365 25

39942 0

15976 8

39942 4

47930 0

23965 5

2917795300 Days

24

11671 1827

5835 5911

70027093 7 Hours

60

Ans 4201625623 8 Minutes.

Case 2nd

A number of an inferior name being given to find how many of each superior denomination is contained in it.

Rule 1st

Divide the given number by the number of times that one of its units is contained in an unit of its next superior name. 2nd Divide this quotient by the parts making one of the next name 3d Divide this quotient by the parts making one of the next name. And thus proceed in this manner and till the highest name is contained

4th Then the last quotient & the several remainders will be the parts of the different names contained in the given number.

Examples.

In 79807218 Farthings how many Pounds Shillings & Pence

	Farthings
4	79807218 = 2 fgs
12	19960804 = 1 d
20	1663067 = 3 s
	83195

Ans £83195, 3s. 8½

In 786791231760 Grains how many Pounds, Ounces, Penny-weights &c.

		Grains
24	12	786791231760 = 8 Grains
	2	65565936230
	20	3278290811 5 = 15 Dwts
	12	163914540 5 = 5 Ozs
		13659570 0 lbs

lbs	Ozs	Dwts	Grs
Ans 13659570	0	5	15. 8

In 79867129867 Drams how many Tons, Cwt, Qrs, lbs, Ozs &c

		Drams
16	4	79867129867 = 3 Drs
	4	19966782160 = 2-8-13-11
16	4	4991695540 Ozs
	4	124792385 4
28	4	31198097 0 = 0 lbs
	4	1114217 10 = 2-1-16-20
	4	27855 4 4 = 4 Cwt
20		1392 7 7 Tons

Tons	Cwt	Qrs	lbs	Ozs	Drs
Ans 1392 7 7	4	4	20	0	11

In 7986729846 Pints how many Tuns Pipes Hhds &c.

Pints

8	7 9 8 6 7 2 9 8 4 6	= 6 Pints
7	9 9 8 3 4 1 4 8 0	
9	1 4 2 6 2 0 1 6 4 0	= 8 × 7 = 56 Galls
2	1 5 8 4 6 6 8 4 8	
2	7 9 2 3 3 4 2 4	Pipes
	3 9 6 1 6 7 1 2	Tuns

Tuns P. H. Gall. Pints

Ans 3961671 2 0 0 5 6 6

In 782718 46 Pints how many Qrs, Bush &c

Pints

8	7 8 2 7 1 8 4 6	= 6 Pints
2	9 7 8 3 9 8 0	
4	4 8 9 1 9 9 0	= 2 Pecks
8	1 2 2 2 9 9 7	5 Bushels
	1 5 2 8 7 4	Qrs

Qrs B. P. Gall Pints

Ans 152874 3 2 0 6

In 86072198607 B. Corns how many Miles, Furl., Poles, Yards, Feet &c

B. Corns

3	8 6 0 7 2 1 9 8 6 0 7	
12	2 8 6 9 0 7 3 2 8 6 9	= 9 Inches
3	2 3 9 0 8 9 4 4 0 5	= 2 Feet
11	7 9 6 9 6 4 8 0 1	= 6 Yds = 1 Pole 3 Yd
5 0	7 2 4 5 1 3 4 5 0	
40	1 4 4 9 0 2 6 9	= 11 Poles
8	3 6 2 2 5 6	= 7 Furl.
	4 5 2 8 2	Miles

Miles F. P. Yds Feet I. B. Corns

Ans 452820 7 11 0 2 2 9 0

In 7967217867896 Minuks how many Yeai Days &c

60 | 7907217867896 | 27 Minuks
6 | 132869644649 | 3
24 | 4 | 22131160774161 = 176 = 613 = 9H
| 5532790186 5

365,25 | 5532790186 5,00 | 15147953 9 Years

365,25
1800,29
1826,25
5404,0
3652,5
1751,51
1461,00
290,518
255,675
34,8436
32,8725
1971,15
1826,25
144,90,0
109,57,5
35,32,50
32,87,25
2,46,25

Years 6 Days 11 +

Ans 15147953 9 24 7 11 9 27

Case 3rd

To reduce a vulgar fraction to its equivalent decimal fraction.

Rule

To the numerator annex one or more ciphers divide this by the Denominator & the quotient will be the fraction sought. If the division does not end when 0 figures are found in the quotient the work need not be carried any further.

Examples

Reduce $\frac{12}{78}$ to its equivalent decimal fraction.

78)12,00 00 0(,1538 Ans.

78
420
390
300
234
660
624
360
312
48

6 0 3
3
3
4

Reduce $\frac{226}{728}$ to its equivalent Decimal Fraction

728)226,0000 0(,310439 Ans.

2184
760
728
3200
2912
2000
1824
1760
1456
3040

0 7 2
1
3
7

WILLIAMS'S 33 1/2 PATENT

Reduce $\frac{681}{924}$ to its equivalent Decimal fraction

924)681,000000(,737013 Ans^{r}

6468

3720

3696

2400

1848

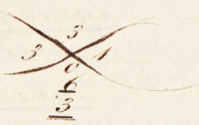
5520

4620

9000

8316

684



Reduce $\frac{8647}{12677}$ to its equivalent Decimal fraction

12677)86470000(,682101 Ans^{r}

76062

101080

101416

26640

25354

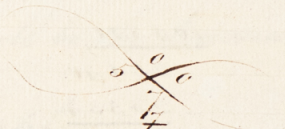
12860

12677

10300

12677

5623



Reduce $\frac{7067}{98067}$ to its equivalent Decimal Fraction

98067) 7,000000 (,0722200 Ans.

784536

.224400

196134

.282660

196134

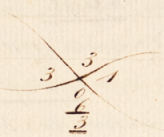
.865260

784536

.807240

784536

.22704



Reduce $\frac{0647}{12677}$ to its equivalent Decimal Fraction.

12677) 8,000000 (,0502101 Ans.

70062

108040

101416

26640

25354

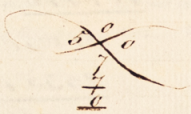
.12860

12677

183 00

12677

5623



WILLIAMS'S 33 1/2 PATENT

Case 4th

To reduce the customary parts of Coin, Weight, Measure &c to their equivalent decimal value.

Rule

Write the given parts under each other for dividends proceeding orderly from the least name to the greatest, on the left hand side of each name write the number of times which that name is contained in the next greater for a divisor

Divide the uppermost number or least name by the divisor which stands against it & write the quotient as a decimal fraction on the right hand of the number next below it.

Divide the next number together with the quotient of the former division, by the divisor which stands against it annexing the quotient as a decimal fraction to the number next below it & the last quotient will be the decimal fraction required.

Examples

Reduce 11^{h}. 7^{d}. 7/12 to the decimal fraction of a £

4	3
12	7,75
20	11,61 563
	0,58 2291 Ans^{r}

Reduce $^{Oz}9 \,„\, ^{Dwt}19 \,„\, ^{Gr}22$ to the decimal fraction of a lb. Troy Weight

24	6	22,
	4	3,666666
	20	19,916666
	12	9,995833
		0,832986 Answer

Reduce $^{Cwt}18 \,„\, ^{Qrs}3 \,„\, ^{lb}27 \,„\, ^{Oz}12 \,„\, ^{Dwt}10$ to the Decimal fraction of a Ton Avoirdupoise Weight.

16	4	10,
	4	2,5
16	4	12,625
	4	3,15625
28	4	27,709062
	7	6,947265
	4	3,992466
	20	13,998116
		0,949905 Answer

Reduce $^{Tyrs}A \,„\, ^{Hhds}1 \,„\, ^{Gall}12 \,„\, ^{Pints}5$ to the Decimal fraction of a Ton Wine Measure

8		3,
63	9	12,375
	7	1,375
	2	1,169428
	2	1,596214
		0,799107 Ans^{r}

WILLIAMS'S 83 1-2 PATENT L.D

Bush^s Pec^s Gall^s Pint^s
Reduce 3 „ 1. 0 „ 7 to the decimal fraction of a Dr Dry Measure

8	7,
2	0,875
4	1,75
8	3,5
	0,4 19921 - Ans^r

Miles Furl^s Poles Yds Feet Inc^s & Cents
Reduce 2 „ 7 „ 39 „ 1 „ 2 „ 11 „ 2 to the Decimal Fraction of a League Long Measure

3	2,
12	11,666666
3	2,972222
5½	4,990740
11	0,998158
40	39,907378
8	7,997684
3	2,999723
	0,999907 - Ans^r

Case 5th

To find the value of a decimal fraction in customary parts of the Integer

Rule

Multiply the given fraction by the number of times that the next inferior denomination is contained in the integer & point off as many figures from the right hand side of the product as there are in the given fraction.

Multiply the figures so pointed off by the number of times that the next inferior denomination is contained in this & point off as before.

Proceed in this manner until the lowest name is arrived at, or until the figures pointed off are all ciphers & the several denominations standing on the left hand side of the points form the answer.

Examples

Find the value of £.99579 1 of a £ Sterling.

20

£ 19.9582 0

12

D 10.7984 0

4

Qrs 2.9936 0

Ans 19s 10d½

WILLIAMS'S 83 1/2 PATENT

Find the value of ^{lb} 0,832906 of a lb Troy

12

Ozs 9,995832

20

Drchs 19,916640

24

3666560

1833280

Grs 21,999360

Ans 9 lb 19 Ozs 21 Drchs 0 Grs

Find the value of ^{Ton} 0,979869 of a Ton Avoirdupoise

20

Cwt 19,597380

4

Qrs 2,389520

20

3,111760

7,779040

lbs 10,906560

16

5459360

90560

Ozs 14,504960

10

3029760

50496

Drs 8,079360

Ans 19 Cwt 2 Qrs 16 lbs 14 Ozs 8 Drs

Find the value of $0,672186$ of a Quarter.

8

Bus, 3 7 7 4 8 8

4

Pck 1, 6 0 9 9 5 2

2

Gall 1, 0 1 9 9 0 4

8

Pints 0, 1 5 9 2 5 2

Bu Pck Gall Pints

Ans. 5 . 1 . 1 . 0

Find the value of $0,007460$ of a Mile

Miles

8

, 0 5 9 7 4 4

40

Poles 2, 3 8 9 7 6 0

5,5

1 9 4 8 8 0 0

1 9 4 8 8 0 0

Yds 2, 1 4 3 6 8 0 0

3

, 4 3 1 0 4 0 0

12

In 5, 1 7 2 4 8 0 0

3

5 1 7 4 4 0 0

Poles Yds Ft In Barleycorns

Ans. 2 . 2 . 0 . 5 . 0

WILLIAMS'S 83 1-2 PATENT

Find the value of Tuns 0,6721066 of a Tun Wine Measure

2

Pipes 1,3443372

2

Hhds 0,6887441

63

2066232

4132464

Galls 43,3908872

8

Pints 347269776

Ans 1 Pipe 0 Hhds 43 Galls 3 Pints

Find the value of Years 0,672606 of a Year.

365,25

3363030

1345212

3363030

4035636

2017818

Days 245,6934150

24

2677366000

1338683000

Hours 16,0641960000

60

Mts 3,8517600000

60

Seconds 231,1056000000

Days Hours Mts Seconds

Ans 245 16 3 51

Compound Addition

Compound Addition is the method of adding compound quantities as Money, Weights & Measures &c &c.

Rule

Write every figure under that of the same name or value. Ex. Add the least denomination or right hand column, reduce the sum to that denomination which the next column bears, write down the remainder & carry the number obtained by Reduction to the next column; add this & reduce the sum to the next greater denomination as before. Thus proceed with every column only the last or greatest wanting no reduction may be added as integers.

Examples

Add the following sums of Money together viz

£	s	d
79	05	7¼
59	12	5½
72	10	3¼
24	17	8¾
52	18	5½
93	13	5¾
82	15	3¼
67	14	2½
59	16	5¾
Answer 593	12	9½

WILLIAMS'S 83 1/2 PATENT

Add the following packs of Troy weight beg.^d viz

	lbs	Ozs	Dwts	Qrs
1	2	8	10	19	13
2	4	8	6	11	15	17
5	2	4	3	9	12	10
5	7	2	4	10	14	15
3	2	1	0	11	13	14
5	2	1	7	10	14	17
2	1	3	7	8	12	16
8	2	4	3	11	10	18
5	2	8	6	6	14	19
Ans.^r	41	8	39	0	16	19

Add the foll.^g packs of Avoirdupoise Weight beg.^r viz.

	Tons	Cwt	lbs	Ozs	Drs
3	6	2	5	13	14	16	12	15
	2	4	1	3	2	10	11	6	5
3	1	2	4	1	9	2	14	12
	2	5	3	4	13	14	12	15
	4	3	1	2	11	13	13	13
5	1	3	4	6	2	17	2	2
7	9	5	2	4	5	2	1	4
3	5	9	8	6	19	12	3	13
	2	5	9	8	11	11	10	12
Ans.^r	57	2	5	21	0	103	0	41

Add the foll^{g} parcels of Wine Measure log^{a} viz

	Tons	Pipes	Hhds	Gall^{s}	Pints
	654321	9	0	1	61 7
	34568	0	1	28	5
	76543	6	1	0	49 6
	63126		1	0	50 4
	2561		0	1	59 3
	56812		0	1	60 2
	2345		1	0	30 1
	65310		0	0	00 0
	648		1	0	40 6
	2167		0	1	20 5
Ans^{r}	753619	6	1	1	23 7

Add the foll^{g} parcels of Dry Measure log^{a} viz

	Bush^{s}	Pecks	Gal Pints
	67588	6	2 0 4
	856	7	2 1 5
	2345	2	1 0 2
	5689	3	3 0 1
	698	6	2 1 6
	6735	6	2 1 5
	89835	7	1 0 7
Ans^{r}	182921	1	0 0 6

WILLIAMS'S 3 1 2 PATENT

Add the following part of Long Measure together

Miles	Furl	Poles	Yds	Feet	In	Bleens
7907	7	39	5	0	11	2
2076	6	38	4	1	10	1
2712	5	37	3	2	9	2
7129	4	36	2	1	8	1
8076	3	35	1	2	7	2
2712	2	34	5	1	6	1
8071	1	33	4	2	5	2
2967	0	32	3	1	4	1
1806	7	31	2	2	3	2
9007	6	30	1	1	2	1
Ans^{r} 53349	1	31	3	0	10	

Add the following portions of Time toge^{r}

Years	Days	Hours	′	″
9076	361	23	59	59
9764	363	22	58	49
8067	237	21	48	47
8079	227	20	47	46
6721	98	19	46	44
9180	222	18	43	42
8067	221	17	44	41
7219	220	16	42	40
6210	219	15	21	22
7918	218	14	22	21
Ans^{r} 81107	205½	0	11	51

Compound Subtraction

To subtract compound quantities, as of money, weight or measures.

Rule.

With the less quantity below the greater so that each number may stand under that of the same name or value. Subtract as in simple subtraction, only where you have occasion borrow as many as are equal to ten the next place for which you must add 1 to the next lower figure.

Examples.

	£	s	d
From	9	8	7 6 4
Take	7	9	8 7 5
Rem^{r}	1	8	8 8 8

	lbs	Oz	Drch	Qrs
From	9	6 2 7	12	0
Take	8	9 1 8	6 7	8
Rem^{r}		6 7	8 4	4

	Tons	Cwt	Qr	lbs	Oz
From	9	8 7 6 2 7	17	1	20
Take	8	9 8 7 1 8	18	2	27
Rem^{r}		8 8 7 8	18	2	20

WILLIAMS'S 31 & 2 PATENT

	Tuns	Pipes	Hhds	Galls	Pints
From	8023	7	0	0	2
Take	7234	9	1	1	62
Rem^{s}	788	7	0	0	21

	Lbs	Bulkes	Galls	Pints
From	279106	2	1	0	2
Take	180797	7	3	1	7
Rem^{s}	98308	2	1	0	3

	Miles	Fur	Poles	Yds	Feet	Ins	Bc
From	798762	2	22	2	1	2	1
Take	679873	7	39	4	2	11	2
Rem^{r}	118888	2	22	2½	1	2	2

	Years	Days	Hours	′	″
From	6897	32	17	20	32
Take	3986	364	16	52	48
Rem	2910	325	10	27	44

	°	′	″	‴
From	6986	7912	32	21	19
Take	5878	9027	49	32	39
Rem	1107	8884	42	48	20

	Miles	Acres	Roods	Poles
From	79867	9	220	1	22
Take	67978	0	639	3	39
Rem	11889	8	220	1	23

Compound Multiplication

To multiply compound quantities as Pound, Shilling, pence &c.

Rule

First multiply the least denomination or right hand figures reduce the product to the next greater & write down the rem^r. Then multiply the next denomination to the product add the number obtained by reduction reduce the sum to the next greater denomination & write down the rem^r. Thus do with every denomination only the greatest may be multiplied as integers.

When the multiplier exceeds 12. If you can take two or more such numbers that their product will be the multiplier first multiply by one of them & that product by another &c the last product will be the same as if you had multiplied by the product of all those multipliers at once. If you cannot find numbers to produce the given multiplier take such as will come as near equal to it as you can & to the product add or subtract as many times the given multiplicand as the product of those numbers fall short of or exceed

WILLIAMS'S 83 1/2 PATENT

the given multiplier. To multiply by a fraction. Multiply by the numerator & divide the product by the denominator

To multiply by a fraction & a whole number &c. Multiply by each and their products by each the Sum will be the product required

Examples

L S D

Multiply 1463 „ 16 „ 8½ by 8

8

Ans 11699 „ 13 „ 8

lbs Ozs Drs Grs

Multiply 262 „ 8 „ 16 „ 20 by 9

9

Ans 2364 „ 7 „ 11 „ 12

Tons Cwt Qrs lbs Ozs Drs

Multiply 2842 „ 19 „ 2 „ 27 „ 11 „ 15 by 12

12

34113 „ 16 „ 3 „ 27 „ 3 „ 4

Tuns Pipes Hhds Gall Pints

Multiply 56524 „ 1 „ 1 „ 62 „ 6 by 16

2

112649 „ 1 „ 1 „ 62 „ 4

8

Ans 901199 „ 1 „ 1 „ 59 „ 4

Wegs Lbs Lbs Sacks Galls Pints

Multiply 28045 " 9 " 6 " 3 " 1 " 0 by 24

4

112183 " 9 " 3 " 3 " 1 " 0

6

Ans^{s} 673103 " 6 " 7 " 1 " 0 " 0

Miles Furl Poles Yds Feet Inl Barns

Multiply 79877 " 7 " 39 " 4 " 2 " 10 " 2 by 128

8

639023 " 7 " 39 " 0½ " 2 " 1 " 1

8

5112191 " 7 " 33 " 3½ " 1 " 10 " 2

2

Ans^{s} 10224383 " 7 " 27 " 2½ " 0 " 9 " 1

Years Days Hours ' "

Multiply 79872 " 222 " 22 " 59 " 48 by 300

10

798724 6 " 37½ " 1 " 58 " 00

10

7987246 1 " 14½ " 19 " 40 " 00

3

Ans^{s} 23961738 3 " 30½ " 11 " 0 " 00

WILLIAMS'S 83 1-2 PATENT L.B

Compound Division.

To divide compound quantities as Pounds Shillings and Pence.

Rule

Divide the greatest denomination as in simple division reduce the rem^{r} to the next less add the given number of the same name to the product divide the same by the same divisor & reduce the rem^{r} as before. Thus proceed until you have brought it to its lowest terms or until nothing remains. Each quotient is of the same name with its respective Dividend & so is its rem^{r} When the Divisor does not exceed the number 12 you may work by short division keeping the work in memory & writing down only the quotient figures. If you can take two or more such numbers that their product will be the divisor you may divide by one of them & that quotient by another &c. The last quotient will be the same as if you had divided by the product of all those divisors. viz by the original divisor.

To divide by a fraction. Multiply by the denominator & divide the product by the numerator.

To divide by a fraction & a whole number Multiply the whole number by the Denominator of the fraction to the product add the numerator for a new divisor multiply the dividend by the same denominator for a new dividend divide & the quotient will be what req^d.

Examples

£ S D

Divide 116990 „ 13 „ 8 by 8

£ S D

8/116990 „ 13 „ 8

Ans^r 14623 „ 16 „ 8½

lbs Oz^s Drs Gr^s

Divide 2364 „ 7 „ 11 „ 12 by 9

lbs Oz^s Drs Gr^s

9/2364 „ 7 „ 11 „ 12

Ans^r 262 „ 8 „ 16 „ 20

Tons Cwt Qrs lbs Oz^s Drs

Divide 3411135 „ 10 „ 3 „ 27 „ 3 „ 1 by 12

Tons Cwt Qrs lbs Oz^s Drs

12/3411135 „ 16 „ 3 „ 27 „ 3 „ 4

Ans^r 284279 „ 19 „ 2 „ 27 „ 11 „ 14

WILLIAMS'S 83 1-2 PATENT L.B

Tuns Pipes Hhds Gall Pints

Divide 901199 „ 1 „ 1 „ 59 „ 0 by 16

Tuns Pipes Hhds Gall Pints

4 / 901199 „ 1 „ 1 „ 59 „ 0

4 / 225299 „ 1 „ 1 „ 62 „ 0

Ans^{rs} 56324 „ 1 „ 1 „ 62 „ 6

Weys Lasts Bus Pecks Gall Pints

Divide 79280 „ 7 „ 4 „ 3 „ 1 „ 6 by 216

Weys Lasts Bus Pecks Gall Pints

9 / 79280 „ 7 „ 4 „ 3 „ 1 „ 6

3 / 8817 „ 6 „ 3 „ 0 „ 1 „ 6 } 2

8 / 2825 „ 2 „ 1 „ 0 „ 0 „ 4 } 4

Ans^{rs} 353 „ 1 „ 4 „ 0 „ 1 „ 0

Miles Fur Poles Yds Feet Ins 15 Grains

Divide 1022383 „ 7 „ 27 „ 2 „ 2 „ 3 „ 1 by 128

Miles Fur Poles Yds Feet Ins 15 Grains

8 / 1022383 „ 7 „ 27 „ 2 „ 2 „ 3 „ 1 } 2

8 / 127804 „ 7 „ 7 „ 38 „ 2¼ „ 0 „ 3 „ 1 } 2

2 / 15975 „ 7 „ 7 „ 39 „ 4,4 „ 0 „ 0 „ 1 } 1

7987 „ 7 „ 7 „ 39 „ 4,10 „ 0 „ 0 „ 0

Vulgar Fractions.

A Fraction represents some part or parts of any unit quantity, or number, as one half, two thirds, three quarters; &c. Fractions are generally expressed by two numbers written one above another with a line drawn betⁿ them

For Example { One half is written thus $\frac{1}{2}$ / Two thirds is written thus $\frac{2}{3}$ / Three fourths is written thus $\frac{3}{4}$

The number below the line is called the Denominator & shews how many parts, the unit or thing is divided into.

The number above the line is called the Numerator & shews how many of those parts are expressed by the fraction. Thus $\frac{3}{5}$ shews that if an unit or thing be divided into five parts, 3 of these parts are expressed by the fraction.

Fractions are distinguished into proper, improper, simple & Compound. A proper simple fraction is that which expresses some part of one entire thing or unity having but one Numerator & one Denominator & the Nume-

ralue less than the Denominator. Thus $\frac{2}{3}$ is a proper simple fraction. An improper fraction is that which has it Numerator greater than its Denominator. Thus $\frac{3}{2}$ is an improper fraction. A Compound Fraction denotes the fractional part of some quantity which is greater or less than unity & is expressed by two or more fractions with the word of betⁿ them. Thus $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ is a compound fraction. A mixed number denotes a quantity which consists of one or more entire units together with some part or parts of another the fractional part being written immediately after the Integral part. Thus $5\frac{3}{5}$ is a mixed number.

A multiple of any number or quantity is one which contains that number or quantity some number of times exactly. Thus 18 is a multiple of 6 & contains it three times exactly. The greatest Common measure of any two or more numbers is the greatest number that will divide them all without rem^r. A prime number is that which will not divide by any other number without rem^r.

A Composite Number is one which will divide by some other Number without rem^{r}

Reduction of Vulgar Fractions.

By Reduction Fractions are reduced to their lowest terms, whole & mixed numbers to improper fractions, improper fractions into whole & mixed numbers, fractions of one denomination to fractions of another retaining the same value.

By it also the customary parts of Coin, Weight, Measure &c are reduced into vulgar fractions. & the value of vulgar fractions found in customary parts of the integer.

Case 1st

To find the greatest common measure of any two numbers. Rule Divide the greater of the two given numbers by the less the divisor by the rem^{r} & so on until there be no rem^{r}. & the last divisor is the greatest that will divide both numbers without rem^{r}

WILLIAMS'S 83 1/2 PATENT L.B

Find the greatest common measure of 369 & 768.

369)768(2
738
30)369(12
30
69
60
9)30(3
27
3)9(3
9
0

The greatest common measure is 3.

Find the greatest common measure of 214 & 2642.

214)2642(12
214
502
428
74)214(2
148
66)74(1
66
8)66(8
64
2)8(4
8
0

The greatest common measure is 2.

Find the greatest common measure of 764 & 966?

764)966(1
764
202)764(3
606
158)202(1
158
.44)158(3
132
.26)44(1
26
18)26(1
18
.8)18(2
16
The greatest common measr. 2)8(4
8
0

Find the greatest common measure of 986 & 874?

986)874(8
7888
856)986(1
856
130)856(6
780
.76)130(1
76
.54)76(1
54
22)54(2
44
10)22(2
20
.2)10(5
10
.0
The greatest common measure is 2

WILLIAMS'S 83 1-2 PATENT L.S

Case 2nd

To abreviate or reduce fractions to its lowest terms

Rule

Find the greatest common measure to the Numerator & Denominator of the given fraction by Case 1st & divide both Numerator & Denominator by it, the two quotient will be the Numerator & Denominator of the fraction in its lowest terms.

Examples

Reduce $\frac{9476}{10128}$ to its lowest Denomination

9476)10128(1
9476
652)9476(14
652
2956
2608
348)652(1
348
304)348(1
304
44)304(6
264
40)44(1
40
4)40(10
40
0

The Greatest common measure 4

$\frac{9476}{10128} = \frac{2369}{2532}$ Ans.

Reduce $\frac{7968}{11496}$ to its lowest Denomination

$$
\begin{array}{r|l}
7968 & 11496 \\
\hline
 & 1 \\
7968 & \\
\hline
3528 & 7968 \\
 & 2 \\
7056 & \\
\hline
912 & 3528 \\
 & 3 \\
2736 & \\
\hline
792 & 912 \\
 & 1 \\
792 & \\
\hline
120 & 792 \\
 & 6 \\
720 & \\
\hline
72 & 120 \\
 & 1 \\
72 & \\
\hline
48 & 72 \\
 & 1 \\
48 & \\
\hline
24 & 48 \\
 & 2 \\
48 & \\
\hline
 & \cdot\cdot
\end{array}
$$

$12)\frac{7968}{11496} = 2)\frac{664}{958} = \frac{332}{479}$ Ans.

The greatest common measure

Reduce $\frac{9369}{78693}$ to its lowest Denomination

$$
\begin{array}{r|l}
9369 & 78693 \\
\hline
 & 8 \\
74952 & \\
\hline
3741 & 9369 \\
 & 2 \\
7482 & \\
\hline
1887 & 3741 \\
 & 1 \\
1887 & \\
\hline
1854 & 1887 \\
 & 1 \\
1854 & \\
\hline
33 & 1854 \\
 & 56 \\
165 & \\
\hline
204 & \\
 & \\
108 & \\
\hline
96 & 33 \\
 & 5 \\
30 & \\
\hline
3 & 6 \\
 & 2 \\
6 & \\
\hline
 & \cdot\cdot
\end{array}
$$

$3)\frac{9369}{78693} = \frac{3123}{26231}$ Ans.

The greatest common measure

Rule 2nd

Fractions may sometimes be abreviated more expeditiously (discovered) by which will divide both Numerator & Denominator without rem.r the following are useful in finding such numbers. 1st If both are even numbers they will divide by two. 2nd If both end with ciphers the fraction may be abreviated by taking an equal number of ciphers from each. 3rd If both end with a 5 or if one end with a 5 & the other with a cypher both will divide by a 5. 4th If the sum of all the digits of a number be divisible by 3 or without rem.r the number itself is divisible by 3 without rem.r 5th If the two last figures of any number be divisible by 4 without rem.r the whole of that number is divisible by 4 without rem.r 6th If the 3 last figures of any number be divisible by 8 without rem.r the whole of that number is divisible by 8 without rem.r 7th After rejecting all the devens the sum of the 1st, 3d, 5th &c figures of any number be equal to the sum of the 2d, 4th that number is divisible by 11.

An Odd number is not divisible by an even one.

Reduce $\frac{17612}{97366}$ to its lowest Denomination.

$$\left.\begin{array}{l} 2 \,/\, \frac{17612}{97366} = \frac{8806}{48683} \end{array}\right\} \text{Ans}^{r}$$

Reduce $\frac{21736}{623424}$ to its lowest Denomination.

$$\left.\begin{array}{l} 8 \,/\, \frac{21736}{623424} = \frac{2717}{77928} \end{array}\right\} \text{Ans}^{r}$$

Reduce $\frac{47964}{94792}$ to its lowest Denomination.

$$\left.\begin{array}{l} 4 \,/\, \frac{47964}{94792} = \frac{11991}{23698} \end{array}\right\} \text{Ans}^{r}$$

Reduce $\frac{978205}{1084695}$ to its lowest Denomination

$$\left.\begin{array}{l} 5 \,/\, \frac{978205}{1084695} = \frac{195641}{216939} \end{array}\right\} \text{Ans}^{r}$$

Reduce $\frac{47454}{98385}$ to its lowest Denomination

$$\left.\begin{array}{l} 3 \,/\, \frac{47454}{98385} = \frac{15818}{29795} \end{array}\right\} \text{Ans}^{r}$$

Reduce $\frac{1870451}{97810922}$ to its lowest Denomination

$$\left.\begin{array}{l} 11 \,/\, \frac{1870451}{97810922} = \frac{170041}{8891902} \end{array}\right\} \text{Ans}^{r}$$

WILLIAMS'S 33 1-2 PATENT L.S

Case 3rd

To reduce an improper fraction to a whole or mixed number

Rule

Divide the numerator by the Denominator & the quotient will be the integral part; put the rem^r, if there be any over the given Denominator; & annex it to the quotient for the fractional part

Examples.

Reduce $\frac{21}{9}$ to a whole or mixed number

$21 \div 9 = 2\frac{3}{9} = \frac{1}{3}$ Ans^r

Reduce $\frac{427}{25}$ to a whole or mixed number

$427 \div 25 = 17\frac{2}{25}$ Ans^r

Reduce $\frac{295}{34}$ to a whole or mixed number.

$295 \div 34 = 8\frac{23}{34}$ Ans^r

Reduce $\frac{4}{3}$ to a whole or mixed number

$4 \div 3 = 1\frac{1}{3}$ Ans.

Reduce $\frac{7}{4}$ to a whole or mixed number.

$7 \div 4 = 1\frac{3}{4}$ Ans.

Reduce $\frac{13}{7}$ to a whole or mixed number.

$13 \div 7 = 1\frac{6}{7}$ Ans.

Reduce $\frac{19}{4}$ to a whole or mixed number

$19 \div 4 = 4\frac{3}{4}$ Ans.

Reduce $\frac{14}{5}$ to a whole or mixed number

$14 \div 5 = 2\frac{4}{5}$ Ans.

Reduce $\frac{22}{13}$ to a whole or mixed number

$22 \div 13 = 1\frac{9}{13}$ Ans.

Reduce $\frac{63}{43}$ to a whole or mixed number.

$63 \div 43 = 1\frac{20}{43}$ Ans.

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Case 4th

To reduce a whole or mixed Number to an improper Fraction.

Rule

Multiply the integral part by the denominator of the fraction & add the numerator of the fraction (if there be one) to it, the sum will be the Numerator, to be put over the given Denominator. If a whole Number be given & no denominator, put Unity for its denominator.

Examples

Reduce $17\frac{9}{18}$ to an improper Fraction

$17 \times 18 + 9 = \frac{9\,315}{18} = \frac{35}{2}$ Ans.r

Reduce 11 to an improper Fraction

$11 = \frac{11}{1}$ Ans.r

Reduce $11\frac{9}{17}$ to an improper Fraction

$11 \times 17 + 9 = \frac{196}{17}$ Ans.r

Reduce $16\frac{11}{19}$ to an improper Fraction

$16 \times 19 + 11 = \frac{315}{19}$ Ans.r

Reduce $12\frac{1}{7}$ to an improper Fraction

$12 \times 7 + 1 = \frac{85}{7}$ Ans.r

Reduce $33\frac{44}{57}$ to an improper Fraction

$33 \times 57 + 44 = \frac{1925}{57}$ Ans.r

Reduce $64\frac{9}{17}$ to an improper Fraction

$64 \times 17 + 9 = \frac{1097}{17}$ Ans.r

Reduce $72\frac{1}{12}$ to an improper Fraction

$72 \times 12 + 1 = \frac{865}{12}$ $\frac{217}{3}$ Ans.r

Reduce $96\frac{9}{18}$ to an improper Fraction

$96 \times 18 + 9 = \frac{1737}{18} = \frac{193}{2}$ Ans.r

Reduce $44\frac{7}{17}$ to an improper Fraction

$44 \times 17 + 7 = \frac{755}{17}$ Ans.r

WILLIAMS' 83 1/2 PATENT

Case 5th

To reduce compound fractions to simple ones of equal value

Rule

Multiply all the Numerators together for the Numerator & all the Denominators together for the Denominator of the simple fraction req.d

Examples

Reduce $\frac{4}{15}$ of $\frac{2}{18}$ of $\frac{4}{18}$ of $\frac{3}{8}$ to a simple Fraction

$$\frac{\overset{1}{\cancel{4}} \times 2 \times \overset{1}{\cancel{4}} \times \overset{1}{\cancel{3}}}{\underset{5}{\cancel{15}} \times \underset{9}{\cancel{18}} \times \underset{9}{\cancel{18}} \times \underset{2}{\cancel{8}}} \quad \frac{1}{90} \Big\} \text{ Ans.r}$$

Reduce $\frac{2}{16}$ of $\frac{1}{7}$ of $\frac{3}{10}$ of $\frac{9}{16}$ to a simple Fraction

$$\frac{\overset{1}{\cancel{2}} \times \overset{1}{\cancel{1}} \times 3 \times 9}{\underset{8}{\cancel{16}} \times 7 \times 10 \times \underset{8}{\cancel{16}}} \quad \frac{243}{4480} \Big\} \text{ Ans.r}$$

Reduce $\frac{4}{10}$ of $\frac{9}{10}$ of $\frac{7}{11}$ to a simple Fraction

$$\frac{\overset{1}{\cancel{4}} \times 9 \times 7}{10 \times \underset{4}{\cancel{16}} \times 11} \quad \frac{63}{440} \Big\} \text{ Ans.r}$$

Reduce $\frac{7}{11}$ of $\frac{4}{9}$ of $\frac{3}{8}$ of $\frac{7}{10}$ to a simple Fraction

$$\frac{7 \times \overset{1}{4} \times \overset{1}{3} \times 7}{11 \times \underset{3}{9} \times \underset{2}{8} \times 10} = \frac{49}{1056}$$ Ans.r

Reduce $\frac{2}{11}$ of $\frac{1}{8}$ of $\frac{3}{7}$ to a simple Fraction

$$\frac{2 \times 1 \times 3}{11 \times \underset{4}{8} \times 7} = \frac{2\cancel{1}}{154}$$ Ans.r

Reduce $\frac{3}{11}$ of $\frac{9}{17}$ of $\frac{4}{6}$ of $\frac{3}{19}$ to a simple Fraction

$$\frac{\overset{1}{3} \times 9 \times \overset{2}{4} \times 3}{11 \times 17 \times \underset{1}{6} \times 19} = \frac{5\cancel{4}}{3553}$$ Ans.r

Reduce $\frac{4}{9}$ of $\frac{3}{8}$ of $\frac{1}{19}$ of $\frac{3}{10}$ to a simple Fraction

$$\frac{\overset{1}{4} \times \overset{1}{3} \times \overset{1}{1} \times \overset{1}{3}}{\underset{3}{9} \times \underset{2}{8} \times 19 \times \underset{5}{10}} = \frac{1}{95}$$ Ans.r

Reduce $\frac{11}{19}$ of $\frac{1}{17}$ of $\frac{6}{19}$ of $\frac{4}{7}$ to a simple Fraction

$$\frac{11 \times 1 \times 6 \times 4}{19 \times 17 \times 19 \times 7} = \frac{1056}{42959}$$ Ans.r

Reduce $\frac{2}{10}$ of $\frac{3}{6}$ of $\frac{7}{12}$ of $\frac{8}{9}$ to a simple Fraction

$$\frac{\overset{1}{2} \times \overset{1}{3} \times 7 \times \overset{2}{8}}{10 \times \underset{1}{6} \times \underset{1}{12} \times \underset{3}{9}} = \frac{7}{30}$$ Ans.r

WILLIAMS'S 83 1/2 PATENT 1/4

Case 6th.

To reduce fractions of different denominators to equivalent fractions of a common denominator

Rule

Multiply each numerator continually into all the denominators except it own for each new numerator & all the denominators together for the common Denominator.

Examples

Reduce $\frac{6}{9}$ & $\frac{19}{22}$ & $\frac{19}{32}$ & $\frac{3}{10}$ to equivalent fractions of a common denominator.

$$\frac{6}{9} = \frac{6\times22\times32\times10}{9\times22\times32\times10} = \frac{12)422\,40}{633\,60} = \frac{3520}{5280}$$

$$\frac{19}{22} = \frac{19\times9\times32\times10}{22\times9\times32\times10} = \frac{12)547\,20}{633\,60} = \frac{4560}{5280}$$

$$\frac{19}{32} = \frac{19\times9\times22\times10}{32\times9\times22\times10} = \frac{12)376\,20}{633\,60} = \frac{3135}{5280}$$

$$\frac{3}{10} = \frac{3\times9\times22\times32}{10\times9\times22\times32} = \frac{12)190\,08}{633\,60} = \frac{1584}{5280}$$

Ans^r

Reduce $\frac{9}{12}$ & $\frac{17}{21}$ & $\frac{13}{29}$ & $\frac{5}{11}$ to equivalent fractions of a common denominator

$$\frac{9}{12} = \frac{9\times21\times29\times11}{12\times21\times29\times11} = \frac{3}{4}\frac{60291}{80388} = \frac{20097}{26796}$$

$$\frac{17}{21} = \frac{17\times12\times29\times11}{21\times12\times29\times11} = \frac{3}{4}\frac{65076}{80388} = \frac{21692}{26796}$$

$$\frac{13}{29} = \frac{13\times12\times21\times11}{29\times12\times21\times11} = \frac{3}{4}\frac{36036}{80388} = \frac{12012}{26796}$$

$$\frac{5}{11} = \frac{5\times12\times21\times29}{11\times12\times21\times29} = \frac{3}{4}\frac{36540}{80388} = \frac{12180}{26796}$$

Ans^{s}

Reduce $\frac{19}{44}$ & $\frac{3}{8}$ & $\frac{5}{12}$ & $\frac{7}{11}$ to equivalent fractions of a common denominator

$$\frac{19}{44} = \frac{19\times8\times12\times11}{44\times8\times12\times11} = \frac{11}{4}\frac{20064}{46464} = \frac{8}{4}\frac{1824}{4224} = \frac{2}{528} = \frac{114}{264}$$

$$\frac{3}{8} = \frac{3\times44\times12\times11}{8\times44\times12\times11} = \frac{11}{4}\frac{17424}{46464} = \frac{8}{4}\frac{1584}{4224} = \frac{2}{528} = \frac{99}{264}$$

$$\frac{5}{12} = \frac{5\times44\times8\times11}{12\times44\times8\times11} = \frac{11}{4}\frac{19360}{46464} = \frac{8}{4}\frac{1760}{4224} = \frac{2}{528} = \frac{110}{264}$$

$$\frac{7}{11} = \frac{7\times44\times8\times12}{11\times44\times8\times12} = \frac{11}{4}\frac{21504}{46464} = \frac{8}{4}\frac{1936}{4224} = \frac{2}{528} = \frac{1224}{264}$$

Ans^{s}

WILLIAMS'S 83 1/2 PATENT 1/8

Rule 2nd

If the less denominator of the two given fractions divide the greater without any, multiply the numerator & denominator of that fraction which has the less denominator by the quotient arising from dividing the greater denominator by the less

Examples

Reduce $\frac{81}{102}$ & $\frac{94}{204}$ to equivalent fractions of a common Denominator

$204 \div 102 = 2$ & $\left.\begin{array}{l} \frac{2}{2} \times \frac{81}{102} = \frac{162}{204} = \frac{81}{102} \\ \frac{2}{204} = \frac{47}{102} \end{array}\right\}$ Ans.r

Reduce $\frac{75}{98}$ & $\frac{151}{392}$ to equivalent fractions of a common Denominator

$392 \div 98 = 4$ & $\left.\begin{array}{l} \frac{4}{4} \times \frac{75}{98} = \frac{300}{392} \\ \frac{151}{392} \end{array}\right\}$ Ans.r

Reduce $\frac{98}{195}$ & $\frac{184}{1170}$ to equivalent fractions of a common Denominator

$1170 \div 195 = 6$ & $\left.\begin{array}{l} \frac{6}{6} \times \frac{98}{195} = \frac{588}{1170} = \frac{294}{585} \\ \frac{2}{1170} = \frac{242}{585} \end{array}\right\}$ Ans.r

Reduce $\frac{94}{108}$ & $\frac{17}{54}$ to equivalent fractions of a common

Denominator $108 \div 54 = 2$ & $\frac{2}{2} \times \frac{17}{54} = \frac{34}{108} = \frac{17}{54}$ } Ans^{r}

$\frac{94}{108} = \frac{47}{54}$ }

Reduce $\frac{26}{789}$ & $\frac{84}{5523}$ to equivalent fractions of a com-

mon Denominator

$5523 \div 789 = 7$ & $\frac{7}{7} \times \frac{26}{789} = \frac{182}{5523}$ } Ans^{r}

$\frac{724}{5523}$ }

Reduce $\frac{75}{965}$ & $\frac{844}{7720}$ to equivalent fractions of a

common Denominator

$7720 \div 965 = 8$ & $\frac{8}{8} \times \frac{75}{965} = \frac{600}{7720} = \frac{150}{1930}$ } Ans^{r}

$\frac{844}{7720} = \frac{211}{1930}$ }

Reduce $\frac{321}{463}$ & $\frac{542}{5093}$ to equivalent fractions of a com-

mon Denominator

$5093 \div 463 = 11$ & $\frac{11}{11} \times \frac{321}{463} = \frac{3531}{5093}$ } Ans^{r}

$\frac{542}{5093}$ }

WILLIAMS'S 83 1/2 PATENT

Rule 3rd

If the denominators of the two given fractions have a common measure let them be divided by their greatest common measure then multiply the Numerator & Denominator of each fraction by the quotient arising from dividing the Denominator of y other by that greatest common measure.

Examples

Reduce $\frac{17}{20}$ & $\frac{15}{36}$ to equivalent fractions of a common Denominator

$36 \div 4 = 9$ Then $\frac{9}{9} \times \frac{17}{20} = \frac{153}{180} = \frac{51}{60}$

$20 \div 4 = 5$ Then $\frac{5}{5} \times \frac{15}{36} = \frac{75}{180} = \frac{25}{60}$

Ans^{rs}

20)36(1
20
16)20(1
16
4)16(4
16
..

Reduce $\frac{14}{98}$ & $\frac{17}{76}$ to equivalent fractions of a common Denominator

$76 \div 2 = 38$ Then $\frac{38}{38} \times \frac{14}{98} = \frac{532}{3724} = \frac{76}{532}$

$98 \div 2 = 49$ Then $\frac{49}{49} \times \frac{17}{76} = \frac{833}{3724} = \frac{119}{532}$

Ans^{rs}

76)98(1
76
22)76(3
66
10)22(2
20
2)10(5
10
..

Reduce $\frac{94}{108}$ & $\frac{17}{24}$ to equivalent fractions of a common Denominator

24 : 12 = 2 Then $\frac{2}{2} \times \frac{94}{108} = \frac{188}{216}$

108 : 12 = 9 Then $\frac{9}{9} \times \frac{17}{24} = \frac{153}{216}$ } Ans^{rs}.

24)108(4
96
12)24(2
24

Reduce $\frac{17}{42}$ & $\frac{32}{49}$ to equivalent fractions of a common Denominator

49 : 7 = 7 Then $\frac{7}{7} \times \frac{17}{42} = \frac{119}{294}$

42 : 7 = 6 Then $\frac{6}{6} \times \frac{32}{49} = \frac{192}{294}$ } Ans^{rs}.

42)49(1
42
7)42(6
42

Reduce $\frac{43}{94}$ & $\frac{17}{38}$ to equivalent fractions of a common Denominator

38 : 2 = 19 Then $\frac{19}{19} \times \frac{43}{94} = \frac{817}{1786}$

94 : 2 = 47 Then $\frac{47}{47} \times \frac{17}{38} = \frac{799}{1786}$ } Ans^{rs}.

38)94(2
76
18)38(2
36
2)18(9
18

Reduce $\frac{19}{36}$ & $\frac{5}{48}$ to equivalent fractions of a common Denominator

36 : 12 = 3 Then $\frac{3}{3} \times \frac{5}{48} = \frac{15}{144}$

48 : 12 = 4 Then $\frac{4}{4} \times \frac{19}{36} = \frac{76}{144}$ } Ans^{rs}.

36)48(1
36
12)36(3
36

WILLIAMS'S 83 LEAD PATENT L.B

Case 7th

To find the value of any given Vulgar Fraction in customary parts of the Integer.

Rule

Multiply the numerator of the given Fraction by the number of times the next inferior denomination is contained in the integer; divide this product by the denominator, & the quotient will be the parts of that denomination.

Multiply the rem.r of there be any by the number of times that the next inferior denomination is contained in this: Divide the product again by the Denominator & proceed thus until you obtain parts of the lowest denomination or until there be no rem.r

Example 1st

What is the value of $\frac{15}{97}$ of a £ Sterling

15
20
97)300(3 S
291
9
12
97)108(1 Penny
97
11
4
97)44(0 Fg.s of a Farthing
97

Ans.r 3 S 1 D $\frac{44}{97}$ of a Fg

Example 2nd

What is ye value of $\frac{15}{95}$ of a Ton Averdupois Weight

18

20

95)360(3 Cwt

285

75

4

95)300(3 Qrs

285

15

28

120

30

95)420(4 lbs

380

40

16

95)640(6 Oz

570

70

16

95)1120(11 Drs

95

170

95

$\frac{75}{95} = \frac{15}{19}$ of a Dram

Ans 2 Cwt 3 Qrs 4 lbs 6 Oz 11 Drs $\frac{15}{19}$ of a Dr

Example 3rd

What is ye value of $\frac{95}{102}$ of a lb Troy

95

12

102)1140(11 Oz

102

120

102

18

20

102)360(3 Dwts

306

54

24

216

108

102)1296(12 Grs

102

276

204

$\frac{72}{102} = \frac{36}{51}$ of a Gr

Oz. dwts Grs

Ans 11 . 3 . 12 $\frac{36}{51}$ of a Grain

WILLIAMS'S 83 LAW PATENT

Example 4th

What is the value of $\frac{42}{96}$ of a Tun Wine Measure

45
2
96)90(0 Pipes
2
96)180(1 Hhd
96
84
63
252
504
96)5292(55 Galls
480
492
480
12
8
96)96(1 Pt
96
00

Hhd Galls Pt
Ans. 1 „ 55 „ 1

Example 5th

What is the value of $\frac{29}{42}$ of a 2^r

29
8
42)232(5 Bushels
210
22
4
42)88(2 Pecks
84
4
2
42)8(0 Gall
8
42)64(1 Pt
42
2)22
42 = $\frac{11}{21}$ of a Pt

lb Oz Dr
Ans. 3 „ 2 „ 0 „ 1 $\frac{11}{21}$ of a Pint

Example 6th

What the value of $\frac{35}{108}$ of a Mile

35
8
108)280(2 Furlongs
216
64
40
108)2560(23 Poles
216
400
324
76
55
380
380
108)4180(3 Yds
324
94
3
108)282(2 Feet
216
66
12
108)792(7 Inches
756
36
3
108)108(1 B Corn
108
...

Ans 2 F 23 P 3 Yds 2 F 7 I 1 B Corn

Example 7th

What the value of $\frac{29}{67}$ of a Circle

29
360
1740
87
67)10440(155^{7}
67
374
335
390
335
55
60
67)3300(49'
268
620
603
17
60
67)1020(15"
67
350
335
15
60
67)900(13'''
67
230
201
29
60
67)1740(25^{IV}
134
400
335
65 of a Fourth
67

Ans 155 5 45 45 15 15 25 $\frac{65}{67}$ of a fourth

WILLIAMS'S 83 1/2 PATENT L.B

Case 8th

To reduce fractions of one denomination to equivalent ones of another denomination.

Rule

If the fraction of an inferior denomination is to be reduced to a fraction of a superior one. Multiply the denominator of the given fraction by the number of times that the less denomination is contained in the greater. but if the fraction of a superior denomination is to be reduced to the fraction of a less multiply the Numerator by that number

Examples

Reduce $\frac{11}{99}$ *of a £ to the fraction of a £*

$$\frac{14}{99} = \frac{14 \times 20}{99} = \frac{£280}{99} \Big\} \text{Ans}^{r}$$

Reduce $\frac{74}{87}$ *of an Oz to the fraction of a lb Troy*

$$\frac{74}{87} = \frac{74}{87 \div 12} = \frac{2/74}{1044} = \frac{lb\ 37}{522} \Big\} \text{Ans}^{r}$$

Reduce $\frac{17}{94}$ of a Ton to the fraction of a Cwt

$$\left.\begin{array}{l} \frac{17}{94}\ \text{Ton} = \frac{17 \times 20}{94} = \frac{340}{94} = \frac{170}{47}\ \text{Cwt} \end{array}\right\} \text{Ans}^{r}$$

Reduce $\frac{17}{56}$ of a Gallon to the fraction of a Hhd

$$\left.\begin{array}{l} \frac{17}{56}\ \text{Galls} = \frac{17}{63 \times 63} = \frac{17}{3528}\ \text{Hhd} \end{array}\right\} \text{Ans}^{r}$$

Reduce $\frac{23}{94}$ of a $\mathfrak{D}^{r}$ to the fraction of a Bushel

$$\left.\begin{array}{l} \frac{23}{94}\ \mathfrak{D}^{r} = \frac{23 \times 8}{94} = \frac{184}{94} = \frac{92}{47}\ \text{Bushel} \end{array}\right\} \text{Ans}^{r}$$

Reduce $\frac{17}{28}$ of a Furl^{g} to the fraction of a Mile

$$\left.\begin{array}{l} \frac{17}{28}\ \text{Furl}^{g} = \frac{17}{28 \times 8} = \frac{17}{224} \end{array}\right\} \text{Ans}^{r}$$

Reduce $\frac{20}{47}$ of a Year to y^{e} fraction of a Month

$$\left.\begin{array}{l} \frac{20}{47}\ \text{Year} = \frac{20 \times 13}{47} = \frac{260}{47} \end{array}\right\} \text{Ans}^{r}$$

Reduce $\frac{49}{72}$ of a Degree to the fraction of a Circle

$$\left.\begin{array}{l} \frac{49}{72} = \frac{49}{72 \times 360} = \frac{49}{25920} \end{array}\right\} \text{Ans}^{r}$$

WILLIAMS'S 83 L & 2 PATENT L^{d}

Case 9th

To reduce the customary parts of Coin, Weights Measures &c to a Vulgar Fraction

Rule

Reduce the given parts into their lowest terms for a numerator & put the number of times which that name is contained in the Integer for a Denominator.

Examples

Reduce 9s 11d 3f to the fraction of a £ Sterling

$$\frac{9\times12+11\times4+3}{20\times12\times4} = \frac{479}{960}\Big\} \text{Ans}^{r}$$

Reduce 5 Ozs 10 dwts 19 Grs to the fraction of a lb Troy.

$$\frac{5\times20+16\times24+19}{12\times20\times24} = \frac{2803}{5760}\Big\} \text{Ans}^{r}$$

Reduce 17 Cwt 3 Qrs 14 lbs 15 Ozs 12 Drs to the fraction of a Ton

$$\frac{17\times4+3\times28+14\times16+15\times16+12}{20\times4\times28\times16\times16} = \frac{4+12732}{573440} = \frac{128183}{143360}\Big\} \text{Ans}^{r}$$

Reduce 1 Pipe 0 Hhd 23 Gall 5 Pints to the fraction of a Tun Wine

$$\frac{1\times2\times0+63\times23+8\times5}{2\times2\times63\times8} = \frac{2+1197}{2016} = \frac{+133}{224} = \frac{19}{32}\Big\} \text{Ans}^{r}$$

Reduce B P Qrs P ls
7 ∴ 3 ∴ 1 ∴ 4 to the fraction of a Quarter

$$\frac{7\times4+3\times2+1\times8+4}{8\times4\times2\times8} \quad \frac{4508}{512} \quad \frac{127}{128} \Big\} \text{Ans}^{r}$$

M. F. P. Yds Ft Ins Barleycorns
Reduce 2 ∴ 7 ∴ 20 ∴ 4 ∴ 2 ∴ 9 ∴ 2 to the fraction of a League

$$\frac{2\times8+7\times40+7\times40+20\times3+4\times3+2\times12+9\times3+2}{3\times8\times40\times55\times3\times12\times3} \quad \frac{5508993}{570240} \Big\} \text{Ans}^{r}$$

Months Weeks D H ′ ″
Reduce 12 ∴ 2 ∴ 4 ∴ 22 ∴ 17 ∴ 19 to the fraction of a Year

$$\frac{12\times4+2\times7+4\times24+22\times60+17\times60+19}{13\times4\times7\times24\times60\times60} \quad \frac{30665039}{31449600} \Big\} \text{Ans}^{t}$$

° ′ ″ ‴ Ⅳ
Reduce 342 ∴ 19 ∴ 47 ∴ 53 ∴ 47 to the fraction of a Circle

$$\frac{342\times60+19\times60+47\times60+53\times60+47}{360\times60\times60\times60\times60} \quad \frac{4436596427}{4666666000} \Big\} \text{Ans}^{r}$$

A R P
Reduce 478 ∴ 2 ∴ 37 to the fraction of a Mile Land Measure

$$\frac{478\times4+2\times40+37}{640\times4\times40} \quad \frac{76597}{102400} \Big\} \text{Ans}^{r}$$

WILLIAMS'S 83 1/2 PATENT L.S

Addition Rule

Reduce compound fractions to simple ones whole & mixed numbers to improper fractions & all fractions to the same Integer & Denominator then add the Numerators together & put their sum over the common Denominator

Examples

Add $\frac{4}{7}$ of $\frac{3}{12}$ of $\frac{9}{17}$ of $4\frac{3}{13}$ of a £ & $\frac{7}{15}$ of $\frac{3}{10}$ of $\frac{7}{19}$ of $4\frac{3}{10}$ of a £ together

$$\frac{4}{7}\text{ of }\frac{3}{12}\text{ of }\frac{9}{17}\text{ of }\frac{55}{13} = \frac{\overset{1}{4}\times\overset{1}{3}\times 9\times 55}{7\times\underset{1}{12}\times 17\times 13} = \frac{\overset{5}{1}\overset{1}{9}\overset{1}{5}}{\underset{1}{1}547}$$

$$\frac{7}{15}\text{ of }\frac{3}{10}\text{ of }\frac{7}{19}\text{ of }\frac{43}{10} = \frac{7\times 3\times 7\times 43}{\underset{2}{15}\times 10\times 19\times\underset{2}{10}} = \frac{\overset{1}{2107}}{\underset{1}{9500}} = \frac{2107}{190000}$$

$$\frac{495}{1547} = \frac{495\times 190000}{1547\times 190000} = \frac{94050000}{293930000}$$

$$\frac{2107}{190000} = \frac{2107\times 1547}{190000\times 1547} = \frac{3259529}{293930000}$$

$$\frac{94050000+3259529}{293930000}$$

$$\frac{97309529}{293930000}\text{ Ans}^{r}$$

Add $\frac{2}{17}$ of $\frac{4}{9}$ of $\frac{7}{16}$ of $7\frac{2}{11}$ of a $\overset{lb}{8}$ $\frac{2}{11}$ of $\frac{17}{24}$ of $\frac{15}{21}$ of $9\frac{3}{5}$ of a Oz Troy $\overset{lbs}{wt}$

$$\frac{2}{17}\text{ of }\frac{4}{9}\text{ of }\frac{7}{16}\text{ of }\frac{306}{11} = \frac{\overset{1}{2}\times\overset{1}{4}\times 7\times\overset{43}{306}}{17\times\underset{1}{9}\times\underset{2}{16}\times 11} = \frac{301}{374}\ lb$$

$$\frac{2}{11}\text{ of }\frac{17}{24}\text{ of }\frac{15}{21}\text{ of }\frac{48}{5} = \frac{9\times 17\times\overset{1}{15}\times\overset{2}{48}}{11\times\underset{8}{24}\times\underset{7}{21}\times\underset{1}{5}} = \frac{306}{77\times 12} = \frac{\overset{11}{306}}{924} = \frac{51}{154}\ lb$$

$$\frac{301}{374} = \frac{301\times 154}{374\times 154} = \frac{46354}{57596}$$

$$\frac{51}{154} = \frac{51\times 374}{154\times 374} = \frac{19074}{57596}$$

$$\frac{46354+19074}{57596} = \frac{65428}{57596} = \frac{\overset{11}{16357}}{14399}$$

$$\frac{1407}{1309}\ \text{Ans}^{r}$$

Add $\frac{4}{5}$ of $\frac{3}{11}$ of $7\frac{5}{8}$ of $\frac{5}{11}$ of a Ton $\overset{T}{}$ $\frac{4}{7}$ of $\frac{2}{11}$ of $\frac{7}{12}$ of $9\frac{4}{13}$ of a Cwt $\overset{cwt}{}$

$$\frac{4}{9}\text{ of }\frac{3}{11}\text{ of }\frac{61}{8}\text{ of }\frac{3}{11} = \frac{\overset{1}{4}\times\overset{1}{3}\times 61\times\overset{1}{3}}{\underset{3}{9}\times 11\times\underset{2}{8}\times 11} = \frac{61}{242}\ \text{Ton}$$

$$\frac{4}{7}\text{ of }\frac{2}{11}\text{ of }\frac{4}{12}\text{ of }\frac{121}{13} = \frac{\overset{1}{4}\times\overset{2}{8}\times\overset{1}{7}\times 121}{\underset{1}{7}\times 11\times\underset{3}{12}\times 13} = \frac{363}{143\times 20} = \frac{363}{2860}\ \text{Ton}^{cwt}$$

$$\frac{61}{242} = \frac{61\times 2860}{242\times 2860} = \frac{174460}{692120}$$

$$\frac{363}{2860} = \frac{363\times 242}{2860\times 242} = \frac{87846}{692120}$$

$$\frac{174460+87846}{692120} = \frac{\overset{2}{262306}}{692120} = \frac{\overset{11}{131153}}{346060} = \frac{11023}{31460}\ \text{Ans}^{r}$$

WILLIAMS'S 33 1-2 PATENT L.S

Subtraction

Rule

. Make the reductions as shall are directed to be made in addition then take the diffs of the Numerators & place it over the common Denominator

Examples

Subtract $\frac{4}{9}$ of $\frac{2}{8}$ of $\frac{4}{7}$ of $9\frac{3}{15}$ of a Sfoon from $\frac{4}{19}$ of $17\frac{3}{8}$ of $\frac{19}{21}$ of $4\frac{3}{8}$ of a £

$$\frac{4}{9}\text{of}\frac{2}{8}\text{of}\frac{4}{7}\text{of}\frac{138}{15} \qquad \frac{\cancel{2}\times\cancel{2}\times\cancel{4}\times\cancel{150}^{16}}{\cancel{9}\times\cancel{8}\times\cancel{7}\times\cancel{15}} = \frac{184}{315\times 20} = \frac{184}{6300}$$

$$\frac{4}{19}\text{of}\frac{3}{17}\text{of}\frac{19}{21}\text{of}\frac{35}{8} \qquad \frac{\cancel{4}\times\cancel{3}\times\cancel{19}\times\cancel{35}}{\cancel{19}\times 17\times\cancel{21}\times\cancel{8}} = \frac{5}{34}$$

$$\frac{184}{6300} \quad \frac{184\times 34}{6300\times 34} \quad \frac{6256}{214200}$$

$$\frac{5}{34} \quad \frac{5\times 6300}{34\times 6300} \quad \frac{31500}{214200}$$

$$\frac{31500-6256}{214200} = \frac{25244}{214200}$$

$$\frac{6311}{53550} \text{ Ans}^{r}$$

Subtract $\frac{3}{11}$ of $\frac{4}{9}$ of $\frac{11}{16}$ of $2\frac{3}{9}$ of an Oz from $\frac{19}{22}$ of $\frac{3}{19}$ of $4\frac{7}{11}$ of $\frac{3}{7}$ of a lb Troy

$$\frac{3}{11}\text{ of }\frac{4}{9}\text{ of }\frac{11}{16}\text{ of }\frac{21}{9} = \frac{3\times\cancel{4}^{1}\times\cancel{11}^{1}\times\cancel{21}^{7}}{\cancel{11}_{1}\times\cancel{9}^{3}\times\cancel{16}_{4}\times 9} = \frac{7}{36\times 12}\,\text{oz} = \frac{7}{432}\,\text{lb}$$

$$\frac{19}{22}\text{ of }\frac{3}{19}\text{ of }\frac{31}{11}\text{ of }\frac{3}{7} = \frac{\cancel{19}^{1}\times 3\times 31\times 3}{22\times\cancel{19}_{1}\times 11\times 7} = \frac{459}{1694}\,\text{lb}$$

$$\left.\begin{aligned}\frac{7}{432} &= \frac{7\times 1694}{432\times 1694} = \frac{11858}{731808}\\ \frac{459}{1694} &= \frac{459\times 432}{1694\times 432} = \frac{198288}{731808}\end{aligned}\right\}\ \frac{198288-11858}{731808} = \frac{\cancel{2}^{1}86430}{731808} = \left.\begin{aligned}&\frac{93215}{365904}\\ &\text{Ans}^{r}\end{aligned}\right\}$$

Subtract $\frac{1}{31}$ of $\frac{3}{11}$ of $\frac{7}{9}$ of $4\frac{7}{14}$ of a Cwt from $\frac{2}{10}$ of $8\frac{2}{11}$ of $\frac{17}{19}$ of $\frac{4}{13}$ of a Ton

$$\frac{1}{31}\text{ of }\frac{3}{11}\text{ of }\frac{7}{9}\text{ of }\frac{63}{14} = \frac{1\times\cancel{3}^{1}\times\cancel{7}^{1}\times\cancel{63}^{7}}{\cancel{31}_{1}\times 11\times\cancel{9}_{1}\times\cancel{14}_{2}} = \frac{7}{8\times 20}\,\text{Cwt} = \frac{7}{160}\,\text{Ton}$$

$$\frac{9}{16}\text{ of }\frac{90}{110}\text{ of }\frac{17}{19}\text{ of }\frac{4}{13} = \frac{9\times\cancel{90}^{45}\times 17\times\cancel{4}^{1}}{\cancel{16}_{2}\times 11\times 19\times 13} = \frac{6885}{5434}\,\text{Ton}$$

$$\left.\begin{aligned}\frac{7}{160} &= \frac{7\times 5434}{160\times 5434} = \frac{38038}{869440}\\ \frac{6885}{5434} &= \frac{6885\times 160}{5434\times 160} = \frac{1101600}{869440}\end{aligned}\right\}\ \frac{1101600-38038}{869440} = \frac{\cancel{2}^{1}063562}{869440} = \left.\begin{aligned}&\frac{531781}{434720}\\ &\text{Ans}^{r}\end{aligned}\right\}$$

WILLIAMS & SONS, PATENT

Multiplication Rule

Reduce whole & mixed numbers to improper fractions & compound fractions to simple ones then multiply the numerators of the given fractions together for a numerator & the Denominators together for the Denominator of the product reduced.

Examples.

Multiply $\frac{1}{7}$ of $\frac{9}{11}$ of $2\frac{2}{3}$ of $\frac{9}{11}$ by $\frac{2}{3}$ of $\frac{9}{18}$ of $\frac{4}{7}$ of $3\frac{9}{11}$?

$$\frac{1}{7}\text{ of }\frac{9}{11}\text{ of }\frac{8}{3}\text{ of }\frac{9}{11} = \frac{1\times 9\times 8\times \overset{3}{9}}{7\times 11\times \underset{1}{3}\times 11} = \frac{864}{1309}$$

$$\frac{2}{3}\text{ of }\frac{9}{18}\text{ of }\frac{4}{7}\text{ of }\frac{51}{14} = \frac{\overset{1}{2}\times \overset{1}{9}\times \overset{2}{4}\times \overset{17}{51}}{\underset{1}{3}\times \underset{1}{18}\times 7\times \underset{7}{14}} = \frac{34}{49}$$

$$\frac{864}{1309} \times \frac{34}{49} = \frac{29376}{64141}$$ Ans^{r}

Multiply $\frac{1}{9}$ of $\frac{3}{5}$ of $\frac{2}{7}$ by $\frac{4}{11}$ of $\frac{5}{25}$ of $\frac{3}{6}$

$\frac{1}{9}$ of $\frac{3}{5}$ of $\frac{2}{7} = \frac{4 \times 3 \times 2}{9 \times 5 \times 7} = \frac{8}{105}$

$\frac{4}{11}$ of $\frac{5}{25}$ of $\frac{3}{6} = \frac{4 \times 5 \times 3}{11 \times 25 \times 6} = \frac{2}{55}$

$\frac{8}{105} \times \frac{2}{55} = \frac{8 \times 2}{105 \times 55} = \frac{16}{5775}$ Ans^{r}

Multiply $\frac{2}{13}$ of $\frac{2}{9}$ of $\frac{3}{7}$ by $\frac{3}{6}$ of $\frac{5}{9}$ of $\frac{3}{7}$

$\frac{2}{13}$ of $\frac{2}{9}$ of $\frac{3}{7} = \frac{3 \times 2 \times 3}{13 \times 9 \times 7} = \frac{6}{91}$

$\frac{3}{6}$ of $\frac{5}{9}$ of $\frac{3}{7} = \frac{3 \times 5 \times 3}{6 \times 9 \times 7} = \frac{5}{42}$

$\frac{6}{91} \times \frac{5}{42} = \frac{6 \times 5}{91 \times 42} = \frac{5}{637}$ Ans^{r}

WILLIAMS & 2 PATENT

Division

Rule

Reduce whole & mixed numbers to improper fractions & compound fractions to simple ones then divide the Numerator & Denominator of the dividend by the Numerator & Denominator of the divisor if they will divide without rem^r & the quotient will be the Numerator & Denominator of the quotient sought. But if they will not divide without rem^r multiply the Numerator of the Dividend by the Denominator of the divisor for the Numerator & the Denominator of the dividend by the Numerator of y^e divisor for the Denominator of the quotient req^d.

Divide $\frac{3}{14}$ of $\frac{6}{7}$ of $\frac{9}{27}$ by $\frac{14}{17}$ of $\frac{19}{28}$ of $\frac{13}{16}$

$$\frac{3}{14} \text{ of } \frac{6}{7} \text{ of } \frac{9}{27} = \frac{8 \times 6 \times 9}{14 \times 7 \times 27} = \frac{3}{49}$$

$$\frac{14}{17} \text{ of } \frac{19}{28} \text{ of } \frac{13}{16} = \frac{14 \times 19 \times 13}{17 \times 28 \times 16} = \frac{247}{544}$$

$$\frac{3}{49} \div \frac{247}{544} = \frac{3 \times 544}{49 \times 247} = \frac{1632}{12103}$$ Ans^r.

Divide $\frac{27}{36}$ of $\frac{6}{14}$ of $\frac{14}{27}$ By $\frac{16}{19}$ of $\frac{27}{32}$ of $\frac{42}{54}$.

$$\frac{27}{36} \text{ of } \frac{6}{14} \text{ of } \frac{14}{27} = \frac{\cancel{27} \times \cancel{6} \times \cancel{14}}{\cancel{36}_{6} \times \cancel{14} \times \cancel{27}} = \frac{1}{6}$$

$$\frac{16}{19} \text{ of } \frac{27}{32} \text{ of } \frac{42}{54} = \frac{\cancel{16} \times \cancel{27} \times \overset{21}{\cancel{42}}}{19 \times \cancel{32}_{2} \times \cancel{54}_{2}} = \frac{21}{38}$$

$$\frac{1}{6} \div \frac{21}{36} = \frac{1 \times \overset{19}{\cancel{38}}}{\cancel{6}_{3} \times 21} = \frac{19}{63} \Big) \text{ Ans}^{r}$$

Divide $\frac{9}{16}$ of $\frac{17}{19}$ of $\frac{16}{36}$ By $\frac{19}{21}$ of $\frac{14}{16}$ of $\frac{12}{38}$.

$$\frac{9}{16} \text{ of } \frac{17}{19} \text{ of } \frac{16}{36} = \frac{9 \times 17 \times \cancel{16}}{\cancel{16} \times 19 \times \cancel{36}_{4}} = \frac{17}{76}$$

$$\frac{19}{21} \text{ of } \frac{14}{16} \text{ of } \frac{12}{38} = \frac{\cancel{19} \times \overset{2}{\cancel{14}} \times \overset{3}{\cancel{12}}}{\cancel{21}_{7} \times \cancel{16}_{4} \times \cancel{38}_{2}} = \frac{1}{4}$$

$$\frac{17}{76} \div \frac{1}{4} = \frac{17 \times \cancel{4}}{\cancel{76}_{19} \times 1} = \frac{17}{19} \Big) \text{ Ans}^{r}$$

WILLIAMS'S 83 1/2 PATENT LB

The Rule of Three or the Rule of Proportion

This rule is so called because three numbers are given to find a fourth which bear the same proportion to the third as the second does to the first. State the terms of the question by placing the three given numbers in one line & that number must be placed last which is of the same kind with the answer to the question. Consider next whether the answer to the question will be less or greater than this number; if it will be greater; place the greater of the remaining numbers in the middle & the less first, but if it will be less, place the less number in the middle & the greater first. Reduce the last number to the least name contained in it, or to the fractional expressing of its name; & the 1st & 2nd to the least name contained in either of them or to the fractional expression of their greatest name. Multiply the 2d & 3d terms together & divide the

product by the 1st term and the quotient will be the answer in that name which the third number was reduced to

Example 1st

When a bushel of wheat cost 6s 8d the penny white loaf ought to weight 9oz 4 Dwts what ought it to weigh when wheat is at 5s 4d a bushel.

s d	s d	Oz Dwts
As 5 4 :	6 8 : :	9 4
12	12	20
64d	80d	184
		80

64)14720(230 (10 dwts
128
192
192
0

29
110z

Oz Dwts
Ans 11.10

Decimally

£	s	d	£	s	d	Ozs	Dwts
As	5	4		6	8	9	4

12 | 4 → 5,333333 ℔

12 | 8 → 6,666666 ℔

20 | 4 → 9,2

6,666666

9,2

13333332

59999994 Oz

5,33333) 61,333327 2 (11,50

5,33333 — 20 dwts

8,000 02 — 1000

5,33333

2,666697

2,666665

. . . 322

Ozs Dwts

Ans.r 11. 10

By Vulgar Fractions.

	s	d		s	d	Ozs Decs
As	5	4	: :	6	8	9 4

$$\text{As } \frac{5\times 12+4}{12} = \frac{64}{12} = \frac{16}{3}^{\text{d}}$$

$$\therefore \frac{6\times 12+8}{12} = \frac{80}{12} = \frac{20}{3}^{\text{s}}$$

$$\therefore \frac{9\times 20+4}{20} \quad \frac{184}{20} \quad \frac{46}{5}^{\text{Ozs}}$$

$$\therefore \frac{20}{3} \times \frac{46}{5} \div \frac{16}{3} :: \frac{\cancel{80}^{5}+\cancel{46}^{23}+3}{8+\cancel{8}+\cancel{16}^{2}} = \frac{23}{2}^{\text{Oz}} = 23 : 2 = 11\tfrac{1}{2}^{\text{Ozs}}$$

Ozs Decs

Ans^{r} 11. 10

Example 2nd

I have a room which is 22½ feet broad & the turner charged 92½ yds of matting for covering it, the matting being 2¼ F. wide what is the length of the room.

Feet	Inches	Feet	Inches	Yds	Ft	In
As	22	6	2	3	92	1	6

12 × 22 = 264 + 6 = 270 Inches

12 × 2 = 24 + 3 = 27 In

3 × 92 = 276 + 1 = 277 Ft

277 × 12 = 3324 + 6 = 3330 In

3330 × 27 = 89910

270 | 89910 | 333 | 9 In

810 | 3 | 270 Ft

891 | 2 Yds

810

810

810

0

Ans. Yd Ft In

2 . 0 . 9

Decimally

	Feet In		Feet In		Yds F I
As	22. 6	: :	2. 3	: :	92. 1. 6

12 | 6, / 22, 5

12 | 3, / 2,25

12 | 6, / 3 | 1, 5 / 92, 5

92. 5

2,2 5

4625

1850

1850

225) 208125 (9.25 yds

2025 — 3

562 — 75

450 — 12

1125 — 900

1125

. . . .

Yds F I

Ans. 9. 0. 9

WILLIAMS'S 83 1/2 PATENT L.B

By Vulgar Fractions

Fo J Fo F Yds F J

As 22 6 : 2 3 :: 92 1 6

As $\frac{22 \times 1216}{12} = \frac{6/270}{12} = \frac{\overset{FF}{45}}{2}$

$\frac{2 \times 1213}{12} = \frac{3/27}{12} = \frac{\overset{F.F}{9}}{4}$

$\frac{92 \times 311 \times 1216}{3 \times 12} = \frac{9/3330}{36} = \frac{2/370}{4} = \frac{\overset{Yds}{185}}{2}$

$\frac{9}{4} \times \frac{185}{2} : \frac{45}{2} = \frac{8 \times \cancel{10}\overset{37}{\cancel{2}} \times \cancel{4}\overset{1}{\cancel{2}}}{4 \times 8 \times \cancel{2}_{5}} = \frac{37}{4} = 37 : 4 = \overset{Yds\ F.\ J\ In}{9.\ 0.\ 9}$

Yds F J

Ans^{r} 9. 0. 9

Examples

When a bushel of wheat cost 5s 4d the penny loaf ought to weigh $\overset{Oz}{11}.\overset{Dwt}{10}$ what is the price of wheat p^r Bushel when the penny loaf weighs $\overset{Oz\ Dwt}{9}.4$

Oz Dwt	Oz Dwt	£ s d
As 9..4	11..10	5..4
20	20	12
184 dwt	230 dwt	64 d

64

920

1380

184) 14720 (80 d

12

1472 6..8

...0

s d

Ans^r 6..8

WILLIAMS'S 83 L. & 2 PATENT L.B

Decimally

£ Duk	£ Duk	£ d
As 9.4	11.10	5.4

20 | 4 — 9.2 — 05

20 | 10 — 11.5 — 05

12 | 4 — 5.33 — 2

9.2 . . . . . 11.5 . . . . . 5.33

11.5

2665

533

533

92 | 61.295 | 6.66

552	12
609	7.92
552	4
575	3.68
552	
23	

s d

Ans: 6. 7 3/4

By Vulgar Fractions

Ozs	Dwts	Ozs	Dwts	s	d
As	9.4	11.10		5	4

As $\frac{9\times20+4}{20}$ $\frac{184}{20}=\frac{46}{5}$

$\frac{11\times20+10}{20}=\frac{250}{20}=\frac{25}{2}$

$\frac{5\times12+4}{12}=\frac{64}{12}=\frac{16}{3}$

$\frac{16}{3}\times\frac{25}{2}\div\frac{46}{5}=\frac{\overset{8}{\cancel{16}}\times\overset{1}{\cancel{25}}\times5}{3\times\cancel{2}_{1}\times\cancel{46}_{23}}=\frac{20}{3}=20\div3=6.8$ 5 4

Ans^{r} 6. 8

s d

Example 4th

What length of a mahogany board which is 1 F. 9 In broad will it take to make a table the length of which is 5 F. 3 In and breadth 3 F. 6

	F.	In		F.	In		F.	In
As	1	9	:	3	6	:	5	3

12 × 21 = 252

12 × 42 = 504

12 × 63 = 756

42 × 126 = 5292

21) 126 (6 In

21

54

42

126

126

...

Inf. In

Ans. 10 6

Decimally

	£	ʃ		£	ʃ		£	ʃ
As	1	9		5	6		5	3

12 | 9. | 12 | 6. | 12 | 3.
1,75 | 3,5 | 5,25

5,25
3,5
2625
1575
1,75) 18,375 (10,5 £
175 12
875 12,0
875
...

£ ʃ
Ans: 10. 6

WILLIAMS'S 83 L.2 PATENT L.B

By Vulgar Fraction

£ s d £ s d £ s d

As 1 9 . . . . . 3 6 . . . . . 5 3

As $\frac{1 \times 12 + 9}{12} = \frac{3/21}{12} = \frac{7}{4}$ 3 7

$\frac{3 \times 12 + 6}{12} = \frac{6/42}{12} = \frac{7}{2}$ 3 7

$\frac{5 \times 12 + 3}{12} = \frac{3/63}{12} = \frac{21}{4}$ 3 7

$\frac{7}{2} \times \frac{21}{4} \div \frac{7}{4} = \frac{7 + 21 + 7}{2 + 4 + 7} = \frac{21}{2} = 21 \div 2 = 11 \text{ s } 6$ £ s d

Ans: 10 s 6

Example 5th

When a bushel of wheat cost £. 3 d the penny loaf will weigh 9 Oz what is the price of wheat when the penny loaf weighs 12. 10 Oz Drs.

Oz Drs	Oz	£ d
As 12. 10	9	£. 3
20	20	12
250	180	75

75
900
1260
250 / 13500 / 254 (6 d
1250 4 s
1000
1000
....

Ans. £ 4. D 6

Decimally.

Ozs dwts	Ozs	£ s d
As 12.10	9	6. 3

20 | 10,
12,5

20 | 9
0,45

12 | 3,
6,25

6,25
,45
3125
2500
12,5) 28125 (,225
250 20
312 4,500
250 12
625 6,000
625
. . .

£ s d
Ans^r 4. 6

By Vulgar Fractions.

Ozs Duets : Ozs : : s d
As 12.10 . . . . . 9 . . . . . 6.3

As $\frac{12+20+10}{20} = \frac{250}{20} = \frac{25}{2}$ Ozs

$\therefore \frac{9+20}{20} = \frac{180}{20} = \frac{2/18}{2} = \frac{9}{1}$

$\therefore \frac{6+12+3}{12} = \frac{3/75}{12} = \frac{25}{4}$ s

$\therefore \frac{25}{4} \times \frac{9}{1} : \frac{25}{2} = \frac{25+9+3}{2+1+2} = \frac{9}{2} = 9:2 = 4.6$ d

4 D

Ans^{r} 4.6

WILLIAMS'S 83 L.2 PATENT L.B

Example 6th.

What will 26 bags of hops cost each of which weighed Cwt 2r 16lb. 4c. 1q. 17lb. when one weighed Cwt 2r 16lb 3c. 3q. 19lb cost £ 16. 9. 3

Cwt	qr	lb	Cwt	qr	lb	£	s	d
3	3	19	4	1	17	16	9	3

4 15 20 129 31 439

4 17 20 113 35 493 26 2958 958 12818 3951 12818 6409 0 115362 38454

20 329 12 3951

439)5064391 8(12)115362(6d
439 20)9613(13s
674 480£
439
2353
2195
1589
1317
2721
2634
878
878
...

£ s d
Ans. 480. 13. 6

Decimally

Cwt 2qrs lbs
As 3.. 3.. 19

28 | 7 | 19
4 | 2,714285
4 | 3,678571
3,919642

Cwt 2qrs lbs
4. 1. 17

28 | 7 | 17
4 | 2,428571
4 | 1,607142
4,401785

L s D
16.9.3

12 | 3
20 | 9,25
16,4625

4,401785
26
2641071 0
880357 0.
114,446410
16,4625
57223205 0
22889282 0
6866784 60
457785640
6866784 60
114446410

3,919642 / 18840741024 6250 (480, 6730

15687856 8
. 3162472 2
31357136
.. 2645864 6
2351785 2
. 294079 42
274374 94
. 19704485
19598210
. 106275 0

L 20
S 13,5000
12
D 6,0000

Ans. L s D
480.13.6

WILLIAMS'S 33 1/2 PATENT L.B

By Vulgar Fractions

Cwt Q^rs lbs	Cwt Q^rs lbs	L s d
1 3 3 19	4 1 17	16 9 3

$$1 \text{ Cwt } \frac{3+4\times13+28+19}{4+28} = \frac{439}{112} \text{ Cwt}$$

$$\frac{4+4\times1+28+17}{4+28} = \frac{2}{112}\frac{128}{18} = \frac{6409}{56} \text{ Cwt}$$

$$\frac{16+20\times19+12+3}{20+12} = \frac{3}{240}\frac{3931}{} = \frac{1317}{80} \text{ L}$$

$$\frac{6409}{56} \times \frac{1317}{80} \div \frac{439}{112} = \frac{6409+\overset{3}{1317}\times\overset{8}{112}}{\overset{1}{56}+\underset{40}{80}+539} = \frac{19227}{40}$$

$19227 \div 40 = 480.13.6$ L s d

Ans^r 480 13 6

The Double Rule of Three

In this rule five numbers are given to find a sixth, which is the answer to the question. A question of this nature is best resolved by two operations of the single Rule of Three; & in stating the 1st operation, that number must be placed last which is of the same kind with the answer to the question, & either two of the other four, which are of the same kind, may be first or second, according as the answer in the 1st stating will be less or greater than the third number. In stating the 2nd Operation the answer to the 1st must be made the 3rd number & those two numbers which were not used in the 1st Operation must now be made the 1st & 2nd numbers the same rules being still observed for determinating which of them must be 1st

WILLIAMS' 83 1/2 PATENT L.B

Example 1st

If 100 gain £. 10 interest in 12 months in what time will 218. 14 gain 17. 11. 5 ½ interest

L S | L S D

As 4 | 10 ..... 17. 11. 5 ½ ..... 12 months

20 | 20
90 | 354
12 | 12
1080 | 4253
4 | 4
4320 | 17014
12
43) 204168 (47
172 80
31360
30240
1120

Ans. 47 months

2nd Operation

L S | L

As 218. 14 ..... 100 ..... 47 months

20 | 20
4374 | 2000
47
14000
8000
4374) 94000 (21 months
8748
6520
4374
2146

Ans. 21 months

Decimally

L l
As 4. 10 . . . . . 17. 14. 5 ½ . . . . . 12 months
20 | 10.
4.5

L S D
17. 14. 5 ½
4 | 2.
12 | 5.5
20 | 14.455033
17.722916
17.722916

12 months
45)212.674992(47.26440
180
326
315
117
90
274
270
49
45
49
45
42

Ans 47 months

2nd Operation

L S
As 218. 14 . . . . . 100 . . . . . 47 months
2,0 | 14
218.7

L
47
700
100
218.7)4700.00(21
4374
3260
2187
1173

Ans 21 months

WILLIAMS'S 33 1/2 PATENT L.B

By Vulgar Fractions

1st Operation

L S L S D

As 1 10 ..... 17 11 5½ ..... 12 months

As $\frac{1 \times 20 + 10}{20} = \frac{90}{20} = \frac{L\ 9}{2}$

$\frac{17 \times 20 + 11 \times 12 + 5 \times 12}{20 \times 12 \times 4} = \frac{2\,37\,014}{960} = \frac{L\ 8507}{480}$

$\frac{12}{1} = \frac{12}{1}$: 10

$\frac{12}{1} \times \frac{8507}{480} \div \frac{9}{2} = \frac{12 \times 8507 \times 2}{1 \times 240 \times 9} = \frac{8507}{100}$ 8507 : 100 = 47 : 10

Ans^{r} 47 months

2nd Operation

L S L

As 218 14 ..... 100 ..... 47 months

As $\frac{218 \times 20 + 14}{20} = \frac{2\,374}{20} = \frac{L\ 2187}{10}$

$\frac{100}{1} = \frac{L\ 100}{1}$

$\frac{47}{1} = \frac{47}{1}$: 10

$\frac{47}{1} \times \frac{100}{1} \div \frac{2187}{10} = \frac{47 \times 100 \times 10}{1 \times 1 \times 2187} = \frac{47000}{87}$ Months 21 Ans^{r}

Example 2nd

How far may 126. 3. 14 Cwt 2 Qrs 6 lb be carried for 2 £ 7 sh. 6 d when the carriage of 72 ½ Cwt for 30 miles cost £ 2. 2. 0

1st Operation

£	sh	d		£	sh	d		Miles
As 2.	2.	6	: : :

2nd Operation

Cart 2'66	Cart 1'	Miles F. Poles Yds
Ans 126. 3. 14.	7. 2. . . .	391. 3. 11. 4.2
4	4	8
507	30	3131
28	28	40
4060	240	125251
1015	60	55
14210	840	626257
		626259
		6888847
		840
		2755538800
		551107760

14210)5786631400(407220
56840
102631
99470
31614
28420
31940
28420
35280

11	407220
5,5 5	37020
10	7404 | 4 Poles
8	185 | 1 Fur
	23

Miles F. Poles Yds
Ans 23. 7. 4. 0

Decimally
1st Operation

L	S	D
As	2	2 6
12	6	
20	2,5	
	2,125	

L	S	D
	27	14 6
12	6	
20	14,5	
	27,725	
	30	

Miles
30

M

2125) 831,750,000000 (391,411764

6375	8
19125	3,294112
19125	40
30000	1176400
2125	55
8750	3822400
8500	3822400
2500	4,2046400
2125	
3750	
2125	
16250	
14875	
13750	
12750	
10000	
8500	
1500	

Miles Fs Pds

Ans 391. 3. 11. 4. 2

WILLIAMS'S 83 1/2 PATENT L.S

2nd Operation

Cwt Qrs lbs Cwt Qr Miles Furl Poles Yds

As 126 3 14 ..... 1 2 ..... 391 3 11 4 2

20 | 7 | 14
   | 4 | 2
   | 4 | 3, 5
   | 126, 875

4 | 2
  | 7, 5

55 | 5 | 4, 2
   | 11 | 8, 4
   | 40 | 11, 76365
   | 8 | 3, 29404
   | 391, 41761
   | 75

19570388805

2739823927 . M

126, 875 | 29355882075 | 23, 1376

253750	8
398088	1, 1008
380625	40
174632	4, 0320
126875	55
47757 70	1600
380625	1600
96945 7	1, 7600
88 8125	
8 1332 5	
761250	
52075	

. Miles 70 Yd

Ans. 23. 1. 4. 1

By Vulgar Fractions

1st Operation

L S D	L S D	Miles
As 2..2..6	27..14..6	30

$$As \quad \frac{27\times 20+2\times 12+6}{20\times 12} = \frac{540}{240} = \frac{351}{24} = \frac{17}{8}$$

$$\frac{27\times 20+14\times 12+6}{20\times 12} = \frac{6654}{240} = \frac{3327}{120} = \frac{1109}{40}$$

$$30 \ldots\ldots \frac{30}{1}$$

$$\frac{30}{1}\times\frac{1109}{40} \div \frac{17}{8} = \frac{6\ 30\times 1109\times 8}{17\times 40\times 17} = \frac{6654}{17} \qquad 6654\div 17 = 391\ \tfrac{7}{17}$$

Miles	F	Yds
391.	3..	11.4.2

Miles F P Yds
Ans: 391. 3.. 11.4.2

2nd Operation

Cwt Qr Lbs	Cwt Qr	Miles F P Yds
As 126. 3. 14	7. 2	391. 3.. 11.4.2

$$As \quad \frac{126\times 4+3\times 28+14}{4\times 28} = \frac{7142210}{112} = \frac{2\ 2030}{16} = \frac{1015}{8}$$

$$\frac{7\times 4+2\times 28}{4\times 28} = \frac{230}{4} = \frac{15}{2}$$

$$\frac{391\times 8+3\times 40+11\times 55+42}{8\times 40\times 55} = \frac{6888847}{1760}$$

$$\frac{6888847}{1760}\times\frac{15}{2} \div \frac{1015}{8} = \frac{6\ 888847\times 15\times 8}{1760\times 2\times 1015} = \frac{2066541}{89320}$$

$$2066541\div 89320 = 23\ \tfrac{122941}{89320} \qquad \text{Ans: } 23..1..4..6$$

Example 3d

If the fourpenny white loaf weighs 16 Oz 3 d when wheat is at 12 s p Bushel what should the twelve penny white loaf weigh when wheat is at 16 the Bushel

1st Operation

£	£	16 Oz
As 16	12	3 d

16

52 Oz

12

16) 624 (39 (2 lb

48 32

144 10 Oz

144

2nd Operation

£	£	16 Oz
As 1	12	2 d 7

16

39 Oz

12

1) 468 (117 (7 lb

112

5 Oz

Decimally

Sqs 16 ..... Sqs 12 ..... lbs 3.. Ozs 1

16 { 4 4 / 4 1 / 3,25

3,25

12

16) 390000 (lbs 24375

32 — 16

70 — 26250

64 — 4375

60 Ozs 7,000 0

48

120

112

80

80

Ans^{o} lbs 2.. Ozs 7

2do Operation

4 1 ..... 4 12 ..... lbs 2.. Ozs 7

16 { 4 7 / 4 1,75 / 2,4375

2,4375

12

4) 292500 (lbs 73125

16

107 50

3125

Ozs 5,0000

Ans^{o} lbs 7.. Ozs 5

WILLIAMS'S 35 L. 2 PATENT L.S

By Vulgar Fractions

1st Operation

£ £ lb Oz
Ans 16 . . . . . 12 . . . . . 3 4

Ans $\frac{16}{1} = \frac{16}{1}$

$\frac{12}{1} = \frac{12}{1}$

$\frac{3 \times 16 + 4}{16} = \frac{192}{16} = \frac{lb\ 12}{4}$

$\frac{13}{4} \times \frac{12}{1} \div \frac{16}{1} = \frac{13 \times 12 \times 1}{4 \times 1 \times 16} = \frac{39}{16} = 39 : 16 = 2 \frac{lb\ Oz}{7}$

lb Oz
Ans: 2 7

2nd Operation

D D lb Oz
Ans 4 . . . . . 12 . . . . . 2 7

$\frac{4}{1} = \frac{4}{1}$

$\frac{12}{1} = \frac{12}{1}$

$\frac{2 \times 16 + 7}{16} = \frac{lb\ 39}{16}$

$\frac{39}{16} \times \frac{12}{1} \div \frac{4}{1} = \frac{39 \times 12 \times 1}{16 \times 1 \times 4} = \frac{117}{16} = 117 : 16 = 7 \frac{lb\ Oz}{5}$

lb Oz
Ans: 7 5

Example 4th

If 26 cwt be carried 28 miles for £2..7 what must be paid for the carriage of 184 cwt 84 miles

1st Operation

cwt	cwt	£ s
As 46	184	2..7
	47	20
	1288	47 s

736
46)1288(28 s
92
368
368
...

£ s
Ans: 9..8

2nd Operation

Miles	Miles	£ s
As 28	84	9..8
		20
		188
		84
		752
		1504

28)15792(564(4 s
140 20
179 28 s
168
112
112
...

£ s
Ans: 28..4

WILLIAMS'S 83 L. 2 PATENT L.S

Decimally

1st Operation

Cwt	Cwt	lb Oz
As 46	184	2 7

20 | 7 | 2,35

184

940

1880

235

16 | 4 3 2 40 | £ 9,40

114 | 20

184 | £8,00

184

...6

Ans. £ 9.8

2nd Operation

Miles	Miles	£ s
As 28	84	9.8

20 | 8 | 9,4

84

376

752

28 | 78 96 | £ 28,2

56 | 20

229 | £4,0

224

56

56

..

Ans. £ 28,4

By Vulgar Fractions

1st Operation

Cwt	Cwt	£ s
As 46	184	2. 7

cwt

As 46 . . . . . $\frac{46}{1}$

cwt

184 . . . . . $\frac{184}{1}$

$\frac{2+20+7}{20} = \frac{\pounds 47}{20}$

$\frac{47}{20} \times \frac{184}{1} : \frac{46}{1} = \frac{47 \times 184^{4} \times 1}{20 \times 1 \times 46_{1}} = \frac{\pounds 47}{5} = 47 \div 5 = 9\frac{2}{5}$

$= \pounds 9.8$

Ans $\pounds$ 9. 8

2nd Operation

Miles	Miles	£ s
As 28	84	9. 8

M

As 28 . . . . $\frac{28}{1}$

M

84 . . . . . $\frac{84}{1}$

$\frac{9+20+8}{20} \quad \frac{2188}{20} = \frac{47}{5}$

$\frac{47}{5} \times \frac{84}{1} : \frac{28}{1} = \frac{47 \times 84^{3} \times 1}{5 \times 1 \times 28} = \frac{141}{5} = 141 \div 5 = 28\frac{1}{5}$

$\pounds$ s

28. 4

Ans $\pounds$ 28. 4

WILLIAMS'S 33 1-2 PATENT L.S

Example 5th

How many Persons will £6. 18. 3½ maintain one year when £16 will maintain 8 Persons 7 Weeks

1st Operation

As £16 ..... £6. 18 3½ ..... 8 Persons

20 / 156 l — 20 / 1738 l

12 / 1872 D — 12 / 20859 D

4 / 7488 fgs — 4 / 83438 fgs

8

7488) 667500 (89,143162

59904
67392
74880
32320
29952
23680
22464
12160
7488
46720
44928
17920
14976
29442

Ans 89 Persons

2nd Operation

As 52 Weeks ..... 7 Weeks 89,143162

52) 624021 ¾ (12,0004

52
104
104
213
208

Ans 12 Persons 52 / 52 / 2

Decimally

1st Operation

L S . . . . . L S D . . . . . Persons

As 7 - 16 . . . . . 86 - 18 - 3½ 8

20 | 16
7.8

2,
3,5
18; 29166
86,914583

8

78) 695316664 (89,14316

624
713
702
.111
78
336
312
246
234
126
78
484
468
.16

Ans^r 89 Persons .16

2nd Operation

Weeks . . . . . Weeks

As 52 . . . . . 7 89,14316

7

52) 624,002 (12,00004

52
104
104
.212
208
.4

Ans^r 12 Persons

WILLIAMS'S 83 1-2 PATENT L.B

By Vulgar Fractions

1st Operation

As £7 16 ..... £86 18 3½ ..... 8 Persons

As $\frac{7+20+16+12+4}{20+12+4} = \frac{9}{7}\frac{480}{960} = \frac{8}{8}\frac{24}{0}\frac{2}{10}\frac{7}{0}\frac{8}{5}\frac{39}{5}$

$\frac{86+20+18+12+3+4+2}{20+12+4} = \frac{7}{9}\frac{8}{6}\frac{34}{0}\frac{38}{0} \quad \frac{41719}{480}$

P

∴ 8 ..... $\frac{8}{1}$

$\frac{8}{1} \times \frac{41719}{480} \div \frac{39}{5} = \frac{8+41719+8}{1+480+39} = \frac{41719}{468} = 41719$

468 = 89 Ans: 89 Persons

2nd Operation

As 52 Weeks ..... 7 Weeks ..... 89 Persons

As 52 ..... $\frac{52}{1}$

7 ..... $\frac{7}{1}$

$\frac{89,14316}{1}$

$\frac{89,14316}{1} \times \frac{7}{1} \div \frac{52}{1} = \frac{22\ 20379}{89,14316+7+1} = \frac{156.00053}{13}$

12 Persons Ans: 12 Persons

Example 6th.

How long will £6. 18. 3½ maintain a family of 12 persons when £7. 16 will maintain 8 persons 7 Weeks

1st Operation

£	s	£	s	d	
As 7	16	6	18	3½	7 Weeks
20		20			
156		138			
12		12			
1872		2859			
4		4			
7488		83438			
		7			

7488) 584066 (78 Weeks

52416

59906

59904

2

Weeks

Ans. 78

2nd Operation

Persons	Persons	Weeks
As. 12	8	78
		8

12) 624

52 = 1 Year

Ans. 1 Year

Decimally
1st & 2nd Operation

£	s	d
As	7	16
20	16	
	7	8
		12
		936

£	s	d	
86.	18.	3½	Weeks
4	2.		7
12	3.	5	
20	18.	291666	
	86.	911583	
		8	
		695,3466	
		7	
936	4867216	18	52,000 17
	4680		
	1872		
	1872		
		166	
		936	
		7288	
		6552	
		736	

Ans^r 52 Weeks or 1 Year

1st Operation By Vulgar Fractions

£	s	d		
As	7	16		Weeks
			86. 18. 3½	7
			£	

As $\frac{7 \times 20 + 16}{20} = \frac{156}{20} = \frac{39}{5}$

$\frac{86 \times 20 + 18 \times 12 + 3 \times 4 + 2}{20 \times 12 \times 4} = \frac{83138}{960} = \frac{41719}{480}$

£

$\therefore 7 \ldots \frac{7}{1}$

$\therefore \frac{7}{1} \times \frac{41719}{480} \div \frac{39}{5} = \frac{7 \times 41719 \times 5}{1 \times 480 \times 39} = \frac{292033}{3744} = 292033 : 3744$

96

78 Weeks

Ans^r 78 Weeks

2nd Operation

	Persons	Persons	Miles
As	12	8	78

As 12 ..... $\frac{12}{1}$

: 8 ..... $\frac{8}{1}$

:: 78 ..... $\frac{78}{1}$

$\frac{78}{1} \times \frac{8}{1} : \frac{12}{1} :: \frac{78+8+12}{1+1+2} = \frac{32}{1} = 52 = 1$ Year

Ans.r 1 Year

WILLIAMS 33 1/2 ATLANTIC ST

Of Powers and their Roots.

The power of a number is the product which arises by multiplying that number by itself; this product by the same number again & so on to any number of multiplications whatever. The given number is called the 1st power or Root.

The product of the 1st power by itself is called the 2nd power or square. The product of the 2nd Power by the 1st is called the 3rd power or Cube. The Product of the 3rd Power by ij first is called the 4th power or Biquadrate. The following table contains the 1st 2nd & 3rd powers of the nine digits

Roots	1	2	3	4	5	6	7	8	9
Squares	1	4	9	16	25	36	49	64	81
Cubes	1	8	27	64	125	216	343	512	729

To find the 2nd Power or Square of 3697 Factors

3697
25879
33273
22182
7 1 1 4
11091

2nd Power or Square 13667809

To find the 3rd Power or Cube of 87654 Factors

87654
350616
438270
525924
613578
701232
0 3 3 0

2nd Power or Square 7683223716 Factors

87654
307328941864
384161185880
460993422296
537825660012
614657897728
0 0 3 0

3rd Power or Cube 673465291602264

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To find the 4th Power or Biquadrate of 7963 Roots

7963
23889
47778
71667
55741

2nd Power or Square 63409369

7963
190228107
380456214
570684321
443865583

3rd Power or Cube 504928805347

7963
15147864116041
30295728320082
45443592481123
35345016374129

4th Power or Biquadrate 4020748076978161

To Extract the Square Root of a given number

Definition

The square Root of a given number is such a number as will when multiplied by itself produce the number given & the method of finding it is called extracting the square root. Rule 1st Draw a line on the right hand side of the given number & divide it into periods of two figures each by putting a point over the units place & another over the place of hundreds & so on both ways if the given number be a mixed number & if there be not complete periods in the fractional part the deficiency must be made up by writing cyphers on the right hand side of it. The figure under a point & that which stands on the left hand side of it form a period. There will be as many fractional places in the root as there are periods in the fractional part of the given number. Find by means of the table of powers the

greatest Square which is contained in the first period set its root on the right hand side of the given number subtract the square from the 1st period bring down the next period to the rem.r & call the result the resolvend. Double the root & set it as a divisor on the left hand side of the resolvend seek how often this number is contained in the resolvend exclusive of the right hand figure set the result on the right hand of both the root & the divisor. Multiply this increased divisor by the last found figure in the root subtract the product from the resolvend bring down another period to the rem.r & repeat the operation until all the periods are made use of. If there be a rem.r after all the periods are brought down the root may be extended to any number of fractional places which may be thought necessary by annexing periods of two cyphers each to the remainder & repeating the operation.

Extract the Square Root of 79,6432,14,94,6310?

79,6432,14,94,6310 (8924,304 Root

64

169)1564

1521

1782) . 4332

3564

17844) . 76814

71376

178483) . 5438,94

535449

1788601) . . 84,456310

71394416

13,061894 Rem^{s}

8924,304 } Roots
8924,304 }

35697216

26772912 0

356 97216 . . .

17848608 . . . .

8631873 6 . . . .

7139443 2 . . . . .

79643201,884416 Product

13,061894 Rem^{s}

79643214,946310 Proof

WILLIAMS & LEA PRINTERS

Extract the square Root of 3765834876549375

3765'8348'7654'9375 / 19405.7591 Root

1

29 / 276
261

384 . 1538
1536

38805 . 223487
194025

388107 . 2946265
2716749

3881145 . 22651649
19405725

38811509 . 3845924380
3493035681

38811,5181 . . 5.28885775
3,88115181

1,40770594 Rem.

19405,7591 } Root
19405,7591

194057591
1746518319.
970287955.
135840313737. . .
97028795 5 . . .
7762303640. . . . .
1746518319. . . . . .
194057591. . . . . .
3765834876247232 81 Product
1,40770594 Rem.
3765834876549375 Proof

3 7 5
8 1
7

Extract the Square Root of 39784567825578464

61 39784567825578464 (19946.0692 Root

1

29 297

261

389 3684

3501

3984 18356

15936

39886 242078

239316

399206 2762557

2393256

39892129 36873218 4

35902916 1

39892138 2 9703023 64

7978427 64

1724596 00 Rem^s

19946.0692 } Root

19946.0692 }

398921384

179514622 8.

11967641 52..

119676415 20...

79784276 8.....

179514622 8.....

179514622 8.....

19946.0692.....

3978456761 5311 8864 Product

1724596 00 Rem^s

29784567 72557 8464 Proof

WILLIAMS & SONS 83 1/2 & 85 PENT. L^d

To Extract the Cube root of a given number.

Definition

The Cube Root of a number is such a number as will produce the given one by multiplying it by itself & multiplying the product again by the same number & ye method of finding it is called extracting the Cube root

Rule Divide the given number into periods of three figures each by putting a point over the units place & another over the place of thousands & so on both ways if ye given number be a mixed number & if there be not complete periods in ye fractional part ye deficiency must be made up by writing cyphers on ye right hand side of it. The figure under a point & ye two which stands on the left hand side of it form a period. There will be as many fractional places in ye root as there are periods in the fractional part of the Cube. Find by means of ye table of powers the greatest Cube which is contained in ye first period write its

wot in the quotient & subtract the Cube from
the 1st period bring down the next period
to the rem.r & call y.e result the resolvend
Square the quotient multiply that
square by 300 add 30 times y.e quotient
to this product & try as in division how
often this sum is contained in the resolvend the result will be another figure
in y.e root. Multiply 300 times y.e square
of what was in y.e Root before by the thus
found & multiply 30 times the said root by
the square of that figure add these two
products to y.e & the Cube of the last found
figure subtract their sum from the resolvend bring down another period to y.e rem.r
& repeat y.e operation until all the periods
are brought down If there be a rem.r
after all y.e periods are brought down the
root may be extended to any number
of fractional places which may be thought
necessary by annexing periods of three ciphers each to the rem.r & repeating the operation

WILLIAMS'S 83 1/2 PATENT L.B.

Extract the Cube Root of 659264,925436

659264,925436 (87,50 Root

1	1		18	1	
30	5592		18	1	
30	2400		144	300	
64	1920		18	300	
120	512		324	8	
188	4032	3 0 0	300	2400	
1920	760647	5	97200		
	684000	0	7		
18	26460		680400		
30	343				
540	707203			187	
49	33444925			187	
4860	52453500			1309	
2160	140250			1496	
26460	125			187	
187	52593857			34969	
30	851050436 Rem^s			300	
5610				10490700	
25	18750			5	
28050	18750 Roots			52453500	
11220	93750				
140250	131250..	3 0 3		1875	
	150000...	0		1875	
	18750....			9375	
	3515625000 Square			13125	
	18750			15000	
	17578125000			1875	
	2460937500..	0 0 3		3515625	
	28125000000...	0		300	
	35156250000....			1054687500	
	659179687500 0 Product				
	851050436 Rem^s				
	659264925436 Proof				

Extract the Cube Root of 35768579,676523?

35,768,579,676,523 (32948 Root

3	27			3
30	8768			3
90	5400			9
4	360			300
360	8		8	2700
32	4768		8 0	2
30	3000579		7	5400
960	2764800		8	32
81	77760		8	32
960	77729			64
7680	2843284			96
77760	157290,676			1024
329	129889200			300
30	15792 0			307200
9870	064			9
16	13004,7184			2764800
59220	2724349523			3294
9870	2604,046400			3294
157920	000512			13176
	2047,371392			29646
3294	1196,121131	Rems		65808
30				9882
98820				108504,36
0064	32940	Root		300
395280	32940			32551308000
592920	263584			08
6324480	131792		1	2604,046400
	29633 2		6	
	6589 6		1	
	9884			
	1085570704	Square		
	32948			
	8604565632			
	434228216		8	
	97701363360		1 8	
	21711408		8	
	3256712112			
	35767383533392	Product		
	1196,121131	Rems		
	35768579,676523	Proof		

WILLIAMS'S 83 1/2 PATENT LP

Extract the Cube Root of 524369,723684297.

80	524369,723684297	(80,639 Root	8
30	512		8
2409	12369,723		64
36	11520,000		300
14400	846,400		19200
7200	216		80
86400	1166,616		80
806	763,107684		6400
30	584,672400		300
24180	,000027		1920000
0009	584,890047		6
217620	170,217637297		11520000
	175,532216300		
8063	,019593090		806
30	,000600729		8063
241890	175,551910119		4836
000081	.2665727178 Rem^{s}		64480
241890			6496,36
1935120	80,639	Root	300
19359090	80,639		19489,0800
	725751		03
	241917		584,672400
	483834..		6083
	645120...		6083
	6502,648321 Square		24189
	80,639		48370
	5852383488 9		645040
	1950794463.		6501,1969
	390158892 6..		300
	52021186568 0...		19503,590700
	524367,057957119 Product		009
	2665727178 Rem^{s}		175,3231 6300
	524369,723684297 Proof		

Of Numeral Series.

A rank of three or more numbers that increase or decrease by an uniform progression is called a Numeral Series.

If the progression is made by equal differences, that is by a constant addition or subtraction of y same number, the Series is called an Arithmetic Progression.

If the progression is made by a constant multiplication or Division with y same number the Series is called a Geometric Progression. The common multiplier or divisor is called the ratio. In any Series of terms in Arithmetic Progression the sum of any two terms considered as extremes is equal to the sum of any two terms taken as means equally distant from the extremes. In any Series of terms in a Geometric Progression the product of any two terms considered as extremes is equal to the product of any two intermediate equidistant terms considered as means.

WILLIAMS'S 83 1/2 STRENT L.

In any arithmetic progression ye sume of any two terms be lessened by ye 1st term or their diffce increased by ye 1st terme will be a terme in that Series. In any Geometic progression the product of any two terms divided by ye 1st term or the quotient of any two terms multiplied by ye 1st term will give a term also in that series

If two Series of terms in Geometic progression be written & series of terms in Arithmetic progression ye 1st term of which is 0 the common diffce 1 term for term then any terme in ye Arithmetic Series will show how far it corresponding termin ye Geometic Series is distant from ye 1st terme. The termsin ye Arithmetic Series are indices to ye term of ye Geometric Series

Problem 1st

To find any Arithmetic progression given y^e 1st term y^e common diff^ce & y^e number of terms Req^d y^e last term Rule Subtract 1 from y^e number of terms multiply y^e rem^r by y^e common diff^ce to y^e product add y^e 1st term & y^e sum will be the last term

Example 1st

Suppose 1879 to be y^e 1st & 2^nd terms of an Arithmetic progression of 1074 terms what was y^e last term

1074 N^o of terms	9 2^nd term
−1 Ratio	1 1^st term
1073 Rem^r	8 Com diff
×8 Com diff^ce	
8584 Product	
+1 1^st term	
8585 last term	

Example 2nd

A person agrees to discharge a certain debt in a Year by weekly payments viz y^e 1st week 5^s y^e 2^d 7^s 8^d constantly increasing each week 3^s what was y^e last payment

52 N^o of terms	8 1^st term
−1 Ratio	5 2^nd term
51 Rem^r	3 Com diff
×3 Com diff	
153 Product	
+5 1^st term	
158 = 7^l 18^s last term	

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Problem 2nd

In an Arithmetic Progression given ye 1st term last term & the number of terms Req. ye sum of all ye terms. Rule Add the 1st & last terms together ye sum multiplied by half ye number of terms gives ye sum of all ye terms.

Example 1st

Req. ye sum of ye first 1000 numbers in their natural order

1000 last term
+1 1st term
1001 Sum
× 500 ½ No. of terms
500500 Sum Req.

2)1000 No. of terms
500½ No. of terms

Example 2nd

A debt was to be discharged in a year by weekly payments equally increasing ye 1st to be 5 ℔ ye last 7. ℔ How much was ye debt

158 No. of terms
+5 1st term
163 sum
× 26 ½ No. of terms
978
326
29)4238 - £211. 18 s. the debt

2)52 No. of terms
20 ½ No. of terms

Example 3rd

Suppose a basket & 500 stones were placed in a straight line a yard distant from one another. Req.d in what time a man could bring them 1 by 1 to the basket allowing him to walk at the rate of 3 miles an hour

500 last term
1 1st term
501 Sum
250 ½ No. of terms

500 No. of terms
250 ½ No. of terms

25050
1002
125250 Sum of all the terms
2
1760)250500,000(142.329 Miles
1760
7450
7040
4100
3520
5800
5280
5200
3520
16800
15840
960

60
26,500
60
34,800

Ans.r 47. 26. 34 nearly

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Problem 3rd.

In a Geometric Progression Given ye 1st term the ratio & ye last term Req.d ye sum of all ye terms Rule Multiply ye last term by ye common ratio, from ye product subtract ye 1st term for a Dividend. Subtract 1 from ye ratio for a divisor then divide & ye quotient will be ye sum of all ye terms

Example 1st

Suppose the 1st term of a series to be 3 the ratio 3 & ye last term 6561 Req.d ye sum of all ye terms

6561 last term	3 Ratio
×3 Ratio	-1
19683 Product	2 Divisor
-3 1st term	
19680 Dividend	
9840 the sum of all ye terms	

Example 2nd.

Let ye 1st term be 62 ye 2nd term 10 & ye last term be 156250 Req.d ye sum of all ye terms

156250 Last term	5/0
×5 Ratio	5 Ratio
781250 Product	1
-2 1st term	4 divisor
781248 Dividend	
195312 ye sum of all ye terms	

Problem 14th

In a Geometric Progression Given ye ratio ye 1st term & ye number of terms Req. the last term. Rule. Write down for 1 of ye leading terms in ye Geometric Series & over them their Indices. Add log. ye most convenient Indices to make an index less by unity than ye No. of terms sought. Multiply log. ye terms of ye Geometric Series belong to those Indices which were added, make ye product a dividend. Raise ye 1st term to a power whose index is less than ye No. of terms multiplied make ye result a divisor by ye formed Dividend and ye quotient will be ye term sought.

What is ye 12th Term of a Geometric Series ye 1st term of which is 3 & the 2nd term 6

Now $\frac{6}{3} = 2$ is ye com. ratio

And { 0, 1, 2, 3, 4, 5, 6 &c Indices
{ 3, 6, 12, 24, 48, 96, 192 &c Geometric Series

192
96
1152
1728
3 | 18432 Dividend
6144 is ye 12th term of ye given series

6
+5
11 index to ye 12th term

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Example 2nd

A person being asked to dispose of a Spnd horse said he would sell him on Condition of having 1fg for ye 1st nail in his shoes 2 fgs for ye 2nd nail &c doubling the price of every nail to 32 the number of nails in the four shoes. how much would that horse be sold for.

Note { 0, 1, 2, 3, 4, 5, 6, 7, 8 &c Indices
{ 1, 2, 4, 8, 16, 32, 64, 128, 256 &c Geometricians

8	256	
+8	×256	
16	1536	
+8	1280	
24	512	
+7	65536	is ye 17th term
31 is ye index of ye 32nd term	×256	
	393216	
	327680	
	131072	
	16777216	25th term
	128	
	134217728	
	33554432	
	16777216	
	1621474836 48	32nd term
	×2	Ratio
	4294967296	Product
	−1	1st term
4	4294967293	(3/4 ye sum of ye terms or ye price
12	1073741823	3d (of ye horse is fgs
20	89478485	5s
	4473924. 5	

£ SD

Ans 4473924. 5. 3/4

Logarithms

Logarithms are numbers so contrived that by using them instead of y^e natural numbers multiplication may be performed by Addition Division by Subtraction y^e raising of powers by multiplication & y^e extraction of roots by division. These artificial numbers are exponents to a series of numbers in a Geometric Progression y^e 1^st term being unity & y^e ratio of any number whatever so there be an infinite number of different kinds of Logarithms

Thus 0, 1, 2, 3, 4, 5 &c are Logarithms
1, 6, 36, 216, 1296, 7776 are numbers to them
Or
0, 1, 2, 3, 4, 5 &c are Log^ms
1, 10, 100, 1000, 10000, 100000 are N^os to them

Now taking y^e 1^st example it must be obs^d that 216 × 36 = 7776 y^e term which has 5 for its exponent or Log^m & the sum of 2 & 3 y^e exponents or Log^ms of 36 & 216 is 5 y^e Log^m of 7776 their Product y^e same will hold good of any other term in either of y^e series. But the Log^ms most convenient for common use

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are those in ye 2nd series where they are adapted to a Geometric Progression in which ye ratio is 10 ye scale of ye natural numbers in this progression it may be obs.d that ye log.m of 10 est consequently ye log.ms of all numbers betn 1 & 10 must fall betn these limits & therefore will be proper fractions. It may further be obs.d that ye log.ms of all ye numbers betn 10 & 100 must fall betn 1 & 2 therefore will be mixed numbers consisting of 1 & some fraction & that ye log.ms of all ye numbers betn 2 and 3 & therefore will be mixed numbers consisting of 2 & some fraction. Hence it appears that ye integral part of ye log.m of any number is always less by unity than ye integral figures contained in that number but fractional parts being more difficult to find have been computed & inserted in a table from whence they may be taken by ye following Rule

Problem 1.st

A. Number being given to find its log.m

If ye. Number consists of 2 integral figures only its log.m will be found in ye 1.st page of ye table where ye number & its log.m stands in ye adjoining columns opposite each

other the number being distinguished by ye
letter N & ye log^m by ye word log.
If ye number consists of integral figures
seek for that page of ye table whose one of ye
numbers which stand at ye top of it is less & ye
other greater than ye given number.
Find ye 3 first figures in ye left hand column
of that page & ye fourth at ye top of it & the
log^m of that number will be found opposite ye 3 first figures & under ye fourth.
If ye log^m consists but of 3 figures find those
three figures in the left hand column
of the table & the log^m stands opposite to
them in the column which has 0 at
the top & bottom
If the number consists of 5 figures find ye
log of ye four left hand figures as before & take
ye difference betwixt it & ye next greater multiply this
difference by ye 5^th figure divide the pro
duct by 10 & add ye quotient to the loga
rithm of ye four first figures the sum
will be ye logarithm of five figures.
As the index or integral part of a
Logarithm is less by unity than the
number of integral figures contained

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in the number which the Log. answers belongs to it follows that

The index of	78649	will be	4
	78649		3
	78649		2
	78649		1
	78649		0 or 10
	78649		9 or $\bar{1}$
	78649		8 or $\bar{2}$
	078649		7 or $\bar{3}$
	0078649		

But the fractional part of y^{e} log^{m} found in the table will be the same to each of them.

Find the Log^m of 32479

Ans^r 3,511611 Logarithm

Find the Log^m of 734873

Ans^r 4,866211 Logarithm

Find the Log^m of 1034729

Ans^r 8 or 2,514066 Logarithm

Find the Log^m of 4372,32

Ans^r 3,641071 Logarithm

Find the Log^m of 378,324

Ans^r 2,577806 Logarithm

Find the Log^m of 42734

Ans^r 3,630771 Logarithm

Find the Log^m of 1032173

Ans^r 8 or 2,501749 Log^m

Find the Log^m of 54,5279

Ans^r 1,733817 Logarithm

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Find the Log^m. of 3,687

Ans^r. 3,56667 Logarithm

Find the Log^m. of 1764

Ans^r. 1,24656 Logarithm

Find the Log^m. of ... 68764

Ans^r. 2,83736 Logarithm

Find the Log^m. of ... 87645

Ans^r. or 1,94272 Logarithm

Find the Log^m. of ... 62796

Ans^r. 3,79793 Logarithm

Find the Log^m. of ... 697.24

Ans^r. 2,84338 Logarithm

Find the Log^m. of ... 8814

Ans^r. or 0,94665 Logarithm

Find the Log^m. of ... 34796

Ans^r. or 1,54153 Logarithm

Problem 2nd

A Log.m being given to find the number which belongs to it. Find that page of y.e table wherein one of the Log.ms which stand at the top of it is less & the other greater than the fractional part of y.e given log. Look down that column of this page which is signed O at y.e top & bottom for y.e next less log.m to the given one against which in the column signed N stand the three first figures of the number sought. Look along y.e same line for the nearest log.m to the given one & annex the figure which stands at y.e top of the column wherein that log.m is found to the former three & you will have the N.o in four places of figures. If the N.o be wanted to 5 places of figures instead of taking the nearest log.m to the given one take the fourth figure from y.e top of y.e column in which y.e next less log.m is found take y.e diff.ce bet.n this log.m & y.e next greater also bet.n y.e next less & y.e given one annex a cypher to y.e latter diff.ce divide by y.e former & y.e quotient will be y.e fifth figure to be annexed to y.e former four. The Index will point out how many figures are integral

Find the N° corresponding to 0,76295

Ans: 5,794

Find the N° corresp^g to . . . 1,7612

Ans: 59,63

Find the N° corresp^g to . . . 2,80123

Ans: 6327

Find the N° corresp^g to . . . 6or4,72897

Ans: ,0005358

Find the N° corresp^g to 7or3,86270

Ans: ,007292

Find the N° corresp^g to 4,67928

Ans: 47783

Find the N° corresp^g to 4,72867

Ans: 53539

Find the N° corresp^g to 9or1,87234

Ans: 7153

Multiplication by Logmes

Rule

Add the Logarithms of the factors & their sum will be the Logm of the product which must be sought for in the tables.

Examples.

Multiply 7,628 By, 371

Multiplicand 7,628 its log is 0,88241
Multiplier ,371 its log is 9,56937
Product 2830 its log is 0,45178

Multiply 89,72 By 6,71

Multiplicand ,8972 its log is 1,95289
Multiplier ..... 6,71 its log is 0,82672
Product 602 its log is 2,77961

Multiply 12,345 By 6729

Multiplicand 12,345 its log is 2,09149
Multiplier 6729 its log is 3,82795
Product 830700 its log is 5,91944

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Division by Logms

Rule.

Subtract the logm of ye divisor from ye logm of ye dividend & ye remr will be ye log of ye quotient which must be sought for in ye tables. If ye dividend be less than ye divisor add ten to ye index of its logm & the quotient will be wholly fractional.

Examples

Divide 67,338 By 89,67

Dividend 67,338 its log is 11,82826
Divisor 89,67 its log is 1,95265
Quotient 7509 its log is 9,87561

Divide ,728 By ,2716

Dividend ,728 its log is 9,86213
Divisor ,2716 its log is 9,43393
Quotient 2,680 its log is 0,42820

Divide 7,863 By 6,729

Dividend 7,863 its log is 3,89562
Divisor 6,729 its log is 0,82795
Quotient 1169 its log is 3,06767

The Rule of Three by Logms.

Rule.

Take the terms of the question in the same manner as in the rule of Three by numbers, but instead of setting them all in one line, put them one under another, prefixing to the 1st term the word As, to the 2nd term the word So, to the 3rd term the words So is, & under them write the word So. Against the 1st term write the Arithmetical complement of its Logm against the 2nd & 3rd write their Logms. Add these three logms logs & their sum rejecting 10 in the index will be the logm of the answer, which must be sought for in the Tables The Arithmetical complement of a number is found thus begin at ye left hand & take every figure in it from 9 except the last which must be taken from 10.

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Examples

The wine expended by 2406 men in a year was 126315 Galls how much will serve 60 men the same times.

As 2406 Ar: Com log 6,61870
To 401 2,60314
So is 126315 5,10145
To 21052 Galls 4,32329

It is known that 15 worth of bread served 46 men a week when wheat cost 48 a Qr how many men will the same 15 worth serve when wheat is at 32 a quarter.

As 32 Ar: Com log 8,49485
To 48 1,68124
So is 46 1,66276
To 69 men 1,83885

How many Horses will 799 Bushels of coon serve for a year when 8789 Bushels were consumed by 396 Horses in that time.

As 8789 Ar: Com log 6,05606
To 799 2,90255
So is 396 2,59770
To 36 Horses 1,55631

The Double Rule of Three by Log^ms

Examples

How many men will reap 68 acres of corn in 14 days when 14 men will reap 8 acres in days

1st Operation.

As 8 Ar: Com log 9,09691
To 68 1,83251
Sois 14 1,14613
To 119 men. ... 2,07555

2nd Operation

As 14 Ar Com Log 8,85387
To 6 0,77815
Sois 119 2,07555
To 51 men. ... 1,70757

If 6 men mow 24 acres of grass in 8 days, in how many days will 24 men mow 288 acres

1st Operation

As 24 Ar Com log 8,61979
To 288 2,45939
Sois 8 0,90309
To 96 days ... 1,98227

2nd Operation.

As 24 Ar Com Log 8,61979
To 6 0,77815
Sois 96 1,98227
To 24 days 1,38021

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Of Powers & their Roots by Log^ms.

Problem 1st

A number being given to find any power of it by Log^ms. Rule. Find y^e log^m. of the given N^o. multiply this log^m. by y^e index of the proposed power, seek the number which corresponds to the product & it will be power required. If y^e given number be wholly fractional the power will be so, & therefore the index will be taken accordingly.

Examples

Find the 2^nd. power or square of 7,62

7,62 it's log is 0,880195

The index is 2

The power req^d. is 58,06 it's log is 1,760390

Find the 7^th. power or 2^nd. sursolid of 9,23

9,23 it's log is 0,965379

The index is 7

The power req^d. is 5578625 it's log is 6,741653

Find the eleventh power or 3^rd. sursolid of 9.

9, it's log is 0,954243

The index is 11

The power req^d. is 31379285711, it's log is 10,496641

Problem 2nd

A number being given to find any root of it by Logms Rule. Find the logm of ye given number & divide it by the number which expresses the root sought the number corresponding to the quotient will be the root req. If ye N° be wholly fractional add as many 10's wanting 1 to the index of the logm as there are units in the number you are to divide by before you begin to divide

Examples

Find the Square root of ,0785

,0785 it log is $^{-}1,894487$

The root req. ,2802 its log is $\overline{1},447443$

Find the Cube root of 871

871 its log is $2,940151$

The root req. 9530 its log is $0,979717$

Find the Cubed Cube root of 5437

5437 its log is $3,735350$

The root req. 559 its log is $0,919282$

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Geometry

Definitions & Principles.

Geometry is a science which treats of the dimensions, properties & relations of magnitudes in general: That is, of things on which length, or length & breadth, or length, breadth, & thickness, are considered. A Point is that which has neither parts nor dimensions. A Line is length without breadth: It is called a Right Line when it is ye shortest dist^ce

A ——— B

betwn two points, as. A B; Or a curved Line when it is not the shortest distance, as C D.

C D

A Line is usually denoted by two letters, viz one at each end, as. A B or C D. A Superficies or Surface is that magnitude which has only length & breadth & is bounded by lines as F G.

F G

A Solid is that magnitude which has length breadth & thickness

A figure is a bounded space the limits or bounds of which may be either lines or surfaces.

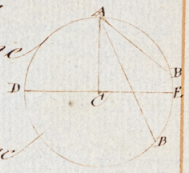
A B C D E

A Plane or Plane Figure, is a superficies which lies evenly, or perfectly flat, betwn its limits & may be bounded by one curve line, but not with less than three right lines, as. A, B, C, D, or E &c

A Circle is a plane figure, bounded by an uniformly curved line called the circumference as A B D which is every equally distant from one point as C within the figure called the center.

A B C D

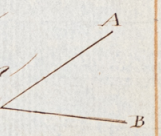
A Radius is a right line drawn from the center to ye circumference as CA, CD, or CE. All the radii of the same circle are equal. An arc is any part of the circumference. As ye arc AB or the arc AD.



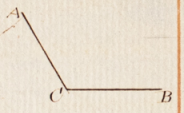
A Chord is a right line joining ye ends of an arc, as AB & is said to subtend that arc; it divides the circle into two parts, called Segments.

A Diameter is a chord passing through the center, as DE, & divides the circle into two equal parts called Semicircles. The circumference of every circle is supposed to be divided into 360 equal parts called Degrees each degree into 60 equal parts called Minutes each minute into 60 equal parts called Seconds &c

A Plane Angle is ye inclination of two lines on the same plane meeting in a point, as ACB

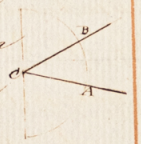


A right lined angle is formed by two right lines. The point where the lines meet is called the angular point, as C



The lines which form the angle are called legs. Thus CA & CB are the legs of the angle ACB.

An angle is usually marked by 3 letters viz one at y^e angular point & one at the other end of each leg but that at y^e angular point is always to be read the middle letter. The measure of a right lined angle is an arc as BA contained betⁿ the legs CB, CA including the angle the angular point C being the centre of that arc.

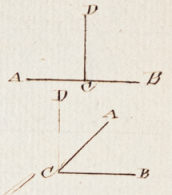


A right angle is that y^e measure of which is the fourth part of the circumference of a circle or 90°

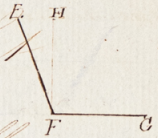


A Perpendicular is that right line which cuts another at right

angles; or which make equal angles on both sides. Thus DC is perpendicular to AB when the angles DCA & DCB are equal or are two right angles. An Acute angle as ACB is that which is less than a right angle as DCB.

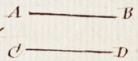


An Obtuse angle as EFG is that which is greater than a right angle as HFG.

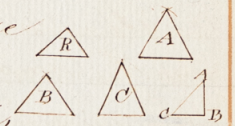


Acute & Obtuse angles are called Oblique Angles.

Parallel Lines are right lines on the same plane which do not incline to one another as AB, CD



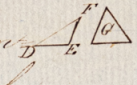
A Triangle is a plane figure bounded by three lines as R



An Equilateral Triangle is that in which the three lines, or sides, are equal, as A.

An Isosceles Triangle is that which has only two equal sides as B or C. A Right angled Triangle, as ABC is that which has one

right angle B. An Obtuse Angled Triangle as D E F has one obtuse angle E. An Acute angled Triangle as G has all its angles acute



A Quadrangle, or Quadrilateral, is a plane figure bounded by four lines or sides.

A Quadrangle is usually expressed by letters at its opposite angles.

A Parallelogram is a quadrangle the opposite sides of which are parallel and equal as P.



A Rectangle is a parallelogram with right angles & in which the length is greater than its breadth as R.



A square is a parallelogram having four equal sides & right angles, as S.



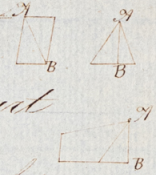
A Trapezium is a quadrangle the opposite sides of which are not parallel as T. The Diagonal of a quadrangle is a line as A B drawn from one angle to its opposite angle.





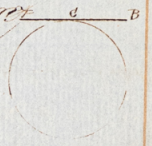
The Base of a figure is the line it is supposed to stand on.

The Altitude or Height of a figure is the perpendicular dist AB betn the base & the vertex or part most remote from the base.

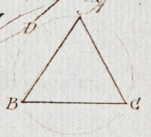


Congruous Figures are those which agree & correspond with one another in every respect.

A Tangent to a circle is a right line as AB touching it's circumference but not cutting & the point C where it's touches is called the point of contact



An Angle BAC in a Segment CADB is when the angular point is in the circumference of the segment & the legs including the angle pass through the ends B & C of the chord CB of the Segment



Such an angle is said is said to be in the circumference & to stand on the arc BC included betn the legs AB & AC of the angle

Right lined figures having more than four sides are called Polygons & have their names from the number of their angles & sides as those of five sides are called Pentagons; of six sides Hexagons of seven sides Heptagons; of eight sides Octagons &c. A regular ..... Polygon is a figure with equal sides & equal angles.

A figure is said to be inscribed in a circle when all the angles of that figure are in the circumference of the circle.

A figure is said to circumscribe a circle when every side of that figure is touched by the circumference of the circle.

A Proposition is something proposed to be considered & requires either a solution or answer or that something be made out or proved.

A Problem is a practical proposition in which something is proposed to be done or effected.

A Theorem is a speculative propo-

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sition or double in which something is affirmed to be true

A Corollary is some conclusion gained from the preceding proposition. A Scholium is a remark on some proposition or an exemplification of the matter which it contains

An Axiom is a self evident truth or principle that every one assents to upon hearing it proposed.

A Postulate is a principle or condition requested, the simplicity & reasonableness of which cannot be denied

Postulates

That a right line may be drawn from any given point to another given point

That a given right line may be continued or long thened at pleasure.

That from a given point, & with any radius a circle may be described.

Axioms

Things equal to the same things are equal to one another. If equal things be added to equal things the sums or wholes will be equal. But if unequals be added the sums are unequal. If equal things be taken from equal things the rems or diffes are equal, but are unequal when unequals are taken.

Things are equal which are double, triple, quadruple, &c or half third part &c of one & the same thing or of equal things

Things which have equal measures are equal. And the contrary. Equal circles have equal radii

Equal arcs in equal circles have equal chords & are the measures of equal angles. And the contrary. Parallel right lines have each the same inclination to a right line cutting them

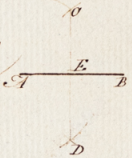
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Geometrical Problems.

Problem 1st.

To bisect, or divide into two equal parts, a given line AB.

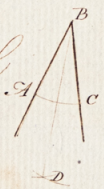
Open the 1st from the ends A & B with one & the same radius greater than half AB, describe arcs cutting in C & D. A line drawn from C & D, gives E the middle of AB as required



Problem 2nd

To bisect a given right lined angle ABC. Open the 1st from B describe an arc AC from A & C with one & the same radius describe arcs cutting in D.

A right line drawn from B to D will divide the angle into two equal parts as req.



Problem 3rd.

To from a given point B in a given right line AF to draw a right line perpendicular to ye given line

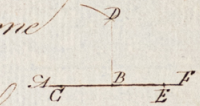
Case 1st

When B is near ye middle of ye line

Oper. 1st On each side of B, take ye equal distances BC & BE,

On C and E. describe with the same radius, arcs cutting in D.

A right line drawn through B & D will be ye perpendicular req.



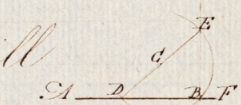
Case 2nd.

When B is at or near ye end of ye given line

Oper. 1st On any convenient point C taken at pleasure with ye distance or radius CB describe an arc DBE. cutting AF in D & B

A line drawn through D & C will cut this arc in E.

A right line drawn through B & C will be the perpendicular req.



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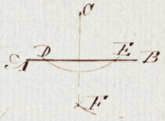
Problem 4th

To draw a line perpendicular to a given right line AB from a point C without that line.

Case 1st

When the point C is nearly opposite to the middle of y^e given line

Oper^n 1st On C with one radius cut AB in D & E.



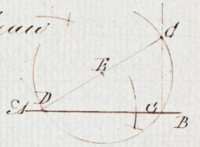
On D & E with one radius describe arcs cutting in F.

A line drawn from C to F will be the perpendicular req^d

Case 2nd

When C is nearly opposite to one end of the given line AB

Oper^n 1st To any point D in AB draw a line CD.



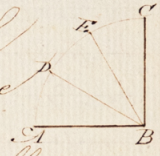
Bisect the line CD in E. On E with the radius EC cut AB in G

Then CG being drawn will be the perpendicular req^d

Problem 5th

To bisect or divide into three equal parts a right angle ABC. Open^n 1st from B with any radius B.A, describe the arc AC cutting the legs B.A, BC, in A & C.

From A, with the radius AB cut the arc AC in E & from C with the same radius cut AC in D

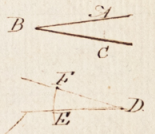


Draw BE, BD & the angle ABC will be divided into three equal parts.

Problem 6th

At a given point D to make a right angle equal to the given right lined angle ABC

Open^n 1st from D & B with the same radius describe y^e arcs E.F & AC cutting y^e legs of y^e given angle in the points A, C.



Transfer the dist^ce AC to y^e arc E.F from F to E. Lines drawn from D through E & F will form the angle EDF equal to y^e angle ABC

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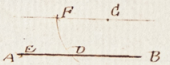
Problem 7th

To draw a line parallel to a given right line AB

Case 1st

When the parallel line is to pass through the given point C

Oper.n 1st From C with any convenient radius describe the arc DF cutting AB in D

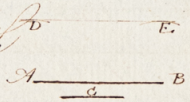


Apply the radius CD from D to E; & from E with the same radius cut the arc DF in F. A Line drawn through F & C will be the parallel line req.d

Case 2nd

When the parallel line is to be at a given dist.ce C from AB

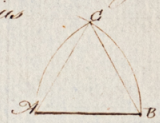
Oper.n From the points A & B with a radius C describe arcs D & E.



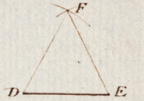
Lay a ruler to touch the arcs D & E. & a line drawn in that position is the parallel req.d

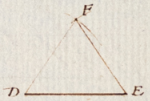
Problem 8th

Upon a given line AB to make an equilateral triangle
Oper. 1st From y points A & B with y radius
AB describe arcs cutting in C
2nd Draw CA, CB & the figure ABC is y
triangle req.



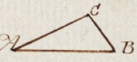
By y like operation, an Isosceles tri
angle DEF may be constructed on a
given base DE with y given equal legs
DF EF either greater or less than y
base DE.

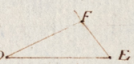




Problem 9th

To make a right lined triangle y sides of which shall
be respectively equal either to those of y given trian
gle ABC or two given lines provided any two of
them so taken be greater than y
third. Oper. 1st Draw a line DE equal
to the line AB. 2nd On D with y rad
ius equal to AC describe an arc F
3rd On E with a radius equal to BC des
cribe another arc cutting the former arc
in F. Draw FD, FE & the triangle DFE
will be that req.





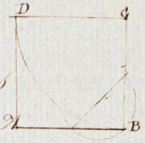
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Problem 10th

Upon a given line AB to describe a square Oper.n 1st Draw BC perpendicular and equal to AB.

2nd On A and C with the radius AB describe arcs cutting in D.

3rd Draw DC, DA & the figure ABCD is the square req.d

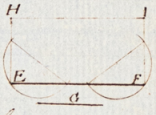


Problem 11th

To describe a rectangle whole length shall be equal to a given line EF & breadth equal to another given line G.

Oper.n 1st At E and F erect two perpendiculars EH and FI each equal to the given line G

Draw HI and the figure EFIH will be the rectangle req.d



Problem 12th

To find the center of a circle.

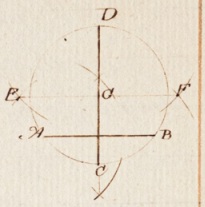
Spec.n 1st Draw any chord AB

2.nd Bisect AB with the chord CD

3.rd Bisect CD with the chord EF &

their intersection G will be the

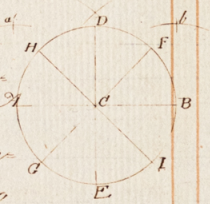
center req.d



Problem 13th

To divide the circumference of a circle into two four eight, sixteen thirty two &c. equal parts

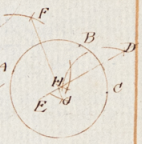
A diameter AB divides the circle into two equal parts. A Diameter DE perpendicular to AB divides the circumference into four equal parts. On A, D, B, describe arcs cutting in a, b then by the intersection a, b, & the center C the diameters FG, HI being drawn divides the circumference into eight equal parts & so on by continual bisection.



Problem 14th

To describe a circle y^e^ circumference of which shall pass through three given points A, B, C, provided they do not lie in one right line

Oper^n^ 1^st^ Bisect the dist^ce^ CB with y^e^ line DE. 2^nd^ Bisect the dist^ce^ AB with y^e^ line FG 3^rd^ On H, the intersection of these lines with y^e^ dist^ce^ to either of y^e^ given points describe the circle req^d^



Problem 15th

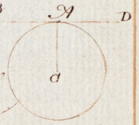
To draw a tangent to a given circle that shall pass through a given point A

Case 1^st^

When A is in the circumference of y^e^ circle

Oper^n^ From y^e^ center C draw the radius C A.

2^nd^ Through A draw BD perpendicular to C A & BD is the tangent req^d^

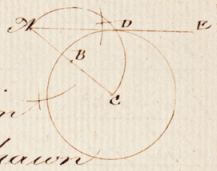


Case 2nd.

When the given point A is without the given circle.

Oper.n 1st. From the centre C draw C A which bisect in B.

2nd. On B with the radius B A cut the given circumference in D 3rd. On D the line A D being drawn will be the tangent req.d



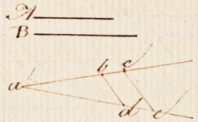
Problem 16th.

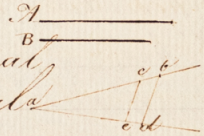
To two given right lines A, B to find a third proportional.

Oper.n 1st. Draw two right lines making any angle & meeting in a.

2nd. In these lines take a b equal first term & a c a d each equal to the second term.

Draw b d & through c draw c e parallel to b d & a e is the third proportional req.d

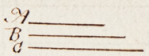




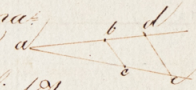
WILLIAMS, BS. 1 & 2 PATENT.

Problem 17th

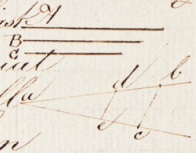
To three given right lines A, B, C, to find a fourth proportional.



Oper.n 1st. Draw two right lines making any angle & meeting in a.

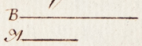


2nd. In these lines take ab equal first A, bc equal second B, & ad equal third C. 3rd. Draw bc & parall.l to it through d, draw de; then ae is the fourth proportional req.d

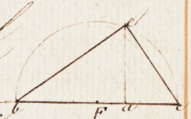


Problem 18th

Between two given right lines AB, to find a mean proportional.



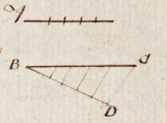
Oper.n 1st. Draw a right line in which take ac equal A, ab equal B. 2nd. Bisect bc in F, & on F with the radius Fb, describe a semicircle bec. From a draw ae perpendicular to bc, then ae is the mean proportional req.d



Problem 19th

To divide a given line BC, in y^e same proportion as a given line A is divided

Oper^n 1st From one end B of BC draw BD making any angle with BC.



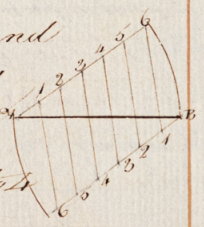
2nd In BD apply from B the several divisions of A, so BD will be equal to A and alike divided.

3rd Draw CD; then lines drawn parallel to CD through the several divisions of BD will divide the line BC in the manner req^d

Problem 20th

To divide a given right line AB into a proposed number of equal parts (suppose 7)

Oper^n From one end of AB, draw AE, to make an angle with AB and from B, the other end draw BF making y^e angle BAF equal to the angle BAE. 2nd Begin at A and B, & set off on these lines, as many equal parts except as AB is to be divided into, viz 1,2,3,4,5,6.

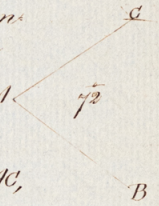


3rd Lines drawn from 1 to 6, 2 to 5, 3 to 4 & so will divide AB as was req^d.

Problem 21st

To make an angle of a proposed number of degrees

Oper.n 1st Take y.e first 60 degrees from y.e scale of chords & with this radius describe an arc BC, the center of which is A. 2nd Take the chord of y.e proposed number of degrees from y.e scale, reckoning from its beginning & apply this dist.ce to y.e arc BC from B to C. 3rd Lines drawn from A through y.e points B and C, will form an angle BAC, the measure of which is the degrees proposed.



Problem 22nd

To measure a given angle BAC.

From the angular A with a chord of 60 degrees describe the arc BC cutting the legs in the points B and C.

Then the dist.ce BC being applied to the chords from the beginning will show the degrees which measure the angle BAC.

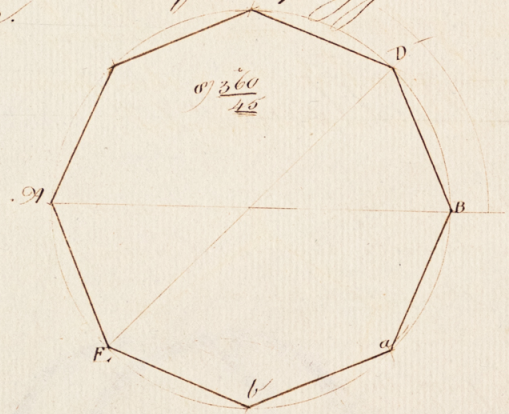


Problem 23rd

To in a given circle to inscribe a regular polygon the number of sides being given.

Oper.n 1st Divide 360 by the number of sides, & the quotient will be the degrees which measure the angle at the center of the circle, subtended by the side of the polygon.

2nd Draw the radius CB, make an angle BCD equal to those degrees & draw a chord DB; then will DB be the side of the polygon req.d which applied to the circumference from B to a, a to b, b to c, &c will give the points to which the sides of the polygon are to be drawn.



WILLIAMS & S.T 2 PATENT

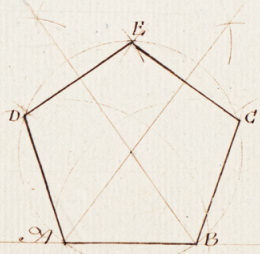
Problem 24th

On a given line AB, to construct a regular polygon of any assigned number of sides.

1st Prop. Divide 360 by the number of sides, subtract the quotient from 180, the rem. will be the degrees which measure the angle made by any two adjoining sides of that polygon and is called the angle of the polygon.

2nd. At the ends A and B of the line AB make angles ABC, BAD equal to the angle of the polygon.

3rd. Make AD, BC each equal to AB. 4th. At the points C, D, make angles equal to that of the polygon as before; & let the sides including those angles be each equal to AB & thus proceed until the polygon is constructed.



Problem 25th

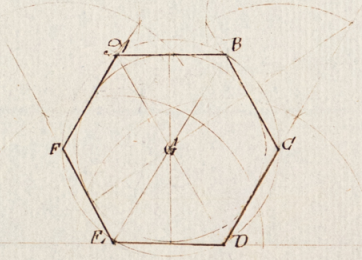
To circumscribe a circle about a given regular polygon or to inscribe a circle within that polygon.

Oper.n 1st. Bisect any two angles F.A B, G B.A with the lines A G B G and the point G where they intersect one another will be the center of the polygon.

2nd. A circle described from G with the radius G.A will circumscribe the given polygon.

Again 1st. Bisect any two sides F E, E D in the points H and I and draw H G, I G at right angles to F E, E D then the point G where they intersect each other will be the center of the polygon.

2nd. A circle described from G with the radius G H will be inscribed in the given polygon.



WILLIAMS & SONS PATENT

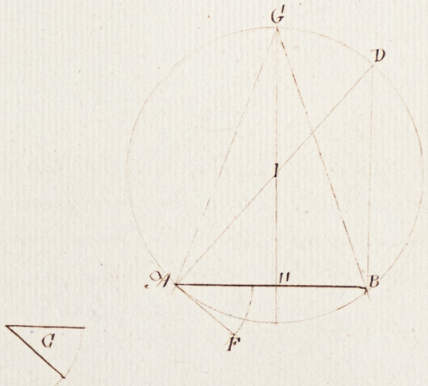
Problem 26th

On a given right line AB to describe a segment of a circle that shall contain an angle equal to the given right lined angle C

Oper.n 1st. Make an angle BAF equal to the given angle C.

2nd. From H, the middle of AB draw HI at right angles to AB, and from A draw AI at right angles to AF cutting HI in I.

3rd. From I with the radius IA describe a circle. Then will the segment AGB contain an angle AGB equall to the given angle C.



Geometrical Theorems

Theorem 1st

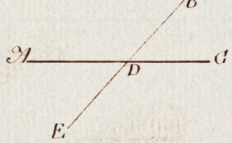
When one right line CD stands upon another right line AB they make two angles BCD, ACD which together are equal to two right angles



Theorem 2nd

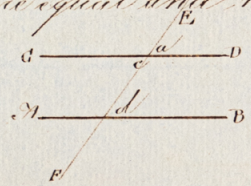
If two right lines AC, EB, intersect each other in D the opposite angles are equal, viz.

∠CDB = ∠ADE, and ∠CDE = ∠ADB



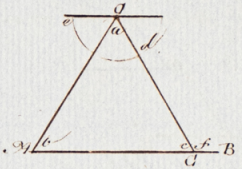
Theorem 3rd

If a right line FE, cut two parallel right lines AB, CB then is the outward angle a equal to the inward and opposite angle d and the alternate angles c and d are equal and the contrary.



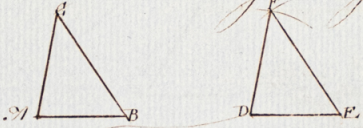
Theorem 4th

In any right lined triangle ABC the sum of y^e three angles A, B, C, is equal to two right angles. And if one side BC be continued, the outward Ex is equal to the sum of the two inward and opposite angles A & B.



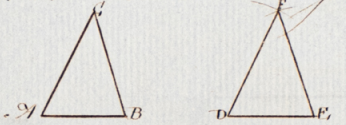
Theorem 5th

If two sides AB, AC & y^e included angle A, in one triangle ABC, are respectively equal to two sides DE, DF, and the included angle D of another triangle DEF each to each then are those triangles congruous.



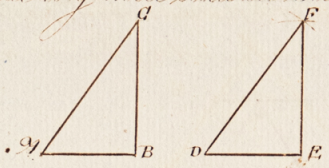
Theorem 6th

If two triangles ABC, DEF, have two angles A, B and the included side AB, in one respectively equal to 2 angles DE, and the included side DE, in the other each to each then are those triangles congruous.



Theorem 7th

Two triangles ABC, DEF, are congr.ous when y.e three sides in y.e one are equal to y.e three sides in the other, each to each.



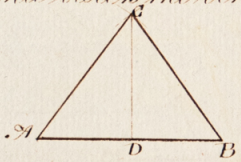
Theorem 8th

Two triangles ABC, DEF are congruous, when two angles A, B, & a side AC opposite to one of them in one triangle, are respectively equal to 2 angles D, E. & a side DF opposite to a like angle in the other triangle, each to each.



Theorem 9th

In y.e Isosceles or equilateral triangle ACB a line drawn from the vertex C to the middle of the base AB is perpendicular to the base and bisects the vertical angle & y.e contrary.



WILLIAMS' 83 1/2 PATENT

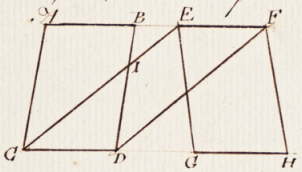
Theorem 10th

In every right lined Triangle ABC the greater angle C is opposite to the greater side AB.



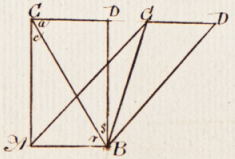
Theorem 11th

Parallelograms ACDB, ECDF, EGHF, standing on the same base CD, or on equal bases CD, GH, and betw the same parallels, GH, AF are equal.



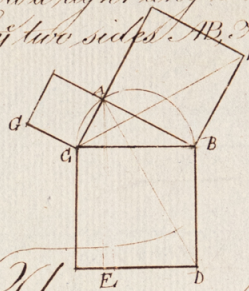
Theorem 12th

A Triangle ABC is the half of the parallelogram AD when they stand on the same base AB & are betw the same parallels AB, CD.



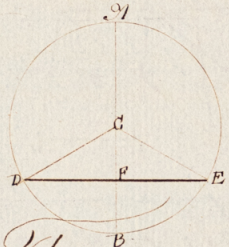
Theorem 13th

In every right angled triangle BAC the square on ye side BC opposite to a right angle A is equal to the sum of ye squares of ye two sides AB AC containing the right angle.



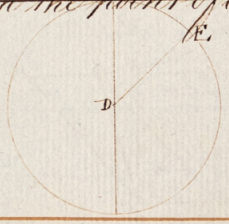
Theorem 14th

In any circle, a diameter AB drawn perpendicular to a chord DE, bisects that chord & its subtended arc DBE.



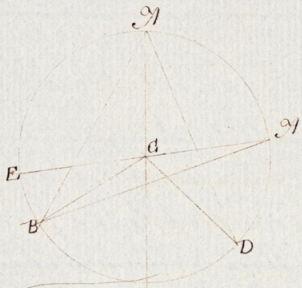
Theorem 15th

A tangent AB to a circle is perpendicular to a diameter DE drawn from the point of contact C.



Theorem 16th

An angle BCD, at the centre of a circle is double of the angle BAD at the circumference when those angles stand on the same arc BD.

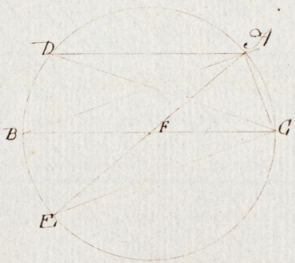


Theorem 17th

An angle BAC, in a semicircle is a right one.

An angle DAC, in a segment less than a semicircle is obtuse.

An angle EAC, in a segment greater than a semicircle is acute.



Of Proportion.

Definitions and Principles

One quantity A, is said to be measured or divided by another quantity B, when A contains B some number of times, exactly. When two quantities of a like kind are compared togr, the relation which one thom has to the other, in respect to quantity, is called Ratio. Equal ratios are those where the antecedents are like multiples or parts of their respective quantity. Ratio of equality is, when the antecedent is equal to the consequent. Four quantities are said to be proportional, which when compared togr by two and two, are found to have ratios. So that when four quantities of the same kind are proportional, the ratio betn the first and second is equal to the ratio betn the third and fourth; & this proportionality is called direct. Also the ratio betn the first and third is equal to the ratio betn the second and fourth; and this proportionality is called alternate. Similar, or like, right lined figures, are those which are equiangular, that is, the several angles of which are equal

one to the other; & also, the sides about the equal angles are proportional.

Like arcs, chords, or tangents, in different circles, are those which subtend, or are opposite equal angles at the center.

Quantities, and their like multiples, have the same ratio. Quantities, and their like parts, have equal ratios. Quantities, and their like multiples, or like parts are proportional.

If quantities are equal, their like multiples, or like parts are also equal.

Then are the antecedents and consequents in a ratio of equality. If the parts of one quantity are proportional to the parts of another quantity, they are like parts of their respective quantities.

Secondly like parts are proportional to their wholes. Ratios which are equal to the same ratio are equal to one another. Proportions, which are the same to the same proportion, are the same to one another. If two ratios or products are equal, their like multiples either by the same or

by equal quantities or by equal ratios are also equal. Equal quantities A and B, have the same ratio or proportion to another quantity C. And any quantity has the same ratio to equal quantities. Hence, when the antecedents are equal, the consequents are equal, and the contrary. Quantities are equal, which have the same ratio to another quantity: or to like multiples or parts of a said quantity.

Thus, if A : C :: B : C. Then A = B.

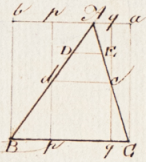
If two or more sets of proportional quantities, A : B :: C : D. Then the Rectangle or Product of the two extremes is equal to the Rectangle or Product of the two means. Hence, if the Rectangle or product of two quantities is equal to the Rectangle or product of other two quantities those four quantities are proportional. If four quantities are proportional then will either of the extremes, and the ratio of the product of the means to the other extreme be in a ratio of equality: And either mean, and the ratio of the product of the extremes to the other extreme will be also in a ratio of equality.

WILLIAMS'S 83 1/2 PATENT L.B

Theorem 18th

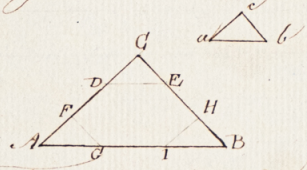
In any plane Triangle ABC, any two adjoining sides AB, BC, are cut proportionally by a line DE, drawn parallel to the other side BC, viz

AD : DB :: AE : EC.



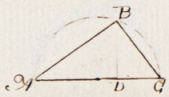
Theorem 19th

In equiangular Triangles ABC, abc the sides about the equal angles are proportionals; and the sides opposite to equal angles are also proportional



Theorem 20th

In a right angled Triangle ABC if a line BD be drawn from the right angle B perpendicular to the opposite side AC then all the Triangles ABD, BCD on each side the perpendicular BD similar to the whole ABC and to one another.



WILLIAMS'S 83 1/2 PATENT L.B

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WILLIAMS'S 83 1/2 PATENT LB

WILLIAMS'S 83 1-2 PATENT L.B

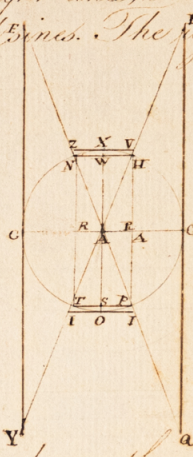
Plane Trigonometry.

Definitions & Principles.

Plane Trigonometry is an art which shews how to find the measures of the sides and angles of plane Triangles, some of them being already known.

A Triangle consists of six parts; namely, three sides & three angles. The sides of plane triangles are denoted, & estimated by measures of length; such as Feet, Yards, Fathoms, Furlongs, Miles, Leagues, &c. The angles of triangles are estimated by circular measure, that is, by arcs containing Deg. Minutes, Seconds, &c and for convenience these ar-

cular measures are represented by right lines, called right sines, tangents, secants, & versed sines. The right sine of an arc, is a right line drawn from one end of the arc perpendicular to a radius drawn to the other end: Or it is half the chord of the double of that arc.



The Tangent of an arc is a right line touching one end of the arc & continued till it meets a right line drawn from the center through the other end of that arc.

The Secant of an arc, is a right line drawn through the center & one end of the arc & produced till it meets the tangent drawn from the other end.

The Versed Sine of an arc is that part of the radius intercepted betn the arc & its right sine

The Complement of an arc is what that arc wants of 90°. The Supplement of an arc is what that arc wants of 180°. The Co-sine of an arc is the right sine of the complement of that arc.

The Co-tangent of an arc is the tangent of that arc's complement

The Co-secant of an arc is the secant of its complement. The Versed sine of an arc is the versed sine of its complement. The greatest right-sine is the sine of 90°. & the sines to arcs less than 90° serve equally

for arcs, as much greater than 90°. When an arc is greater than 90° the sine tangent secant of the supplement is to be used. The angles of two triangles may be respectively equal although their sides may be unequal.

Therefore in a triangle among the things given in order to find the rest one of them must be a side. In Trigonometry the three things given in a triangle must be either.

1st Two sides & an angle opposite to one of them.
2nd Two angles & a side opposite to one of them.
3rd Two sides & the included angle.
4th 5th 6th Three sides.

Trigonometrical Theorems.

1st In every right angled plane triangle, if the hypothenuse be made radius, the legs will be the sines of their opposite angles or if one of the legs be made radius the hypothenuse will be the secant of the angle adjacent to the leg made radius & the other leg will the tangent of the said angle.

2nd In every plane triangle, the sides are in proportion to the sines of their opposite angles.

3rd In every plane triangle the sum of any two sides is to their diffce as the tangent of ½ the sum of their opposite angles to the tangent of ½ the diffce of those angles.

4th In every plane triangle the Base is to the legs as

the diffce of the legs is to the diffce of the segments of the Base made by letting fall a perpendicular to the base from the opposite angle.

5th In every plane triangle As the product of any two sides is to the square of the radius so is the product of the ½ sum & ½ difference of the third side & the difference of those two sides, to the square of the sine of the angle contained by those two sides

6th In any two quantities their ½ difference added to their ½ sum gives the greater & the ½ difference subtracted from the ½ sum gives the less.

Therefore if the ½ sum & ½ diffce of any two quantities can be found the quantities themselves are also found.

WILLIAMS'S 83 1-2 PATENT

Problem 1st

To find the logarithmic sine, tangent, secant, co-sine, cotangent, cosecant, to a given number of Degrees, Minutes, Seconds. &c.

Rule

Take the difference betn the logarithm of the Degrees & Minutes next less & the logarithm of those which are next greater than the Degrees Minutes & Seconds given.

Then as 60 is to the Seconds in the given number so is the diffce of the logms to a fourth number. Add this number to the logm of the Degrees & Minutes next less than the given number when the logms are increasing but subtract it from that logm when they are decreasing & the sum or remr will be the logm required.

Find the log sine to 79° 18′ 37″

The log sine to 79° 19′ is 9,99241
The log sine to 79° 18′ is 9,99238
The difference is 3

Then As 60″ : 37″ :: 3 : 2
The log sine to 79° 18′ is 9,99238
+2
The log sine to 79° 18′ 37″ is .. 9,99240

Find the log tangent to 31° 45′ 23″

The log tangent to 31° 46′ is 9,79185
The log tangent to 31° 45′ is 9,79156
The Diff^ce is 29

Then As 60″ : 23″ :: 29 : 11

The log tangent to 31° 45′ is .. 9,79156
+11
The log tangent to 31° 45′ 23″ is .. 9,79167

Find the log secant to 53° 16′ 31″

The log secant to 53° 17′ is .. 10,22340
The log secant to 53° 16′ is .. 10,22323
The Diff^ce is 17

Then As 60″ : 31″ :: 17 : 9
The log secant to 53° 16′ is 10,22323
+9
The log secant to 53° 16′ 31″ is .. 10,22332

Find the log cosine to 55° 17′ 19″

The log cosine to 55° 17′ is . . 9, 755551
The log cosine to 55° 18′ is . . 9, 755533
The Diffce is 18

Then As 60″ : 19″ :: 18 : 6
The log cosine to 55° 17′ is . . 9, 755551
−6
The log cosine to 55° 17′ 19″ is 9, 755545

Find the log cohangent to 37° 15′ 36″

The log cohangent to 37° 15′ is . . . 10, 11110
The log cohangent to 37° 16′ is . . . 10, 11084
The Diffce is 26

Then As 60″ : 36″ :: 26 : 16
The log cohangent to 37° 15′ . . . 10, 11110
−16
The log cohangent to 37° 15′ 36″ is 10, 11094

Find the log cosecant to 7° 13′ 27″

The log cosecant to 7° 13′ is 10, 01669
The log cosecant to 7° 14′ is 10, 01666
The Diffce is 3

Then As 60″ : 27″ :: 3 : 1
The log cosecant to 7° 13′ is 10, 01669
−1
The log cosecant to 7° 13′ 27″ is . . 10, 01668

Problem 2nd

A Logarithmic sine tangent secant cosine cotangent cosecant being given to find the degrees minutes & seconds which belongs to it.

Rule

Seek in the tables in that column which has the given name at the top or bottom for the two nearest Logms to the given one, one being greater & the other being less than it & take out the degrees & less number of minutes belonging to either of those two logms

Take the diffce between those two logms & also the diffce between the logms of the degrees & minutes taken out & the given logm then say As the diffce between the two logms taken out of the Table is to the diffce between the logm of the degrees & minutes taken out & the given logm so is 60" to a fourth number of seconds which being placed to the right hand of the degrees & minutes before taken out will shew the Degrees Minutes & seconds req.d

To find the Degrees Minutes & Seconds belonging to the log sine of 9,74085

log sine to 33° 25′ is ... 9,74093
log sine to 33° 24′ is ... 9,74074
Diffce ... 19
Given log sine is ... 9,74085
log sine to 33° 24′ is ... 9,74074
Diffce ... 11

Then As 19 : 11 :: 60 : 35

Ans. 33° 24′ 35″

To find the Degrees Minutes & Seconds belonging to the log tangent of 7,78481.

log tangent to 0° 21′ is ... 7,78595
log tangent to 0° 20′ is ... 7,76476
Diffce 2119
Given log tangent is ... 7,78481
log tangent to 0° 20′ is ... 7,76476
Diffce ... 2005

Then As 2119 : 2005 :: 60 : 57

Ans. 0° 20′ 57″

To find the Degrees Minutes & Seconds belonging to the log secant of 10,73071

log secant to 79° 18′ is ... 10,73127
log secant to 79° 17′ is ... 10,73060
Diffce ... 67
Given log secant is 10,73071
log secant to 79° 17′ is ... 10,73060
Diffce ... 11

Then As 67 : 11 :: 60 : 10

Ans. 79° 17′ 10″

Find the Degrees Minutes & Seconds belonging to the log cosine of 9,74358

log cosine to 56° 21′ is ... 9,74360
log cosine to 56° 22′ is 9,74341
Diff^ce 19

Given log cosine is 9,74358
log cosine to 56° 22′ is 9,74341
Diff^ce 17

Then As 19 : 17 :: 60″ : 54″ Ans 56° 22′ 54″

Find the Degrees Minutes & Seconds belonging to the log colangent of 10,54325

log colangent to 15° 58′ is 10,54346
log colangent to 15° 59′ is 10,54298
Diff^ce 48

Given log colangent is 10,54325
log colangent to 15° 59′ is 10,54298
Diff^ce 27

Then As 48 : 27 :: 60″ : 34″ Ans 15° 59′ 34″

Find the Degrees Minutes & seconds belonging to the log cosecant of 10,39714

log cosec^t to 23° 37′ is 10,39727
log cosec^t to 23° 38′ is 10,39698
Diff^ce 29

Given log cosec^t is 10,39714
log cosec^t 23° 38′ is 10,39698
Diff^ce 16

Then As 29 : 16 :: 60″ : 33″ Ans 23° 38′ 33″

WILLIAMS'S 83 1-2 PATENT

Rules for the Solution of the four cases of right angled plane triangles deduced from the first Trigonometrical Theorem.

N.B. The cases are distinguished by the parts given it being understood that in each case all the unknown parts are required to be found.

Rule 1st When the hypothenuse is made radius either of the acute angles may be made the center & the leg opposite to that angle which is taken for the center, will be the sine of it & the leg adjacent will be its cosine.

2nd When either of the Legs is made radius the acute angles adjacent to that leg must be the center the Hypothenuse will be the secant of that angle & the other leg will its tangent but they will be the cosect & cotangent of the other acute angle.

To make out the Proportions.

N.B. No notice is ever taken of the right angle

3d When two sides are given & an angle is

wanting the first term must be that side which is made radius the second term will be the radius, the third term must be the other known side & the fourth term will be the name of that side which belongs to the reqd angle.

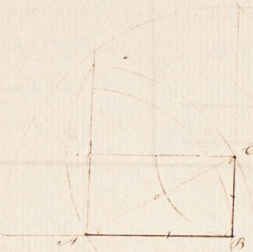
4th To a side be wanted begin with that name of the known side which belongs to a known angle make the known side the second term that name of the reqd side which belongs to the known angle the third term & the side sought will be the fourth term.

5th When two sides are given the unknown side cannot first be made radius.

NB. When one side & the angles are given to find the other two sides if the given side be made radius the first term must be radius the second must be the given side the third should be that name of the reqd side which belongs to a known angle & the side sought will be the fourth term.

WILLIAMS'S 83 1/2 PATENT L'S

Given { AB 63 m / BC 34 m } Req^d the rest



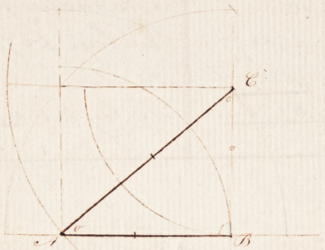
∠A Radius to find ∠A.	BC radius to find ∠C
As log AB 63 m . . 8,20006	As log BC 34 m . . 8,56852
To radius 90.00 . . 10.	To radius 90.00 . . 10.
So log BC 34 m . . 1,53148	So log AB 63 m . 1,79934
To tan ∠A 28.21 . . 9,73214	To tan ∠C 61.38 . 10,26786
AB Rad to find AC with AB	BA rad to find AC with BC
As radius 90.00 . . 10.	As tan ∠A 28.21 . . 9,72795
To log AB 63 m . . 1,79934	To log BC 34 m . 1,53148
To sec ∠A 28.21 . 10,05549	To sec ∠A 28.21 . 10,05549
To hyp AC 71.59 m 1,85483	To hyp AC 71.60 m 1,85492
BC rad to find AC with AB.	BC rad to find AC with BC
As tan ∠C 61.38 . . . 9,73235	As radius 90.00 . . . 10.
To log AB 63 m . . 1,79934	To log BC 34 m . 1,53148
To sec ∠C 61.38 . . 10,32320	To sec ∠C 61.38 . 10,32320
To hyp AC 71.60 m 1,85409	To hyp AC 71.56 m 1,85468

AC rad. to find AC with the AB	AC rad. to find AC with the
As cosine 42° 21′ ... 0.055 19	As cosine 64° 38′ ... 0.337 31
To log AB 63 m. ... 1.799 31	To log BC 31 m. ... 1.531 40
So radius 90° 00′ ... 10.	So radius 90° 00′ ... 10.
To hyp AC 71 59 m. ... 1.85 183	To hyp AC 71 58 m. 1.85 179

Ans. { A 42° 21′ / C 64° 38′ / AC 71 58 }

WILLIAMS'S 85 1/2 PATENT

Given { AB 72m / AC 97m } Req.d the rest



A rad.s to find ∠ A	AC rad.s to find ∠ C
As leg AB 72m ... 8.41267	As hyp.e AC 97m ... 8.01323
To rad.s 90.00 ... 10.	To rad.s 90.00 ... 10.
So hyp.e AC 97m ... 1.98677	So leg AB 72m ... 1.85733
To sec.t ∠ A 42.01 ... 10.12944	To sine ∠ C 47.59 ... 9.87056
AC rad to find BC with AB	**AB rad to find BC**
As cosine ∠ A 42.01 ... 9.87056	As rad.s 90.00 ... 10.
To leg AB 72m ... 1.85733	To leg AB 72m ... 1.85733
So sine ∠ A 42.01 ... 9.82607	So tan.t ∠ A 42.01 ... 9.95545
To leg BC 64.98m ... 1.81278	To leg BC 64.98m ... 1.81278
AC rad to find BC	**AB rad to find BC w.h the AC**
As rad.s 90.00 ... 10.	As sec.t ∠ A 42.01 ... 9.87062
To hyp AC 97m ... 1.98677	To hyp AC 97m ... 1.98677
So sine ∠ A 42.01 ... 9.82607	So tan.t ∠ A 42.01 ... 9.95545
To leg BC 64.98m ... 1.81284	To leg BC 64.98m ... 1.81284

A Rad to find B C with A C	A B rad to find B C with A B
As radius 90.00 ... 10.	As radius 90.00 ... 10.
To hyp A C 97m ... 1.98677	To leg A B 72m ... 1.85733
So sume ∠ A 12.04 ... 9.82607	So tan ∠ C 47.55 ... 9.95371
To leg B C 64.99 ... 1.81284	To leg B C 65.02 ... 1.81104

Ans ∵ { ∠ A 12.04 ; ∠ C 47.55 ; B C 65.02 }

WILLIAMS'S 83 1/2 PATENT

Given $\left\{\begin{matrix} AB\ 42m \\ \angle A\ 53.00 \end{matrix}\right\}$ Req^{d} the rest



AB radius to find BC	AB radius to find AC.
As radius 90.00 ..... 10.	As radius 90.00 ..... 10.
To leg AB 42m 1.62325	To leg AB 42m 1.62325
To tang ∠ C 53.00 ... 10.12289	To sec ∠ A 53.00 ..... 10.22054
To leg BC 55.74 ..... 1.74614	To hyp AC 69.79 ..... 1.84379

AC radius to find BC.	BC radius to find AC.
As radius 90.00 ..... 10.	As radius 90.00 ..... 10.
To hyp AC 69.79m 1.84379	To leg BC 55.74m 1.74614
To sine ∠ A 53.00 .... 9.90235	To cosec ∠ A 53.00 ..... 10.09765
To leg BC 55.74 ..... 1.74614	To hyp AC 69.79m. 1.84379

BC rad to find AB with AC	AC rad to find AB with BC
As cosec ∠ A 53.00 ... 9.90235	As sine ∠ A 53.00 ..... 9.90765
To hyp AC 69.79m 1.84379	To leg BC 55.74m 1.74614
As radius 90.00 ..... 10.	As radius 90.00 ..... 10.
To leg BC 55.74m. 1.74614	To hyp AC 69.79m. 1.84379

Ans^{r} $\left\{\begin{matrix} BC\ 55.74 \\ AC\ 69.79 \end{matrix}\right\}$

Given { AC 73 m / ∠A 44°.00 } Req.ᵈ the rest

∠A 44.00
∠B 90.00
134.00
180.00
∠C 46.00



A radius to find BC	A rad.ᵈ to find AB
As radius 90°.00 ..... 10.	As radius 90°.00 ..... 10.
To hyp.ˢ AC 73 m 1.86332	To hyp.ˢ AC 73 m 1.86332
So sine ∠A 44°.00 ..... 9.84177	So cosine ∠A 44°.00 ..... 9.85693
To leg BC 50.71 ..... 1.70509	To leg AB 52.51 ..... 1.72025
A rad.ᵈ to find AB with BC	**A rad.ᵈ to find AB with BC**
As cosine ∠A 44°.00 ..... 9.85693	As sine ∠A 44°.00 ..... 9.85823
To leg AB 52.51 ..... 1.72024	To leg BC 50.71 ..... 1.70509
So sine ∠A 44.00 ..... 9.84177	So cosine ∠A 44°.00 ..... 9.85693
To leg BC 50.71 ..... 1.70500	To leg AB 52.51 ..... 1.72025
AB rad.ᵈ to find BC with AC	**BC rad.ᵈ to find AB with AC**
As sec.ᵗ ∠A 44°.00 ..... 9.85693	As cosec.ᵗ ∠A 44°.00 ..... 9.84177
To hyp.ˢ AC 73 m 1.86332	To hyp.ˢ AC 73 m 1.86332
So tan.ᵗ ∠A 44°.00 ..... 9.98184	So tan.ᵗ ∠A 44°.00 ..... 10.01516
To leg BC 50.71 ..... 1.70509	To leg AB 52.51 ..... 1.72025

Ans.ʳ { ∠C 46°.00 / BC 50.71 / AB 52.51 }

WILLIAMS'S 83 1-2 PATENT

Rules for the Solution of the four Cases of Oblique angled plane triangles deduced from the last five of the Trigonometrical Theorems

Rule 1st In any plane triangle, if among the things given there be a side & its opposite angle to find the rest.

Then As a given side,
To the sine of its opposite angle,
So is any other given side,
To the sine of its opposite angle.

2nd Therefore to find an angle begin with a side opposite to a known angle. Also, As the sine of a given angle,
To its opposite side,
So is the sine of another given angle,
To its opposite side.

Therefore to find a side begin with an angle opposite to a known side.

2nd In any plane triangle, if the three things given, be two sides & their contained angle to find the rest; find the sum & diffce of the given sides, also the ½ sum of the unkn. angles.

Then. As the sum of the given sides
To the diffce of those sides
So is the tan. of ½ the sum of the unk. angles
To the tan. of ½ the diffce of those angles.

Add the ½ diffce of the angles to the ½ sum gives the greater angle. Subtract the ½ diffce of the angles from the ½ sum & it will give you the less angle.

3do. In any plane triangle if the three sides be known & the angles req. From the greatest angle suppose a line drawn perpendicular to the opposite side or base dividing it into two segments & the given triangle into two right angled triangles.

Then. As the base or sum of the segments
Is to the sum of the other two sides
So is the diffce of those sides
To the diffce of the segments of the base.

Add the ½ diffce of the segments to ½ the base gives the greater segment. Subtract the ½ diffce of the segments from ½ the base there remains the less segment.

Then in each of the triangles there will be known two sides & a right angle opposite to one of them therefore the angles will be found by Rule 1st.

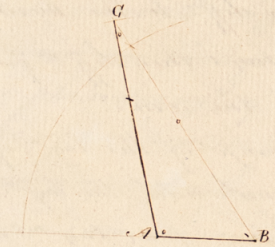
When two of the given sides are equal then a line drawn from the included angle perpendicular

to the other side bisects that side, & the angles being found in one of the right angled triangles will also give the angles of the other, &c

5th When the three sides of a triangle are given to find an angle. Take the diffce betn the two sides which contain the req. angle add this diffce to the third side & take ½ the sum. Subtract this diffce from the third side & take ½ the remr

To the arithmetical compt of the logms of the two sides which contain the req. angle add the logm of the ½ sum & ½ remr ½ the sum of these four logms will be the logarithmic sine of ½ the angle sought

Given { A B 41 m, A C = 92 m, ∠ B 55.00 } Req^d the rest



∠ B = 55.00
∠ C = 21.10
sum = 76.10
180.00
∠ A 103.50

To find ∠ C	To find B C
As side A C = 92 m . . . 9.90864	As sine ∠ B = 55.00 . . 9.90864
To sine ∠ B = 55.00 . . . 9.91336	To side A C = 92 m . . 1.96848
To side A B = 41 m . . . 1.61278	To sine ∠ A = 103.50 . . 9.98722
To sine ∠ C = 21.10 . . . 9.55766	To side B C = 110.2 m . . 2.04234

Ans^r { ∠ C = 21.10, ∠ A = 103.50, B C = 110.2 m }

Given { ∠A = 71.00, ∠C = 82.00, AB = 42m } Req.d the rest

∠A = 71.00
∠C = 82.00
153.00
180.00
∠B = 26.00



To find AC.	To find BC.
As sine ∠C = 82.00 . . . . 9.00326	As sine ∠C = 82.00 . . . . 9.00326
To side AB = 42m . . . 1.62325	To side AB = 42m . . 1.62325
To sine ∠B = 26.00 . . . . 9.64184	To sine ∠A = 71.00 . . 9.97567
To side AC = 18.55m . . . 1.26034	To side BC = 40.01m . . 1.60217

Ans.r { AC = 18.55m, BC = 40.01m, ∠B = 26.00 }

Given { AB = 73m, AC = 42m, ∠A = 50.06 } Req.d the rest.

AB = 73m
AC = 42m
sum = 115m
diff.ce = 31m

180.00
∠A 50.06
2)129.54
64.57 ½ sum ank.s



To find ∠s B and C.		To find BC	
As sum given sides = 115m	7,93921	As sin ½ G = 94.55	9,99851
To diff those sides 31m	1,49136	To side AB = 73m	1,86332
To tan ½ sum ank.s ∠s C.s 57	10,23034	So sin ∠A = 50.06	9,88489
To tan ½ diff those ∠s 29.58	9,79691	To side BC 56,20m	1,74972
∠G 94.55			
∠B 24.59			

Ans.s { ∠G = 94.55, ∠B = 24.59, BC = 56,20m }

Given { AC = 95 m / BC = 42 m / AB = 64 m } Req^d the rest

AB = 64 m
BC = 42 m
sum = 106 m
diff = 22 m
AC 95
175



To find the diff^e of the segments

As the base or sum of the segments 95 m 8,02228
To the sum of the other two sides 106 m 2,02536
So is the diff^e of those sides 22 m 1,34207
To the diff^e of the segments of the base 21,55 m .. 1,30963

17,275
475
AD 59,775
DC 35,225

In the △ ADB To find ∠B	In the △ CDB To find ∠B
As side AB = 64 m 8,19382	As side BC 42 m 8,37675
To sine ∠D = 90.00 10	To sine ∠D = 90.00 10
To side AD 59,775 m .. 1,77652	To side DC 35,225 m .. 1,54706
To sine ∠B = 69.04 9,97034	To sine ∠B 57.00 9,92380
90.00	90.00
∠ 159.04	∠ 33.00

By Rule 4th

To find ∠ B

AB 6 m/m
BC 12 m/m
diff 22 m/m
95
2)117 sum
58.5 ½ sum
95
22
2)73 rem^d
36.5 ½ rem^d

AB 6 m/m Ar Comp^t Log^m 8.19382
BC 12 m/m Ar Comp^t Log^m 8.37675
58.5 ½ sum Sin Log^m 4.76716
36.5 ½ rem^d Sin Log^m 4.56229
2)19.89992
63.02 Log sine of ½ B 9.94996
2
126.04 ∠ B

Ans: ∠ B = 126.04
∠ A = 20.56
∠ C = 33.00

WILLIAMS 83 1/2 & FENTON 14

WILLIAMS'S PATENT

Geography.

Definitions & Principles

Geography is the art of describing the figure magnitude & positions of the ..... several parts of the surface of the Earth.

The Earth is a spherical or globular figure & is usually called the terraqueous globe.

There are two points on the terraqueous globe called the Poles of the Earth which are diametrically opposite to one another one is called the north pole & the other the south pole

The Equator is a great circle on the Earth equally distant from each pole, dividing the terraqueous globe into two equal parts one called the northern hemisphere

in which is the north pole & the other containing the south pole is called the southern hemisphere.

Meridians are imaginary circles on the earth passing through both the poles, & cutting the equator at right angles.

Latitude is the distance of a place from the equator reckoned in Degrees & parts of Degrees on a meridian. On the north side of the equator it is north latitude & on the south side it is south latitude.

As latitude begins at the equator, where it is nothing; so it ends at the poles, where the latitude is greatest or 90 Degrees. Parallels of Lat^e are circles parallel to the Equator. Diff^ce of latitude is an arc of a meridian or the least distance of the parallels of Lat^e of two places; shewing how far one of them is to the northward, or southward, of ye other.

The diff^ce of latitude can never exceed 180 Degrees.

Longitude of any place on the earth is expressed by an arc of the equator, shewing the East or West distance of the meridian of that place from some fixed meridian, where longitude is reckoned to begin.

Diff^ce of longitude is an arc of the Equator, intercepted betw^t the meridians of two places, shewing how far one of them is to the Eastward, or Westward of the other.

As Long^e begins at the meridian of some place, & is counted from thence both eastward, & westward,

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till they meet at the same meridian on the opposite point of the Equator: therefore the diffᵉ of Longitude can never exceed 180 Degrees.

When two places have latitudes both north, or both south, or have longitudes both east or both west; they are said to be of the same, or of like name: but when one has north latitude, & the other, south; or if one has east longᵉ & the other west, then they are said to have contrary, different or unlike names.

The Horizon is that apparent circle, which limits or bounds the view of a spectator on the sea, or on an extended plain the eye of the spectator being always supposed in the center of his Horizon.

The tropic of cancer is a parallel of Latitude about 23½ from the Equator on the Northern Hemisphere

The tropic of capricorn is a parallel of Latᵉ about 23½ from the Equator in the Southern Hemisphere.

The arctic polar circle is a parallel of Latᵈᵉ about 23½ from the North Pole.

The antarctic Polar circle is a parallel of Latᵈᵉ about 23½ from the south pole.

A Zone is a space on the Earth contained between two parallels of Latitude.

The Earth is divided into five zones. viz the torrid zone, two frigid & two temperate zones.

The Torrid zone is that portion of the Earth, over every part of which the sun may be perpendicular at one time of the year or other.

This Zone is about 47° broad as it extends to about 23½ on each side of the Equator, being terminated on the North side by the tropic of cancer & on the South side by the tropic of capricorn.

The Northern Frigid Zone is that region about the North pole where the sun does not rise for some days when it is on the south side of the Equator nor set for some days when it is on the north side of it. This Zone is about 47° in diameter extending about 23½ every way from the north pole & it is terminated all round by the arctic polar circle.

The Northern Temperate Zone is that portion of the Earth which is between the Torrid Zone & the Northern Frigid Zone

This Zone is bounded on the south side by y tropic of cancer & on the north side by the arctic polar circle consequently it is about 43° broad it being about 23° 1/2 from the Equator to the tropic of cancer & about 66½ from the Equator to the arctic polar circle & 23½ taken from 66½ leaves 43° for the breadth of the Zone. The Southern Frigid Zone is that region about the South pole where the sun does not rise for some days when it is on the north side of the equator nor set for some days when it is on the south side of it.

This Zone is about 47° in diameter it extends 23½ every way from ..... the South pole & is termina

ted all round by the antarctic polar circle.

The Southern temperate Zone is that portion of the Earth which is between the torrid zone & the Southern frigid zone. This Zone is bounded on the north side by the tropic of Capricorn & on the south by the antarctic polar circle consequently it is about 43 broad it being about 23½ from the Equator to the tropic of Capricorn & about 66½ from the Equator to the antarctic polar circle & 23½ taken from 66½ leaves 43 for the breadth of the Zone.

A Climate is a Space of Earth which is contained between two parallels of Latitudes where the diffe bet the longest days in the two parallels is half a hour.

Of the natural division of the Earth.

The surface of the Earth is divided into Land & Water Land is devided into Continents & Islands & these have parts called Peninsulas, Isthmuss, Promontories, Points, Coasts & Capes or Headlands & Water is divided into Oceans, Seas, Gulfs, or Bays, Straits, Rivers & Lakes.

A Continent is a large track of Land which comprehends several contiguous, Countries, Kingdoms or States.

An Island is a part of Land of a moderate extent surrounded by Water.

A Peninsula is a part of Land almost surrounded by Water, being joined to some other Land by a very narrow neck.

An Isthmus is a narrow neck of Land which

joins a Peninsula to some other land.
A Promontory is a narrow track of high land which runs a considerable way into the sea. The Coast is the extremity or out line of any land where it is bounded by the sea. A Cape or Headland is a part of high land which runs farther into the sea than the neighbouring land does. It is generally called a point when the land is low. An Ocean is a vast collection of water separating Continents from one another.
A Sea is a branch of the ocean which runs a long way into a Continent or which separates an Island from a Continent. A Gulf or Bay is a part of the Ocean almost surrounded by land.
A Strait is a narrow passage which joins one sea ocean or gulf to another. Rivers are considerable streams of water collected from various sources & running in long narrow channels through the land till they fall into the sea. A Lake is a large collection of water in an inland place which has no communication with the sea unless it be by rivers.
There are two Continents called the Old & new worlds these are generally divided into five parts, the Old world contained three called Europe, Asia & Africa & the new world called North & South America. There are five oceans viz. The Northern, the Atlantic, the Pacific, the Indian & the Southern Oceans.

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The Northern Ocean lies to the northward of Europe, Asia & America about the North Pole.

The Atlantic Ocean has the Northern Ocean & Greenland on the North, Europe & Africa on the East the Southern ocean on the south and America on the West. The Pacific Ocean has Asia & the Northern Ocean on the north, America on the East the Southern Ocean on the South and new Holland and new Zealand on the west.

The Indian Ocean has Asia on the North Asia new Holland and new Zealand on the East the Southern Ocean on the south and Africa on the West.

The Southern Ocean lies to the Southward of Africa and America about the South Pole.

Of the political division of the Earth.

By the Political division of the Earth is understood the different Countries Empires Kingdoms States and other denominations established by men.

Of Europe

Europe is bounded by the Northern Ocean on the North by Asia on the East by the mediterranean sea on the south and by the Atlantic Ocean on the west. It lies between the Lat^s of 36° & 72° north & between 11° of west Long^e and 65° E from Greenwich being about 2200 m from North to South and about 2300 miles from East to West.

The Countries their position with respect to the middle parts of Europe the chief cities principal rivers with their courses the most noted mountains and what part of the country they are in are exhibited in the following table.

Countries	Position	Chief cities	Rivers	Courses	Mountains
E Turkey	S E	Constantinople	Danube	E	Argentum N d
K Poland	Mid	Warsaw	Vistula	N N W	Carpathian S d
E Muscovy	N E	Moscow	Volga	S to S	Boglowy S d
E Russia		Petersburg	Nieper	S	Riphian Wd
K Sweden	N	Stockholm	Dalebarlia	E	Dofrine W d
K Norway	N N W	Bergen	Glama	S	Dofrine W d
K Denmark	N W	Copenhagen	Eyder	W	Carpathian N d
K Hungary	Mid	Presburg	Danube	S E	Alps S d
E Germany	Mid	Vienna	Danube	E	Alps N d
Italy	S	Rome	Po	E	
			Tyber	S	Apennine Mid
R Switzerland	Mid	Bern	Rhine	W	Alps S d
Netherlands	W	Bruſsels	Maeſe	N	
R Holland	W	Amsterdam	Rhine	N N W	
K France	W	Paris	Loire	N to W	Pyrenees S W
			Rhine	S	Alps E d
K Spain	S W	Madrid	Tagus	W	Pyrenees N E
K Portigal	S W	Lisbon	Tagus	W	C Rocca W
K England	W	London	Thames	E	Malvern N W
K Scotland	W	Edenburg	Forth	E	Grampian N d
K Ireland	W	Dublin	Shannon	S W	Knockpatric W

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K. Prussia	&	Berlin
K. Bohemia	&	Prague
K. Sardinia	&	Turin in Italy
K. Scotes	&	Naples in Italy

The principal Seas, Gulfs & Bays in Europe are.
The Mediterranean Sea, having Europe on the N. & Africa on the South.
The Adriatic Sea, between Italy & Turky
The Euxine, or Black Sea in Turky betn Europe & Asia
The White Sea in the N. N.W. parts of Muscovy
The Baltic Sea betn Sweden, Denmark, & Poland.
The German Ocean or Sea, betn England and France
St Georges Channel betn Britain and Ireland
The Bay of Biscay formed betn France and Spain
The Gulf of Bothnia, in the N.E. parts of Sweden
The Gulf of Venice in the N.W. end of the Adriatic Sea.
The Principle Islands in Europe, are
The British Isles, Viz. Great Britain, Ireland, Orkneys & Western Isles.
The Spanish Isles, Majorca, Minorca, Ivica, in the mediterranean Sea
Italian Isles, Italy, Sardenia, Corsica, Sipara, in the Med Sea
Turkish Isles, Candia, Archipelago Isles, in the Med Sea
Swedish Isles, Gothland, Oeland, Man, Rugen in the Baltic Sea.
Danish Isles, Zealand in the Baltic Sea Ireland, Faro Isles, & W. Greenland in the Northern Ocean
Azores Isles in the Atlantic Ocean belonging to Portugal

Of Asia

Asia is bounded by the northern Ocean on the north by the Pacific Ocean & Beherings Straits on the East by the Pacific Ocean & Indian on ye south & by Europe ye Red Sea & ye Indian Ocean on the west. It lies betn ye 1st & 78th Degree of Land & betn N & S betn the Longd of 26° & 126° E from Greenwich & is in length from East to West is about 7800 miles and is in breadth from north to south (exclusive of ye Isles) is about 4680 m. The Countries their positions with respect to the middle parts of Asia the chief Lakes principal rivers with their courses the most noted mountains & what part of the country they are in are exhibited in the following Table.

Countries	Position	Chief Lakes	Rivers	Courses	Mountains
E. China	SE	Pekin Nankin Canton	Yellow R Kiam Ta	N & E E SE.	Herocoram Nd Damasian Wd
K. Korea	E		Yalu	S	Shanalin
Chinese Tartary	E	Chynian	Yamour	N & E	Pongwanshan
Mongalia	Mid	Hudak	Yellow R	N & W	
K. Thibet	Mid	Eskerdu	Yara	E	Kanks
Bukharia or Usbecs	Mid	Samarkand	Amu	N.W	Belurtag
Harazm	Mid	Urgenz	Amu	S.W.	Izder
Kalmucks	Mid		Jekis		
E. Siberia	N	Tobolski Astracan	Oby Jenisha	N N.N.W	Stolp
E. Turkey	W	Smyrna	Euphrates	S.E.	Taurus
K. Syria	W	Aleppo	Euphrates	S	Lebanon
Arabia	SW		Euphrates	S.E.	Gabel el ared
K. Persia	S	Ispahan	Oxus Jaxes	W S.W	Caucasus Tauras
India Westr	S	Agra	Indus	S.W	Caucasus
the Ganges		Delli	Ganges	S.W	Balagat
India East of the Ganges	S	Ava Pegu Siam Cambodi	Domea Meeon Menan Ava	S. S. S. S.	Damascene

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The principal Seas, Gulfs, & Bays in Asia are
Caspian Sea, quite surrounded by Siberia on the north,
Korazm east, by Persia on the South & by Georgia on the W.
Korean Sea, betn Korea & the Islands of Japan.
Yellow Sea, betn China & the Japan Isles.
Gulf of Cochin China, on the borders of Tonquin & Cochin
china. Bay of Siam, formed by the countries of Siam & Malacca.
Bay of Bengal, betn India east, & India west of the Ganges
Gulf of Persia, having Persia on the N.E. & Arabia on the S.W.

The principal Islands, belonging to Asia are
Ladrone, or Marian Isles, whose chief island is Guam

Japan isles	Ch isles	Japan	Bongo	Tonsa
	Ch cities	Jeddo	Bongo	Tonsa
Philippines	Ch isles	Luconia	Mindanao	Samar
	Ch cities	Manilla	Mindanao	
Chinese isles	Ch isles	Formosa	Hinan	Makao
	Ch cities	Tayowanfu	Tan	Makau
Moluccos	Ch isles	Celebes	Gilolo	Ceram
	Ch cities	Macasser	Gilolo	Ambay
Sunda isles	Ch isles	Borneo	Sumatra	Java
	Ch cities	Banjar	Achin	Batavia

The Andaman Isles to the west of Siam.
Nicobar Isles west of Malacca.
Maldive Islands to the S W of Bisnagar.
The Island of Ceylon S E of Bisnagar; the chief city is Candy, or Candy Uta.

Of Africa.

Africa is bounded by the Mediterranean Sea on y North by the red sea & the Indian Ocean on the East by the Southern Ocean on the South & by the Atlantic Ocean on the west. It is situated betn the Lat of 38° North & 35° South & between the Longs of 18° West & 52 East it is about 3200 miles long from east to west & about 2300 miles broad from north to south. The Countries chief cities the most noted rivers & mountains are contained in the following Table.

Countries	Position	Chief cities	Rivers	Course	Mountains
E. Morocco	N W	Fez	Mulvea	N	Atlas
K Algiers	N	Algiers	Saffran	N	Atlas
K Tunis	N	Tunis	Megrada	N	Atlas
K Tripoli	N	Tripoli	Salines	N E	Atlas
K Barca	N	Docra			Mteics
K Egypt	N E	Cairo	Nile	N	Guamadel
E Abyssinia or Ethiopia	E	Ambamgjara	Nile	N	
Ajan	E	Adea	Madadara	S	
Zanguebar	E	Melinda	Cuama	E S E	
Sofala	S E	Sofala	Amara	E	Amara
Tonades Natal	S E	Natal	St Espirit	E	Amara
Cafraria	S	Cape Town	St Christopha	E	Table
Madagascar	S S W		Angri	W	Sunda
K Benguela	S S W	Benguela	Negros	W	Sunda
K Angola	S W	Loando	Coanza	W	Sunda
K Congo	S W	St Salvador	Zaara	S W	Sunda
K Loango	S W	Loango	Zeba	S W	St Espirit
Biafara	S W	Biafara	Camerons	S W	St Espirit
K Benin	S W	Benin	Formesa	S W	

Guenea	S W	Cape Coast	Volta	S	Sierra Leone
Mandingo	W	James Fort	Gambia	W	C. Verd
Janhaga	W	Janhaga	Senegal	N W	
Bildulgerid	Mid E	Darà	Dara	S	Atlas
Zara	Mid	Zuenzega	Nubia	E	
Nubia	Mid	Nubia	Nubia	E	
Negroland	Mid	Tombuk	Nigar	W	
Ethiopia inter	Mid	Chaxume	Nigar	W	Luna
Monomugi	Mid S	Merango	Cuama	E S E	Luna
E Monomotapa	Mid S	Morgar	Amara	S E	Amara

The principal Seas, Gulfs, & Bays in Africa are,
The Red Sea, betw. Africa & Asia: It washes the coast of Arabia on the Asiatic side, & the Coasts of Egypt & Abyssinia on the African side.
Mosambique Sea, betw. Africa & the island of Madagascar eastward.
Saldanna Bay in Caffaria, on the Ethiopic Ocean.
Bight of Benin on y coast of Guinea in y Ethiopic Ocean.

The principal African Islands are

	Chief Isle	Chief Town	Situation
Madeira Isles	Madeira	Princepal	N Atlantic Ocean
Canary Isles	Canaria	Palma	N Atlantic Ocean
C Verd Isles	St Jago	St Jago	N Atlantic Ocean
Ethiopian Isles	St Helena		Ethiopic Ocean
Comora Isles	Johanna	Demani	Indian Ocean
Sokotora Isles	Zocotora	Calansia	Indian Ocean
Almirante Isles	but little known		Indian Ocean

The Island of Madagascar one of y largest in the world lies on y Indian Ocean: It is divided into a multitude of little states, some of them formed by the European privateers & their successors descended from a mixture with y natives.
The Islands of Bourbon & Mauritius lie in y Indian Ocean, Est east of Madagascar: these belong to the French. The Madeiras & Cape de Verd Isles belong to the Portuguese. The Canary Isles to Spain; St Helena to England.

North America

North America is bounded by the Northern Ocean on the N. by the Atlantic Ocean on the E. by the Gulf of Mexico & South America on the S. & by the Pacific Ocean on the W. It is situated bet.n the Lat.des of 10° 30′ & 70° N. & bet.n the long.es of 55° & 167° W. being about 3000 miles long from east to west and about 1200 miles broad from North to South. The Countries their Capital Cities the most noted rivers & mountains are exhibited in the following Table.

Countries	Position	Chief cities	Rivers	Course	Mountains
California	W.	St.a Juan			
New Mexico	S.	St.a Fe	N. River	S.S.E.	
Old Mexico	S.W.	Mexico	Rumuco	E.	
Louisiana	S.	New Orleans	Mississippi	S.	
Florida	S.	St.a Augustine	St.a John	N.E.	Apalachian
Georgia	S.S.E.	Savannah	Alatamaha	E.S.E.	Apalachian
Carolina	S.E.	Charles Town	Ashly	S.E.	Apalachian
Virginia	E.	James Town	Potomack	S.E.	Apalachian
Maryland	E.	Annapolis	Potomack	S.E.	Apalachian
Pensylvania	E.	Philadelphia	Delaware	S.	Apalachian
Jerseys	E.	New York	Albany	S.	
New England	E.	Boston	Connecticut	S.	
Nova Scotia	N.E.	Halifax	St.a John	S.S.E.	Ladies
Canada	Mid.	Quebec	St. Lawrence	N.E.	
New Britain	N. & E.	Fort Rupert	Rupert	W.	
New Wales	N.	York Fort	Nelson	W.	

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Of South America.

South America is bounded by North America, the gulf of Mexico & the caribbean sea on the North; by the Atlantic Ocean on the East; by the Southern Ocean on the South & by the Pacific Ocean on the west.

It is situated between the lat^{de} of 10° N & 56° S. bet^{n} the long^{de} of 35° & 77° west being about 2500 miles long from east to west & about 3000 miles broad from North to South. The several countries the chief cities the principal rivers & mountains are contained in the following Table.

Countries	Position	Chief cities	Rivers	Runs	Mountains
Terra Firma	N	Panama	Oronoque	N E	
Peru	W	Lima	Chuquimayo	W & N	Andes
Chili	S W	St Jago	Valparaiso	W	Andes
Patagonia	S		Defeguadero	S	Andes
La Plata	S E	Buenos Ayres	La Plata	S	Andes
Paraguay	Mid	Assumption	Paraguay	S	
Brasil	E	St Salvador	Rio Real	N E	
Amazonia	Mid		Amazons	E	
Guiana	N E	Surinam	Esquebe	N & E	

The principal Seas Gulfs & Bays in America. The Caribbean Sea bounded by Terra Firma on the South and a range of Islands on the North and East. Gulf of Mexico formed by Old Mexico Louisiana and Floridas.

Problem 2nd

Given the lat^e of one place & the diff^ce of lat bet^n it & another place. Req^r the lat^e of the latter place.

Case 1st

When the given lat^e & diff^ce of lat have the same name

Rule To the given lat^e add the degrees & minutes in the diff^ce of lat^e that sum is the other lat of the same name.

Case 2nd

When the given lat^e & diff^ce of lat^e have contrary names

Rule Take the diff^ce bet^n the given lat^e & the degrees & minutes in the diff^ce of lat^e .... the rem^r is of the other lat^e of the same name with the greater

Example 1st

A Ship from the lat^e 38° 20' N sails N till her diff of lat^e is 20. 56: What Lat^e is she come to

Lat^e from	38. 20	N
Diff^ce Lat^e	20. 56	N^m
Lat in	59. 16	= 59 76

Example 1st

A Ship from the lat^e 56. 15 N sails S till her diff^ce of lat^e is 2. 04: What lat^e is she come to

Lat^e from	56. 15	N
Diff^ce lat^e	2. 04	S
Lat^e in	54. 11	N = 851 m

Example 2nd

A Ship from Glasgow sails N till her diff^ce of lat^e is 2. 57 What lat^e is she come to

Glasgow lat^e	55. 52	N
Diff^ce lat^e	2. 57	N^m
Lat in	98. 49	= 5929

Example 2nd

A Ship from C: Coronation sails N till her diff^ce of lat^e is 20. 15 What lat^e is she come to

C: Coronation lat^e	22. 05	S
Diff^ce lat^e	20. 15	N^m
Lat in	1. 50	= 110

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Problem 3rd

Given the long.s of two places
Req.d their diff.ce of long.e

Rule

If the long.s are of the same name their diff.ce is the diff.ce of long.e req.d
But if the long.s are of different names their sum gives the difference of long.e
And if this sum exceeds 180, take it from 360 & there remains the diff.ce of long.e

Example 1st

What is the diff.ce of long.e betn Caen & Bristol
Caen's long.e 0° 17' W
Bristol's long.e 2° 30' W
Diff.ce Long.e 2° 13' W = 133m

Example 2nd

What is the diff.ce of long.e betn Amsterdam & Batavia
Amsterdam long.e 75° 15' E
Batavia's long.e 106° 45' E
Diff.ce long.e 31° 30' E
60
1890 m

Example 3rd

What is the diff.ce of long.e betn Banks's Isle & Isle Apention
Apention's long.e 14° 16' W
Banks's long.e 172° 40' E
186° 56'
360° 00'
Diff.ce long.e 173° 04' = 10384m

Example 4th

What is the diff.ce of long.e betn Pekin Fort & Pegu
Pekin Fort long.e 64° 10' W
Pegu long.e 96° 58' E
Diff.ce long.e 160° 48' W
60
9480 m

Problem 4th

Given the long.e of one place & the diffce of long.e betn that & another.
Reqr the long.e of the second place.

Rule

If the given long.e & diffce of long.e are of a contrary name their diffce is the long.e sought & is of the same name with the greater.
But if the given long.e & diffce of long.e are of the same name, the sum is the long.e sought of the same name with the given place.
And if this sum is greater than 180° take it from 360, remains the long.e reqd of a contrary name to that of the given place.

Example 1st

A Ship from the long.e 38°.56 E sails Eastward till her diffce of long.e is 13°.46 What long.e is she in.

Long.e from 38.56 E.
Diffce Long.e 13.46 E.
Long.e in 52.42 E. = 3162 m

Example 2nd

A Ship from C. Horn sails W.b till her diffce of long.e is 12°.16 What long.e is she in

C. Horn is long.e 67°.21 W
Diffce long.e 12°.16 W
79°.37 W
360.00
Long.e in 160°.23 E. = 9923 m

Example 3rd

A Ship from the long.e 113°.14 E sails Westward till her diffce of long.e is 13°.04 What long.e is she in.

Long.e from 113.14 E.
Diffce long.e 13.04 W
Long.e in 70.10 E. = 4210 m

Example 4th

A Ship from Ramhead sails Eastward till her diffce of long.e is 2°.13 What long.e is she in

Ramhead long.e 4°.15 W
Diffce long.e 2.13 E.
Long.e in 2.02 W. = 122 m

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Of Winds.

Wind is a stream or current of air which may be felt; it usually blows from one part of the horizon to its opposite ..... part.

The horizon, beside being divided into 360 degrees, like all other circles, is by mariners supposed to be divided into four quadrants, called the north-east, north-west, south-east, south-west quarters; each of these quarters they divide into eight equal parts, called points, and each point into four equal parts, called quarter-points.

So that the horizon is divided into 32 points, which are called rhumbs, or winds; to each wind is assigned a name, which shews from what point of the horizon the wind blows.

The points of North, South, East, & West, are called cardinal points; and are at the distance of 90 degrees, or 8 points, from one another.

Winds are either constant or variable, general or particular. Constant winds are such as blow the same way, at least for one or more days; & variable winds are such as frequently shift within a day.

A General wind is that which blows the same way over a large track of Earth, almost the whole year. A Particular wind is that which blows in any place, sometimes one way, & sometimes another, indifferently.

Of the Tides.

A Tide is that motion of the water in the seas & rivers, by which they are found regularly to rise & fall. Daily experience shews, that all bodies thrown upwards from the Earth, fall down to its surface in perpendicular lines; and as lines perpendicular to the surface of a sphere tend towards the center, therefore the lines, along which all heavy bodies fall, are directed towards the Earth's center. All these bodies apparently fall by their weight, or gravity; therefore the law by which they fall, is called the Law of Gravitation.

Now as bodies by their gravity fall towards the Earth, it is not improper to say the Earth attracts those bodies; & therefore in respect to the Earth, the words gravitation & attraction may be used one for the other; as by them is meant no more than the power, or law, by which bodies tend towards its center.

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The incomparable Sir Isaac Newton, by a sagacity peculiar to himself, discovered from many observations that this law of gravitation or attraction was universally diffused throughout the solar system; and that the regular motions observed among the heavenly bodies were governed by this same principle; so that the Earth & Moon attracted each other, & both of them are attracted by the ... Sun.

He discovered also that the force of attraction exerted by these bodies one on the other, was less & less as the distance increased, in proportion to the squares of those distances; that is, the force of attraction at double the distance was four times less, at triple the distance nine times less, at quadruple the distance sixteen times less, & so on.

Now as the Earth is attracted by the Sun & Moon, therefore all the parts of the Earth will not gravitate towards its center in the same manner as if those parts were not affected by such attractions. And it is very evident that, were the Earth entirely free from such actions of the Sun & Moon, the ocean being equally attracted towards its center, on all sides, by the force of gravity

would continue in a perfect stagnation without ever ebbing or flowing.

But since the case is otherwise, the water in the ocean must needs rise higher in those places where the sun & Moon diminish its gravity, or where the sun & moon have the greatest attraction.

As the force of Gravity must be diminished most of those parts of the Earth to which the Moon is nearest, or in the Zenith; that is, where she is vertical or perpendicularly over, & consequently, where her attractions is most powerful; therefore the waters in such places will rise higher, & it will be full sea or flood in such places.

The parts of the Earth directly under the moon, & also those in her Nadir, viz such places as are diametrically opposite to those where the moon is in the Zenith, will have the flood, or high water, at the same time.

Those parts of the Earth where the moon appears in the horizon, or is 90° distant from the Zenith & nadir, will have the ebbs or low waters.

The time of high water is not precisely at the time of the Moon's coming to the meridian, but about an hour after.

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The tides are greater than ordinary twice every month; that is, about the times of new & full moon: these are called Spring-Tides.

The tides are less than ordinary twice every month; that is, about the times of first & last quarters of the moon: & these are called Neap Tides.

When the Moon is in her Perigæum, or nearest approach to the Earth, the tides increase more than in the same circumstances at other times.

The Spring-Tides are greater about the times of the Equinoxes, that is, about the latter ends of March & September, than at other times of the year; & the neap tides then are less.

The following observations have been made on the rise of the Tides.

1st The morning-tides generally rise higher than the evening-tides.

2nd The new & full moon spring tides rise to different heights.

3d In winter the morning tides are highest.

4th In summer the Evening tides are highest.

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Of Chronology.

Chronology is the art of estimating, & comparing together, the times when remarkable events have happened, such as are related in history.

An Era, or Epocha, is a time from which some memorable transactions occurred; & from which some nations date & measure their computations of time.

	Years of the Julian Period	Years before Christ
Some have dated their events from the creation of the world, & suppose it to have happened	710	4004
Others from the deluge or Flood	2366	2348
The Greeks, from their Olympiads of 4 years each	3938	776
The Romans, from the building of Rome	3961	753
The Astronomers, from Nabonassar king of Babylon.	3967	747
Some Historians, from the death of Alexander the great	4390	324
The Christians, from the birth of Christ	4713	AD
The Mahometans, from the flight of Mahomet, called the Hegira	5335	622

In order to assign the distance between those and other events, the ancients found it necessary to have a long measure of time, the limits of which were naturally pointed out to them by the return of the seasons; & this interval they called a year. The most natural division of the year appeared to them to be the returns of the new moon; & as they observed 12 new Moons to happen within the time of the general return of the seasons, they therefore first divided the year into 12 equal parts, which they called months; & as they reckoned about 30 returns of morning & evening between the times of new Moon & new Moon therefore they reckoned their month to consist of 30 days, & their year, of 12 months, to contain 360 days; & this is what is generally understood by the lunar year of the ancients. But in long they time it was found, that this year did not agree with the course of the sun, the seasons gradually falling later in the year, than they had been formerly observed; this put them upon correcting the method of estimating their year, which they did from time to time, by taking a day or two from the months, as often as they found it too long for the course of the moon; & by adding a month, called an intercalary month, as often as they found 12 lunar months to be too short

per the return of the four seasons and parts of the Earth. This kind of year so corrected from time to time by the priests, whose business it was, is what is to be understood by the lunisolar year, which was anciently used in most nations & is still among the Arabs & Turks.

As a great variety of methods was used in different countries to correct the length of the year, some by intercalating days in every year, & others by inserting months & days in certains, etaens, or periods of years; & those different methods being observed by some eminent men to create a considerable difference in the accounts of time kept by neighbouring nations, introducing a confusion in the chronological order of times, they therefore invented certain periods of years, called cycles, with which they compared the most memorable occurrences.

At length Julius Cæsar observing the confusion that this variety of accounts occasioned, & knowing that his order, as Emperor of the Romans, would be followed by a considerable part of the world; he therefore, about 50 years before the birth of Christ, decreed that every fourth year should consist of 366 days & the three three of 365 days each.

This he did in consequence of the information given him by Sosigenes, an eminent mathematician of Alexandria in Egypt; for at that time the philosophers of the Alexandrian school knew from a length of experience, that the year consisted of about 365 days & a quarter; & this was the reason of ordaining every fourth year to consist of 366 days, thereby compensating for the quarter day omitted to each of the preceding three years. This method, called the Julian account, or Old Style, continued to be used in most ..... Christian States until the year 1582.

The Astronomers, since the time of Julius Cæsar, have found that the true length of the solar year, or common year, is 365 days, 5 hours, 48 minutes, 55 seconds nearly, being less than the Julian, of 365 days, 6 hours, by about 11 minutes, 5 seconds, which is about the 130th part of 86,400 the seconds contained in a day; so that in 130 Julian years there would be one day gained above 130 solar years; & in 400 Julian years there would be gained 3 days, one hour, 33 minutes, 20 seconds, consequently one day

omitted to every 130 common years, would bring the current account of time to agree very nearly with the motion of the Sun.

In the year of our Lord 325, when the Council of Nice settled the day for the celebration of Easter, the Vernal Equinox (that is, the day in the spring when the Sun rose at six & set at six) happened on the 21st of March; but about the year 1580 the Vernal Equinox fell on the 11th of March; making a diff.ce of about 10 days.

Pope Gregory the 13th, who was pope at that time, observing that this diff.ce of time in the falling out of the Equinox would affect the intention of the Nicene Council, concerning the time of the year appointed by them for the celebration of Easter; he therefore, in the year 1581, published a Bull, ordaining that in the year 1582. the 5th of October should be called the 15th & so on, thus the 10 days taken off would cause the time of the Vernal Equinox, to fall on the 21st of March, as at the time of the Nicene Council;

and because a little more than three days were gained in every 400 years by the Julian account; therefore to prevent any future difference, every centenary, or number of 100 years, not divisible by 4, such as 1 hundred, 18 hundred, 19 hundred, &c. should contain only 365 days which by the Julian account should have contained 366 days; and the centuries divisible by 4 such as 16 hundred, 20 hundred, 24 hundred, &c. should be leap years of 366 days and thus the three days would be omitted, which the anticipation of the equinoxes would gain in 400 years; the small excess of 1 hour, 53 minutes, 20 seconds, not amounting to a whole day in less than 5082 years, being rejected as inconsiderable; the intermediate years to be reckoned as they used, in the Julian or Old Style.

This Pope's alteration, called the Gregorian or New Style, was received in most of the Christian States: but some at that time chose to continue the Julian: among whom were the English; & they in the year 1752 reformed their account, & introduced among themselves a new one, that nearly corresponds with the Gregorian

A certain length of the year being once settled, & a regular account of time in consequence of it being so settled, as to correspond invariably to the seasons, it was natural for the States who received such account, to put to it a register of the days in each month, & to note the days when any remarkable occurrence was to be commemorated; & such register has obtained the name of Calendar.

The Calendar now in use among most of the Christian states consists of 12 months called January, February, March, April, May, June, July, August, September, October, November, December, these months are called civil; the number of days in each month may be readily remembered by the following rule.

Thirty days has September, April, June, & November. February hath 28 alone, All the rest have 31.

When the year consists of 365 days. But in every fourth, which consists of 366 days February hath 29. This additional day was intercalated after the 24th day of February, which in the Old Roman calendar was called the sixth of the calends of March, & being this year reckoned twice

over the year was called Bissextile or leap year. Besides the months, time is also divided into weeks, days, hours, minutes, &c. a year containing 52 weeks, a week 7 days, a day 24 hours, an hour 60 minutes, &c. In the Calendars it has been usual to mark the seven days of the week with the seven first letters of the Alphabet, always calling the 1st of January A, the 2nd B, the 3d C, the 4th D, the 5th E, the 6th F, the 7th G, & so on, throughout the year; & that letter answering to all the Sundays for a year, is called the Dominical letter. According to this disposition, the letter answering to the first day of every month in the year will be known by the following rule.

At Dover dwells George Brown Esquire
Good Caleb Finch, & David Frier.

Where the 1st letter of each word answers to the letter belonging to the 1st day of the months in the order from January to December.

A year of 365 days contains 52 weeks & 1 day; & a leap year has 52 weeks & 2 days; therefore 1st & last days of a common year fall on the same week day; suppose 2d Monday; then the next year begins on a Tuesday, the next year on Wednesday, and so on to the eighth year,

which would be on monday again did every year
contain 365 days; Also the Dominical letter would
run backwards throughout all the seven letters:
But this round of seven years is interrupted by
the leap years: For then February having a 29th
day annexed, the 1st Dominical letter in March
must fall a day sooner than in common years;
so that leap year has 2 Dominical letters, the
one supposing G serving for January & Febru-
ary, & the other, which is the preceding letter
F serving for the rest of the sundays in that
year. The Solar Cycle, or cycle of the sun, is a
period of 28 years, in which all the varieties of
the dominical letters will have happened,
& they will return in the same order as they
did 28 years before. At the birth of Christ
9 years had passed in this cycle. For the
changes, were all the years common ones,
would be seven; but the interruption by leap
years being every fourth year; therefore the
changes would be 4 times 7, or 28 years. This
return of the Dominical letter is constant in
the Julian account.

But in the Gregorian, where among the complete centuries or hundredth years, only every fourth is leap year; the other three hundred years, which according to the Julian would be leap years, are by the Gregorian only common years of 365 days in these all the letters must be removed one place forward in addirect order & either year instead of having two Dominical letters (as suppose D, C will have only one (as D, the Dominical letters moving retrograde.

The Lunar Cycle, or cycle of the Moon & sometimes called the Metonic Cycle from Meton, an Athenian, who invented it about 432 years before the time of Christ, is a period of 19 years, containing all the variations of the days on which the new & full moons happen; after which they fall on the same days they did 19 years before.

The Prime or Golden Number, is the number of years elapsed in this cycle. At the birth of Christ, the golden number was 2. For many years after the Nicene Council, it was thought that 19 Solar years, or 228 solar months, were exactly equal to 235 synodical, or lunar months, & that the same yearly golden number set in their calendars against the days when the new moons happened throughout one lunar cycle.

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would invariably serve for the new Moons of corresponding years throughout every successive lunar cycle. But later obsrs shew, that this cycle is less than 19 years, by a little more than 1 hour, 28 minutes; therefore, the new Moons will, in a little less than 311 years, happen a day earlier than by the Metonic account; & consequently all the festivals depending on the new Moons, will in time be removed into other seasons of the year than those which they fell in at their first institution: thus the new Moons in the year 1750 happened above 4½ days earlier than the times shewn by the calendar. But were the Golden Numbers, which once prefixed to the prospective new moon days in a Metonic period, to be set a day earlier at the end of every 310 or 7 years, a pretty regular correspondence might be preserved betn the solar & lunar years.

The Epact of any year is the moon's age at the beginning of that year; that is, the days past since the last new moon. The time betn new Moon & new Moon is in the nearest round numbers 29½ days; therefore the lunar year consists of 12 lunations must be equal to 354 days which is 11 days less than the solar year of 365 days

Now supposing the solar & lunar years to begin together, the epact is 0; the beginning of the next solar year, the epact is 11; the 3d year, the epact is 22; the 4th 33, &c. But when the epact exceeds 30, an intercalary month of 30 days is added to the lunar year, making it consist of 13 months; so that the epact at the beginning of the 4th year is only 3, the 5th 14; the 6th 25, the 7th 36, or only 6, on account of the intercalary month; & so on to the end of the cycle of 19 years; at the expiration of which the same epacts would run over again, were the cycle perfect; & the epact would always be 11 times the prime.

By the Nicene Council it was enacted,

1st That Easter day should be celebrated after the vernal equinox, which at that time happened on the 21st of March.

2nd That it should be kept after the full, or 14th day of that Moon which happened 1st after the 21st of March in common years, & 1st after the 20th of March in leap years. 3rd That the Sunday next foll[owing] the 14th or day of full moon, should be Easter

sunday which must always fall betn the 20th or 21st of March, & 25th of April. The Moons souther-ing at any place is the time when she comes to the meridian of that place, which is every day taken by about ½ of an hour, because 24 hours in a day, being divided by 30, the number of times which she passes the meridian betn new Moon & new Moon give ½ = 12 for the retardation of her passage over the meridi-an in one day. The Sun & Moon come to the meridian at the same time on the day of the change or at new Moon, also the new moon comes to the opposite part of the same meridian, when she is in opposition, or at full moon.

Hence betn new & full ..... she comes to the meridian at midnight, & when past the full, after midnight, or in the morn-ing. The Roman Indiction is a cycle of 15 years, used by the ancient Romans for the times of taxing the provinces. Three years of this cycle were elapsed at the birth of Christ. The Dionysian Period is a cycle of 532 years

arising by multiplying together 28 and 19, the solar & lunar cycles; it was contrived by Dyonysius Exiguus, a Roman abbot about the year of Christ 527, as a period for comparing chronological events. The Julian period contains 7980 years; it arises by multiplying together 28, 19, 15, the cycles of the Sun, Moon, & Indiction.

This was also contrived as a period for chronological matters; & its beginning is put 7 years before the usual date of the creation.

On the principles laid down in the preceding articles depend the solution of the following problems

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Chronological Problems.

Problem 1st

To find whether any given year is leap year.

Rule

Divide the given year by 4 if 0 remains it is leap year if 1 2 or 3 remains it is so many years after. Observing that the years 1800, 1900, 2100 &c are common years.

Example 1st

Is the year 1816 leap year.

4) 1816 (454

Rem^{r} 0 it is leap year

Example 2nd

Is the year 1819 leap year

4) 1819 (454

Rem^{r} 3 years past leap year.

Problem 2nd

To find the years of the Solar Lunar & Indiction cycles

Rule

To the given year add 9 for the Solar 1 for the lunar 3 for the Indiction. Divide the sums in order by 28, 19, 15, the rem^{rs} on each shews the year of its respective cycle.

Example 1st

Req^{d} the years of the solar lunar & indiction cycles for the year 1816

1816	1816	1816
9	1	3
28) 1825 (65	19) 1817 (95	15) 1819 (121

Rem^{r} 5 Solar cycle. Rem^{r} 12 Lunar cycle or golden N^{o} rem^{r} 5 indict cycle

Whereby it appears that the year 1816 is the { 6th year of the 66th Solar cycle / 12th year of the 96th Solar cycle / 5th year of the 122nd indict cycle } since the birth of Christ.

Problem 3rd

To find the Dominical letter till the year 1800.

Rule

To the given year add its fourth part, divide the sum by 7 the rem^r taken from 7 leaves the index of index of the letter in common years, reckoning A 1, B 2, C 3 &c. But in leap year this letter & its preceding one in the retrograde order which these letters take are the Dominical letters.

Example 1st

Find the Dominical letter for the year 1816

1816
454
7 / 2270 324
Rem^r 2

Then 7 − 2 = 5 = E so E is for the Dominical letter

Example 2nd

Find the Dominical letter for the year 1819

1819
454
7 / 2273 324
Rem^r 5

Then 7 − 5 = 2 = C so C is the Dominical letter.

Problem 4th

To find the Epact till the year 1900.

Rule

Multiply the golden number for the given year by 11 & divide the product by 30 from the rem^r take 11 & it will give the Epact. If the rem^r is less than 11 add 19 to it & it gives the Epact

Example 1st

Find the Epact for the year 1816

The Golden N^o 12
Multiply by 11
30 / 132 4
120
Remains 12
Subtract 11
7 Epact

Example 2nd

Find the Epact for the year 1819

The Golden N^o 15
Multiply by 11
30 / 165 5
150
Rem^r 15
Subtract 11
4 Epact

Problem 5th

To find the Moon's Age

Rule

To y^e^ epact add the number of day of the month; the sum if under 30 is the Moons age. But if it be above 30 take 30 from it & the rem^r^ will be the Moon's age, or days since y^e^ last conjunction. The N^o^ of months are monthly epacts the Moons age at the beginning of each month, when the solar & lunar cycles begin together.

	0	2	4	2	3	4	5	6	7	8	9	10
end nov^r^	Jany	Febr	Mar	Apr	May	June	July	Augt	Sept^r^	Octo^r^	Nov^r^	Dec^r^

What is Moons age on the 11th of July 1810

The Epact is	1
N^o^ of Months	5
The day of the month	11
Sum is	17

What is the Moons age on the 5th Sept^r^ then following

The Epact is	4
The N^o^ of months	7
The days of the Month	4
Sum is	15

Problem 6th

The day of the month in any year being given to know on what week day it will fall

Rule

Find the Dominical letter & the week day on which the 1st of the proposed month falls & hence the name of the proposed day of the month will be known; observing that y^e^ 1st, 8th, 15th, 22nd & 29th days of any month fall on the same week days.

On what day of the week does the 11th July 1810 fall on

The Dominical letter	F
The 1st of July is	G
Therefore the 1st of July is on Mond^y^	
Consequently 11th is on	Thursday

On what day of the week does y^e^ 5th Sept^r^ fall on

The Dominical letter	C
The 1st Sept^r^ is	F
Therefore 1st is on	Wednesday
Consequently the 5th is on	Saturday

Problem 7th

To find when Easter-day will fall in any year between 1700 & 1899.

Rule

Find what day that new Moon falls on which is nearest to the 21st of March in common years or to the 20th in leap years; then the Sunday next after the full, or 15th day of that new Moon, will be Easter day.

If the 15th day fall on a Sunday, the next Sunday is Easter day.

When will Easter day fall on in 1810

The Dominical letter is F.
March 21st Moon's age is 25 days
New Moon on March 28th
The 15th day is April 12th
April F. S. G. on Monday
Then Easter Sunday is April 15

When will Easter day fall on in 1819.

The Dominical letters C
March 21st Moon's age 26 days
New Moon on March 25th
The 15th day is April the 9th
April F. S. G. on Thursday
Then Easter Sunday is April 11th

Problem 8th

To find the time of the Moon's southing on a given day

Rule

The Moon's age in days, multiplied by 0.8 gives the time of the southing nearly in hours & tenth parts. That is the, if less than 12 hours, is the time after mid-day. But if greater the excess is the time after last midnight.

At what time does the Moon come to the meridian of Boulogne on the 17th of May 1816

Moon's age . . . . . 21 days
Multiplied by 0.8
Moon So 16h.8 = 16.8

At what time does the Moon come to the meridian of Yarmouth on the 6th Oct. 1819

Moon's age . . . . 18 days
Multiplied by 0.8
Moon So 14h.24 = 14.4

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Problem 9th

To find the time of high-water at any place.

Rule

To the time of the Moons southing add the time the Moon has passed the meridian on the full & change days to make high-water at that place the sum shews the time of high water on the given day.

Example 1st

On the 17th of May 1816 at what time will it be high-water at Boulogne?

Moons southing ... 16h 48'
H.W at Boulogne ..... 10, 30 on syzygies
sum 27, 18
— 24, 00
High water 3, 18 AM on the day propos.

Example 2nd

On the 6th of October 1816 at what time will it be high water at Yarmouth.

Moons southing 14h 24'
H W at Yarmouth 9, 45 on syzygies
sum 24, 09
— 24, 00
High Water 00, 09 AM on the day prop.

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Navigation

Defenitions & Principles.

Navigation is the art of conducting a ship from one port to another & is divided into Plane & Globular Sailing. Plane Sailing is the art of navegating a Ship upon principles deduced from the notion of the Earths being an extended plane. The Mariners Compass is an artificial representation of the Horizon of every place, by the means of a circular piece of paper called a Card, divided like the Horizon into degrees & points which are called Rhumbs. A Rhumb Line is a right line drawn from the center of the

Compass to the Horizon; & is named from the point of the Horizon it falls in with.

The Course is an angle which the rhumb line the ship sails on makes with the meridian & it is sometimes reckoned in degrees & sometimes reckoned in points of the Compass. Distance is the number of leagues, miles &c. intercepted betn two places reckoned on a Rhumb or it is the way or length a ship has gone on a direct course in a given time.

Difference of Latitude is the Distance which a ship has made North or South of the place she set out from & it is reckoned in miles & parts of a mile on a meridian.

Departure is the east or west distance which a ship has made from the meridian of the place she departed from, reckoned on a parallel of latde in which she is supposed to sail, the plane chart it is the same as the diffce of longe.

If a ship sails due north or south she sails on a meridian, makes no departure & her distance & difference of latitude are the same.

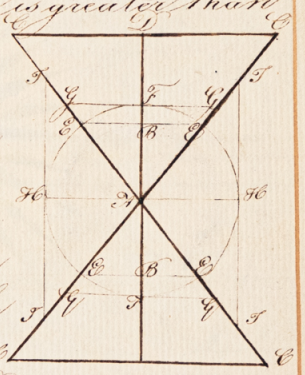
If a ship due east or west she runs on a parallel of latde makes no difference of latde & her dep & distce are the same.

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The diff^ce of latitude & departure always make the legs of a right angled triangle, the hypothenuse of which is the distance the ship has sailed.

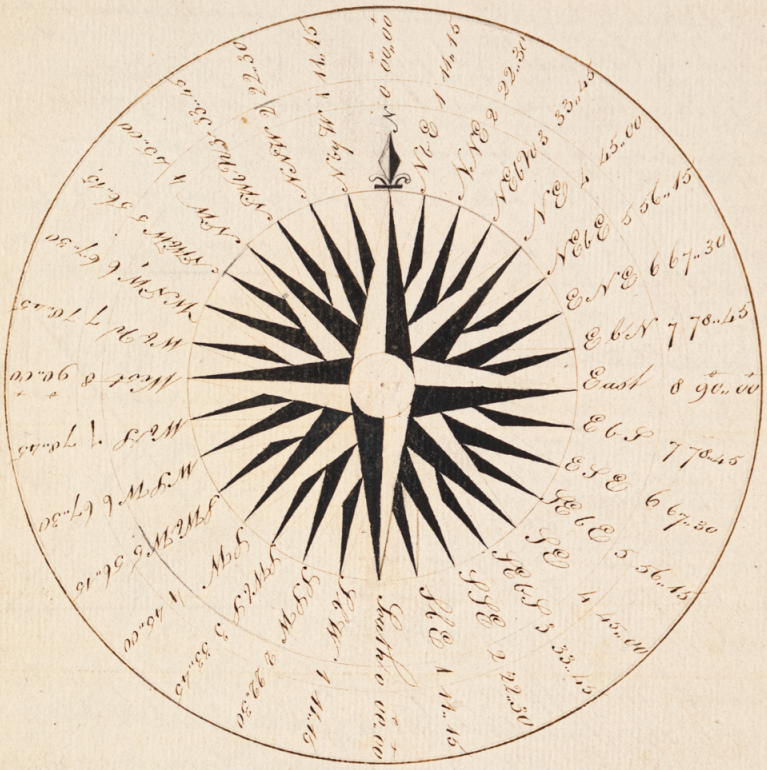
When the course is 45°, or 4 points, the difference of latitude & departure are equal.

When the course is less than 45°, or 4 points, the difference of latitude is greater than the departure. But when the course is greater than 4 points, or 45°, the departure is greater than the difference of latitude.



In the annexed figure suppose a circle I H I H to represent the horizon of the place A from whence the ship sails; A C the rhumb she sails on & C the place come to; then H H represents the parallel of latitude she sailed from, & C the parallel of lat^de arrived in: so that A D becomes the difference of lat^de D C the departure A C the distance sailed, the angle D A C is the course & the D C A is the complement of the course. The particulars will be alike represented whether the ship sails in the N E or N W, S E or S W quarters of the horizon.

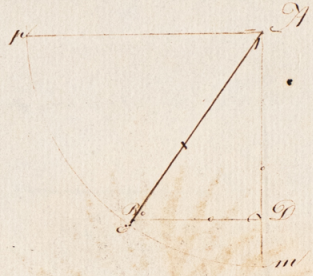
The Mariners Compass.



1	°	′	2.49
½	°	′	5. 37½
¼	°	′	0. 26

Example 1st

A Ship is from the Lat^{e} 47°30 N has sailed S.W.by S 98 miles. What lat^{e} is she in & what departure has she made

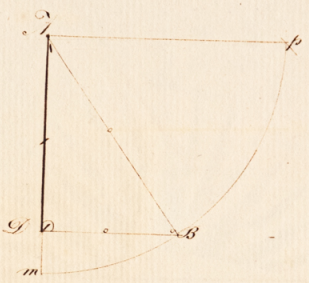


To find the diff lat^{e}	To find the departure
As rad 90°0 ... 10.	As rad 90°0 ... 10.
To dist 98 m 1.99123	To dist 98 m 1.99123
To cos C° 33°15 .... 9.91983	To sine C° 33°15 .... 9.74174
To diff lat 81,8 m. 1.91106	To dep 54,15 m 1.73297

Ans { The diff lat is 81,8 m / The dep is 54,15 ms }

Example 2nd.

A ships has sailed S.E by S. from the lat. 47°30' N. to the lat 46°08' N. What dist. has she run & what departure has she made.



Lat from 47°30' N.
Lat in 46.08 N.
Diff. lat 1.22 N.
60
82 m

To find the Dist.ce	To find the Dep.t.
As cosi Co 33°45' ... 0,08045	As cosi Co 33°45' ... 0,08045
To diff lat 82 m. .. 1,91381	To diff lat 82 m. .. 1,91381
To rad 90.00 .. 10 —	To sine Co 33°45' .. 9,74174
To dist 98,62 m. .. 1,99396	To dep.t 54,79 m. .. 1,73870

Ans { The dist is 98,62 m
The dep.t is 54,79 m }

Example 3rd

A Ship from the lat 57°.30 N sailing S W b y W finds she has made 82 miles of departure; What is her distance run & what lat is she in

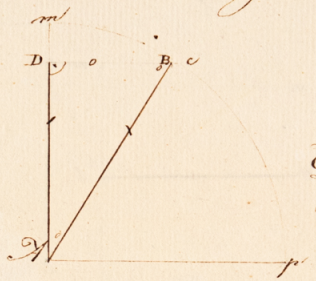


To find the distance		To find the diff lat	
As rad 90.00	10.	As sine C 56.15	9.91815
To dep 82 m	1.91381	To dep 82 m	1.91381
To cosec C 56.15	10.08185	To cosine C 56.15	9.74474
To dist 98.62 m	1.99566	To diff lat 54.79 m	1.73855

Ans. { The dist is 98.62 m
The diff lat 54.79 m }

Example 4th

A Ship from lat 47.20 N sails bet the nor Ne & East 90 miles & is arrived in lat 48.42 N: What course did she steer & what dep has she made



Sat from 47.20 N
Sat in 48.42 N
Diff lat 1.22 N
60
82 m

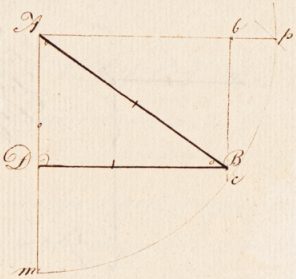
To find the course		To find the dep.	
As dist 90 m	8,00877	As cosC 33.12	9,92258
To rad 90.00	10,	To diff lat 82 m	1,91381
To diff lat 82 m	1,91381	To sine C 33.12	9,73843
To cos C 33.12	9,92258	To dep 53.65 m	1,72955

Ans { The course is 33.12
The dep is 53.65 m }

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Example 5th

From the lat 50° 13′ N a ship in sailing betw the South & East 98 miles, makes her departure 82 miles: What course did she keep & what lat is she arrived at?

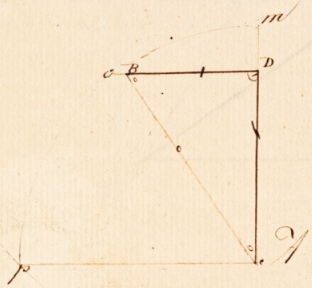


To find the Course		To find the diff lat	
As dist 98 m	8,008877	As rad 90.00	10.
To rad 90.00	10.	To dist 98 m	1,991123
To dep 82 m	1,913881	So co ✠ 56.48	9,738643
To sin ✠ 56.48	9,922258	To diff lat 53,66 m	1,729766

Ans^{r} { The course is 56.48 / The diff^{ce} lat 53,66 }

Example 6th

A Ship sailing from Lat° 48°.32 N is arrived in Lat 49°.54 N & by Obs^n finds herself 54 miles to the westward of her departed meridian: What course did she steer & what is the direct dist^ce she has run.



Lat from 48°.32 N
Lat in 49.54 N
Diff^ce lat 1.22 N
60
82 m

To find the course		To find the dist^ce	
As diff lat 82 m	8,08619	As cosi C° 33.22	9,25964
To rad 90°.00	10,	To depart 54 m	1,73239
So dep 54 m	1,73239	So rad 90°.00.	10,
To tan C° 33,22	9,81058	To dist^ce 98,18 m	1,99203

Ans^r { The Course is 33°.22 / The dist is 98,18 m }

Traverse Sailing.

Traverse Sailing is the art of resolving those cases in plane sailing where two or more diff.t courses & distances are concerned. A Traverse is an irregular track which a ship describes by sailing on several different courses & distances.

To work a Traverse

1st. Make a table of six columns, titles them Course, Distance, N. S. E. W. Begin at the left side, & write the given courses & distances in their proper columns. 2nd. Seek in the traverse table the given courses & distances & let the corresponding differences of lat.e & departure be written in their proper columns in the table made for the question 3rd. Add up the columns of northing southing, easting, westing; then the difference bet.n the sums of the northing & southing will be the whole diff.ce of lat.e of the same name with the greater. & the diff.ce bet.n the sums of the easting & westing will be the whole dep.t of the same name with the greater also 4th. The whole diff.ce of lat.e & dep.t is the Compound course being found the direct course & dist will be found as in Single Courses

Case 6

To construct a Traverse.

1st Describe a circle with the chord of 60° & divide it into 4 quadrants marking the north point with N.

2nd Lay off each course on the circumference, reckoned from its proper meridian & from the center to each point draw so many blank radii; mark these with the proper number of the course. 3rd On the 1st radius lay the 1st distance mark its extremity; through this extremity & parallel to the 2nd radius draw the 2nd distance of its proper length through the extremity of the 2nd distance & parallel to the 2nd radius draw the 3rd dist of its proper length & thus proceed until all the dist^s are drawn.

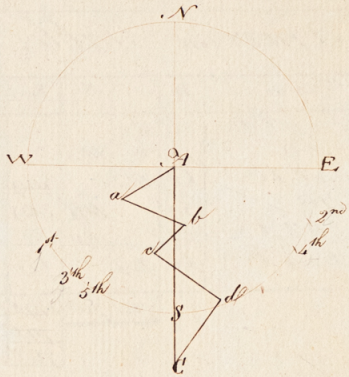
4th A line drawn from the extremity of the last dist to the center of the center of the circle will represent the dist made good; & a line drawn from the same point, perpendicular to the meridian lengthened if necessary will represent the departure

Example 1st

A ship sails from a place in lat^{e} 2° 32′ N^{ly} has run the following courses & distances, viz.
1st S W b W dist 45 m. 2nd E S E dist 50 m. 3rd S W dist 30
4th S E b. E dist 60 m 5th S W b S ½ W dist. 63 m
Req^{d} her present lat^{e} with the direct course & dist^{ce} betw^{n} the place sailed from & the place come to

Traverse Table

Courses	D	N	S	E	W
S W b W	45		23,0		37,1
E S E	50		19,1	46,2	
S W	30		21,2		21,2
S E b E	60		22,3	49,9	
S W b S ½ W	63		56,6		27,5
			142,2	96,1	96,1



Lat from 2° 32′ N.
D. Lat 142 = 2. 22 S
Pres^{t} lat 0 2. 54 N.

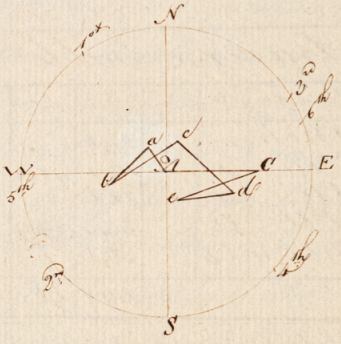
Ans^{r} { Course due South
Pres^{t} lat 2. 54 N }

Example 2nd

A ship from the lat.e 28.32. N has run the following courses, viz.
1st. N.W.b.N. 20m 2nd. S.W. dist 40m 3d. N.E.b.E. dist 60m
4th S.E. dist 55m 5th W.b.S. dist 41m 6th E.N.E. dist 66m
Req.d her present lat.de & the direct course & dist.e bet.n the place sailed from & the place come to.

Traverse Table.

Courses	D	N	S	E	W
N.W.b.N	20	16,6			11,1
S.W.	40		28,3		28,3
N.E.b.E	60	32,3		49,9	
S.E.	55		38,9	38,9	
W.b.S.	41		8,0		40,2
E.N.E.	66	25,3		60,0	
		75,2	75,2	149,8	79,6
				79,6	
				70,2	Dep



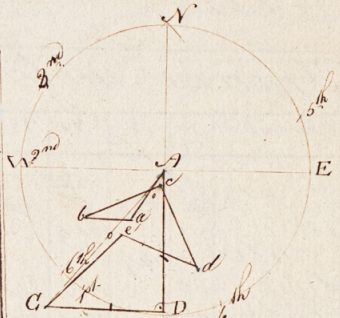
Ans.r { Course is due East
The dep.t is 70,2m }

Example 3d

Suppose yesterday noon we have run the following courses.
1st S.W b S. 20 m. 2d West 16 m. 3d N.W b W. 28 m.
4th S.S.E. 32 m. 5th E.N.E. 14 m. 6th S.W. 36 m.
What diffce lat & depe has the ship made & what is her direct course & distance.

Traverse Table.

Courses	P	N	S	E	W
S.W b S.	20		16,6		11,1
West	16				16,0
N.W b W.	28	15,6			23,3
S.S.E.	32		29,6	12,3	
E.N.E.	14	5,4		12,9	
S.W.	36		25,5		25,5
		21,0	71,7	25,2	75,9
			21,0		25,2
		DLat	50,7	Dep.	50,7



To find the Course	To find the Dist
As the diffce lat & depe are equal the course is 45° or 4 points.	As sine Co 45.00 ... 0,15052 To dep 50,7 m. ... 9,70501 To rad 90.00 ... 10, To dist 71,70 m. ... 10,85553

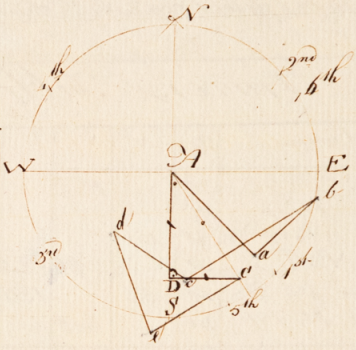
Ans { The Course is 45.00 or SW
The Distance 71,70 m }

Example 4th

The last 24h we have run the follg courses, viz
1st SE 40m 2nd N E 28m 3d S W½W 32m 4th N W½W
30m 5th S.S E 36m 6th S E½E 58m.
Req the diffce Lat & depr, direct course &
distance

Traverse Table

Courses	D	N	S	E	W
S E	40		28,3	28,3	
N E	28	19,8		19,8	
S W½W	32		28,9		12,2
N W½W	30	16,7			24,9
S S E	36		33,3	13,8	
N E½E	58		32,2	10,2	
		36,5	122,7	110,1	68,1
			36,5	68,1	
		D. Lat	86,2	42,0	Dep



To find the Course		To find the dist	
As d. lat 86,2 m	8,064549	As sine Co 25.59	0,364512
To rad 90.00	10,	To dep 42,0 m	1,623250
To dep 42,0 m	1,623250	To rad 90.00	10,
To tan Co 25.59	9,687701	To dist 93,87 m	1,987762

Ans { The course is S 25.59 W
The dist is 93,87 m }

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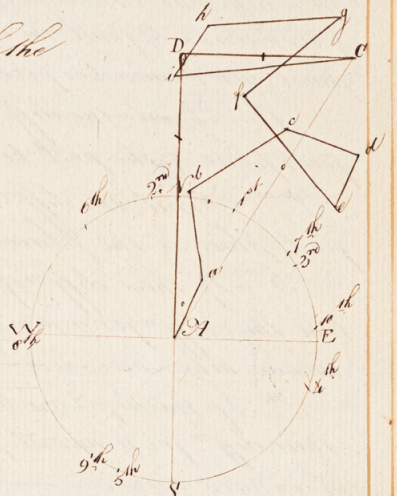
Example 5th

Yesterday noon we were in lat^{e} 3° 18′ S & since then we have sailed on the following courses, viz. 1st N. NE. 22m 2nd N. 6 W. 30m 3rd N E b E 10m 4th E S E 25m 5th S S W 18m 6th N W b. N ½ W 31m 7th N E ½ E 12m 8th N. b. E. ½ W 15m 9th S W. b S 20m 10th & lastly E b. N ¼ E 62m.

Req^{d} our present lat & dep^{e} with the direct course & dist made good.

Traverse Table

Courses	D	N	S	E	W
N. NE	22	20,3		8,4	
N. 6 W	30	29,4			5,8
N E b E	10	9,2,2		3,3,3	
E S E	25		9,6	23,1	
S S W	18		16,6		6,8
N W b. N ½ W	50	38,6			31,7
N E ½ E	12	26,6		3,2,5	
W b S ½ W	45		4,1,4		44,8
S W b. S	20		16,6		11,1
E b. N ¼ E	62	9,1		6,1,3	
		146,2	47,2	158,6	100,2
		47,2		100,2	
		99,0	DLat	58,4	Dep.



Lat I am 3° 18′ S
Dist 99/100 1,20 S
Prest Lat^{e} 1,39

To find 1st Course	To find 2d Dist
As dlat 99,0 m ... 8,00426	As sine C 30° 32′ ... 9,29401
To rad = 90.00 ... 10.	To dep 58,4 m ... 1,76651
To dep = 58,4 m ... 1,76651	To rad 90.00 ... 10.
To tan C = 30. 32 ... 9,77077	To dist 114,9 m ... 2,06052

Ans^{r} { The Course is N 30° 32′ E
The Dist is 114,9 m }

Compound Courses

When a ship is bound from a given place to a known port & on some occasion is obliged to run on one or more different courses not in the direct one to the port & the[n] wants to find the direct course & distance she must run to gain her port observe the following precepts

1st Find the diff.ce lat.e & dep.e bet.n the ships first place & the port bound to.

2nd Find the diff.ce lat.e & dep.e bet.n the ships first place & the place she is come to.

3rd If these diff.ces of lat.e are both of the same name take their diff.ce for a new diff.ce lat.e: But if contrary names take their sum.

4th If the dep.es are both of the same names take their diff.ce for a new dep.e. But take their sum if of contrary names

5th To this new diff.ce lat.e & dep.e find the direct course & distance 6th If bet.n the place sailed from & the place come to there has been more than one Course to each course let the diff.ce lat.e & dep.e be found & let their sum or diff.ce be taken according as they are of the same or of contrary names and this will give the diff.ce lat.e & dep.e between the place sailed from and the place come to.

Then proceed by the 3rd 4th & 5th precepts.

WILLIAMS's 83 L.2 PATENT L.B

To construct a Compd Course

1st Describe a circle, in which draw the meridian & parallel of the place sailed from.

2nd Construct the triangle betn the place sailed from marked A & the place bound to marked B, & let ye intersection of the diffce lat & dep be marked D.

3rd Construct the figure between the place sailed from or A & the place come to, marked C, & let the intersection of this diffce lat & dep be marked E.

4th Through C draw CF parallel to AD, meeting BD continued if necessary in F. & draw CB.

5th Then in the triangle CFB, the lines CF, FB, CB, will represent the diffce lat, dep & distance betn the place come & the place bound to.

6th From the center A draw a line parallel to CB in the direction from C to B, & this shews the course wanting.

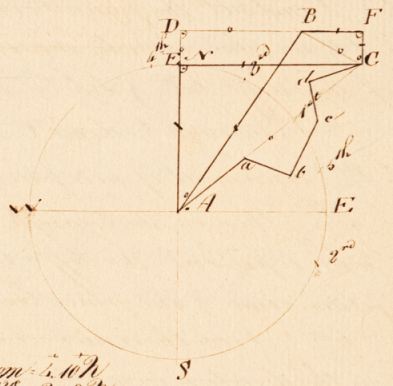
Example 1st

A Ship from the latitude of 4° 40 N being to make an Island that bears N E b N distance 196 miles & having run these courses viz. N E ½ E 73 m. E S E ½ E 42 m. N N E ½ E 32 m. W b N 34 m. E N E ½ E 19 m.

It is desired to know her present Lat^e & the her direct course & distance to the Island.

Traverse Table

Courses	D	N	S	E	W
N E ½ E	73	67,0		56,1	
E S E ½ E	42		14,1	39,5	
N N E ½ E	32	29,6		12,2	
W b N	34	33,3			6,6
E N E ½ E	19	14,2		19,6	
		141	141	166	6,6
		141		6,6	
Diff		128,0	Sal	166,0	Dep



N D = 163 E C = 166 Sal^e Sum = 4.10 N
N C = 128,0 D B = 166,9 D. Sal^e 128 / 60 = 2,08 N
C D = 35,0 B F = 34,1 Prest Lat = 6.18 N

To find 4th Deff Lat^e (In 4th △ A D B) To find 4th Dep^e

As rad = 90.00 10	As cos Co = 32.45 1,07615
To dist = 196 m. 2,29226	To d. lat^e = 163,1 m. 2,21219
To cos Co = 32.45 9,91985	To sine Co = 23.45 9,74471
To d. lat^e = 163,0 m. 2,21211	To dep^e = 108,9 m. ... 2,03708

To find 4th Course (In 4th △ A E C) To find 4th Dist^ce

As dep^e 166,0 m. 7,79588	As sine Co 51.20 9,89764
To rad = 90.00 10	To dep^e = 166,0 m 2,20412
To dist 128,0 m. 2,10721	To rad = 90.00 10
To cot Co 51.20 9,90309	To dist = 204,9 m. 2,31156

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To find ☿ Course & Dist by ☿'s Δ CFB To find ☿ Dist

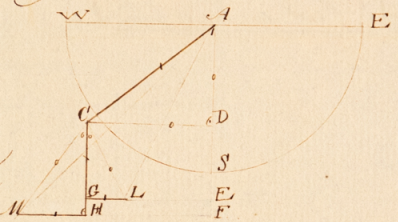
As dep = 311m 2,29138	As cosi Co. 55.36 9,24798
To rad = 90.00 10,	To d. lat = 250 m 1,54407
Sod. lat = 250 m 1,54407	To rad = 90.00 10,
To cot Co. 55.36 9,83365	To dist = 61,95 m 1,79205

Ans { Prod lat S. 18° W
Course N. 55. 36 W.
Dist is 61,95 m }

Example 2nd

Two ships take their departure from the Lizard in lat^{e} 49.57 N; one bound to Saint Michaels which lies 715 m^{s} to the South & 715 m^{s} to the West; the other bound to Lisbon lying 660 m^{s} to the South and 215 m^{s} to the West, reckoning from y^{e} Lizard they sail in company S 60° W. 660 m^{s} & then part:

What is the direct Course & distance of each ship to her port?



AE = 660 m^{s}
AD 287 m^{s}
DC 274 m^{s}

CD = 471.6 m^{s}
EL = 215.0 m^{s}
GL = 256.6 m^{s}

AF = 715 m^{s}
AD = 287 m^{s}
CK = 258 m^{s}

MF = 745.0 m^{s}
HF = 471.5 m^{s}
HM = 273.5 m^{s}

To find y^{e} diff lat^{e} (In y^{e} Δ ADC)	To find y^{e} Dep^{t}
As rad = 90° 00′ 10,	As cot Co = 50° 27′ 30″ 9,92583
To dist^{e} = 660 m^{s} 2,78533	To diff lat^{e} = 287.0 m^{s} ... 2,45771
To cosi Co = 50° 27′ 30″ 9,80236	To rad = 90° 00′ 10
To diff lat = 287.0 m^{s} ... 2,58769	To dep^{t} = 471.6 m^{s} .. 2,67354

To find y^{e} Course (In y^{e} Δ CGL)	To find y^{e} Dist^{e}
As dep^{t} = 256.6 m^{s} 2,40974	As cot Co = 42° 07′ 9,95743
To rad = 90° 00′ 10,	To diff lat = 274 m^{s} 2,43775
To diff lat = 273.5 m^{s} 2,43775	To cosec Co = 42° 07′ 10,16527
To cot Co = 42° 07′ 10,03049	To dist = 373.4 m^{s} ... 2,57445

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To find the Course (In 7th Ch. III) To find the Dist

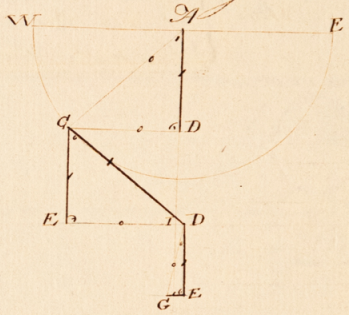
As diff lat 328 m 7.18413	As rad 90.00 ..... 10. —
To rad 90.00 ..... 10. —	To diff lat 328 m 2.21587
To dep 273.5 m 2.43696	As sec Co 29.49 .... 10.11758
To tan Co 39.49 ..... 9.92109	To dist 427.0 m 2.63045

Ans. { Course to Lisbon is S 15.07 E / Course to St. Michaels is S 39.49 W / Dist to Lisbon is 371.4 m / Dist to St. Michaels is 427.0 m }

Example 3d.

A Ship in Latitude 42.20 N Longitude 25.10 W is bound to St. Michaels bearing S.56. W in the Latitude 37.45 N & from thence she is bound to Madeira in Lat^{e} 32.25 N & distant from St. Michaels 511 miles in the South East Quarter Req^{d} the dist to St. Michaels the course from thence to Madeira together with the Course & Distance from Madeira to y^{e} Island of Palma in Lat^{e} 28.30 N Long^{e} 18.00 W

Ships Lat^{e} ... 42.20 N
St. Michaels Lat^{e} 37.45 N
Diff Lat^{e} ..... 3.49 = 225 m
St. Michaels Lat^{e} 37.45 N
Madeira's Lat^{e} 32.25 N
Diff Lat^{e} 5.20 N = 320 m
Madeira's Lat^{e} = 32.25 N
P. Palma's Lat^{e} = 28.30 N
Diff Lat^{e} = 3.49 N = 229
Madeira's Long^{e} 16.00 W
P. Palma's Long^{e} 17.06 W
Diff Long^{e} 01.04 W = 54 m



To find the Dep^{e} (In the $\triangle$ ABC)	To find the Dist
As rad 90.00 10,	As rad 90.00 10,
To d. Lat^{e} 335 m. ... 2,52504	To d. Lat^{e} 335 m. ... 2,52504
To tan Co 56.11 10,01921	To sec Co 56.11 10,16033
To dep^{e} 3591 m. ... 2,54425	To dist 484,6 m. ... 2,68537

To find the Course (In the $\triangle$ CED)	To find the Dep^{e}
As d. Lat^{e} 320 m. ... 7,49485	As rad 90.00 10,
To rad 90.00 10,	To d. Lat^{e} 320 m. ... 2,50515
To dist 511 m. .. 2,71096	To tan Co 51.30 10,09939
To sec Co 51.30 10,20501	To dep^{e} 4033 m. ... 2,60454

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To find the Course (In the D°F6) | To find the Dist

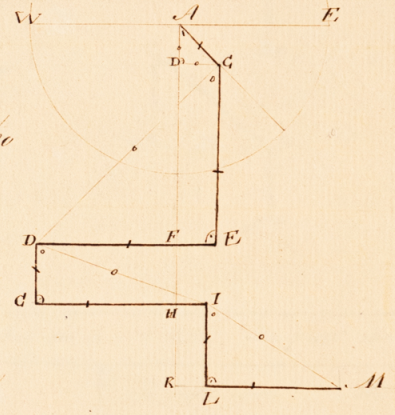
As d. lat 229 m 7.6 9016	As sin Co 13.16 .... 9.62741
To rad 90.00 .... 10.	To dif^{e} 54 m 1.732939
To dif^{e} 54 m 1.732939	To sec^{t} Co 13.16 .... 10.01175
To tan Co 13.16 .... 9.37255	To dist^{e} 2352 m 2.37155

Ans^{r} { Distance to St. Michaels 48½ m
Co from St. Michaels to Madeira S 37.30 E
Co from Madeira to S. Palma S 13.16 W
Dist from Madeira to S. Palma 2355 m }

Example 4th

Departing from St. Marys one of the Azores, & bound to the river Sierra Leon, after running SE 36 hours, at 7 knots an hour, my sea compass was spoiled by a flash of lightning; then after 8 days direct run between the S and W. I met a ship in latitude 20.00 N long.e 26.10 W. & bound to Mayo, one of the cape Verd Isles, to which I sailed in her company: Req.d my course & dist in the 8 days also the same while I sailed in company & the same from Mayo to Sierra Leon.

Ships lat.e 33.50.00 N
Sat in 20.00.00 N m
Diff lat.e 13.50.10 = 828.6
Ships long.e 22.41.10 W
Log in 26.10.00 W
Diff long.e 13.28.50 = 838.80 m
Lat.e in 20.00 N
Mayo lat.e 15.12 N m
Diff lat.e 4.48 = 288
Long in 26.10 W
Mayo long.e 23.11 W m
Diff long.e 12.59 = 779
Mayo lat.e 15.12 N
S. Leon lat.e = 8.20 N
Diff lat.e = 6.46 N = 403 m
Mayo long.e = 23.11 W
S. Leon long.e = 13.09 W
Diff long.e = 10.02 = 602



To find the diff lat.e (In ye Δ ABC) To find ye Dep.e

As rad 90.00 ..... 10.
So dist.e 252 m 2.40140
So cosine 45.00 .... 9.84948
So d.lat.e 178.2 m ... 2.25088

As the C o is 45 or 4 pts the diff lat.e & Dep.e are equal.

WILLIAMS'S 83 L. 2 PATENT L.B

To find the Course (In the S & D's) To find the dist

As d. lat 828.6 ..... 7,076 45	As rad 90.00 10
To rad 90.00 10,	To d. lat 838 m 2,92355
To dep 838,80 m 2,92366	To sec Co 45.00 10,15031
To ta Co 45.00 10,00011	To dist 118,6 m 3,07386

To find the Course (In the S D & W) To find the dist

As d. lat 288 m 7,55961	As rad 90.00 10,
To rad 90.00 10,	To d. lat 288 m 2,45929
To dep 776 m 2,88989	To sec Co 69.39 10,45870
To ta Co 69.39 10,43050	To dist 828.1 m ... 2,91809

To find the Course in the S I L M To find the Dist

As d. lat 103 m 7,39469	As rad 90.00 10,
To rad 90.00 10	To d. lat 103 m 2,00531
To dep 665 m 2,78176	To sec Co 56.20 10,25621
To ta Co 56.20 10,17645	To dist 772 m 2,88152

Ans^{r} { Course on 7th Days ..... S 15.00 W
Dist on 7th Days ..... 1186 m
Co to Mayo ..... S 69.39 E
Dist to Mayo ..... 828.1 m
Co from Mayo to Sierra Leon .. S 56.20 E
Dist from Mayo to L. Leon .. 772 }

Oblique Sailing.

By Oblique Sailing is understood the resolution of such cases in plane sailing as req^e the consideration of Oblique Angled Triangles

To set an object is to observe the rhumb or point of the compass which points to it.

The bearing of one place to another is an angle contained betw^n the meridian & the rhumb line which passes through the two places. The bearing of an object is an angle contained betw^n the meridian & the rhumb line which the object is seen on

In oblique sailing the bearing of a line which does not pass through the center of the Horizon is estimated by the bearing of one which is drawn parallel to it through the

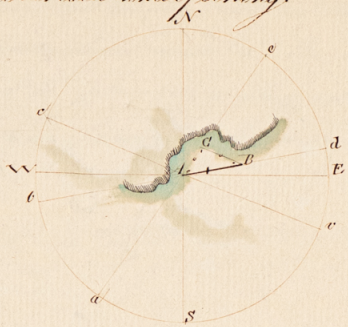
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WILLIAMS' 83 1/2 PATENT

Example 1st

Coasting along a shore a Headland bore N.E b. N. then having run E.S.E. 15 m. the Headland bore W. N.W. Req. the distance from the Headland at each time of setting.

N d - 78.45
N e - 33.45
2 A - 45.00
W c - 22.30
W b - 11.15
2 B - 33.45
S a - 33.45
S e - 67.30
2 C 101 15



To find BC	To find AC
As si 2 C - 78.45 0,00843	As si 2 C - 78.45 0,00843
To side AB - 15 m. ... 1,17609	To side AB - 15 m. ... 1,17609
To si 2 A - 45.00 ... 9,84948	To si 2 B 101.15 ... 9,99574
To side BC 16,81 m. ... 1,02400	To side AC 8,197 ... 0,92026

Ans { The dist at the 1st setting is 8,197 m / The dist at the 2nd setting is 16,81 m }

Example 2nd

Two ships sail at the same time from one port one sails E.S.E & the other S.S.E. Req.d the bearing & distance when each has run 37½ miles.

180.00
L A 46.00
2)135.00
L B ½ 67.30 G

m b = 67.30
N m = 22.30
N b = 45.00



To find B C
As sine L B = 67.30 ..... 0.034128
To side A C = 37.5 m. 1.574103
So sine L A = 45.00 ..... 9.849849
To side B C = 20.7 m. 1.457900

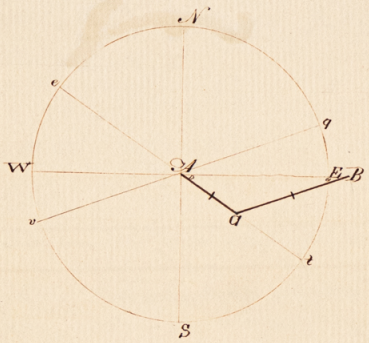
Ans.r B bears from C N 45.00 E

Example 3rd

A Merchant Ship meeting a Ship of war which had sailed I E by E 39 leagues from the latitude 45° 5' N informed them that she had been plundered the day before in the same Lat.e & had since run 67 leag.s on a direct course betw.n the S & W, & the pirates were cruising about the place where they robbed her. What course must the man of war steer to come up with them.

39
2
117 m.

67
2
201 m
∠B = 18° 52'
Course 90. 00
71. 08



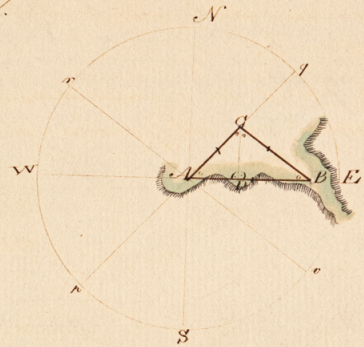
∠A = 33° 15'
∠B = 18. 52
52. 07
180. 00
∠C = 127. 23

To find ∠B	
As side BC = 201 m	7,69670
To sine ∠A = 33° 15'	9,74174
So side AC = 117 m	2,06819
To sine ∠B = 18° 52'	9,50952

The man of wars course is N 71° 08' E.

Example 4th

Two Ships met at sea, found they had sailed from two ports in the same latitude & distant 576m. The ship from the westerna port had run betn the N & E. 328m & the other Ship had sailed 416m. Req. the course steered by each Ship.



∠C = 44.49
∠D = 90.00
134.49
180.00
∠A 45.11

∠C = 55.59
∠D = 90.00
145.59
180.00
∠B = 34.01

328
416
sum 744
diff 88

2)576
288 ½ sum

To find the difference of the segments

As the base or sum of the segments	= 576m	7.23958
To the sum of the other two sides	= 744m	2.87157
So is the diffce of those sides	= 88m	1.94448
To the diffce of the segments of the base	= 112.7m	2.05503

50.8
288.0
BD 344.8
AD 231.2

To find ∠C In ▽ ADC		To find ∠C In ▽ BDC	
As side AC	328m	7.48413	As side BC 416m 7.38094
So si ∠D	90.00	10.	So si ∠D 90.00 10.
So side AD	231.2m	2.36399	So side BD 344.8m 2.53757
So si ∠C	44.49	9.84812	So si ∠C 55.59 9.91848

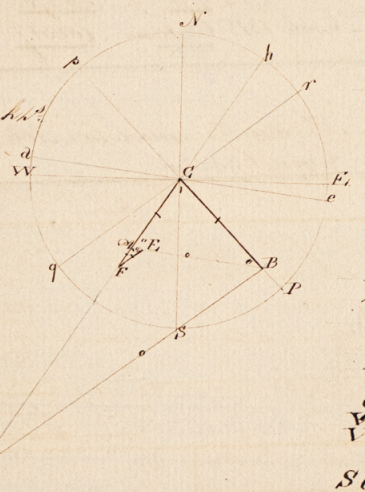
Ans { A Course is N 44.49 E
B Course is N 55.59 W }

WILLIAMS'S 83 L. 2 PATENT L.B

Example 5th.

A Fleet of Ships steering S W b S four miles an hour on sail, descried a cruiser, which gave chase N E 5 leagues at the rate of 7 miles an hour, comes up with & takes the chase. After adjusting matters on board the prize she steers for the fleet, which still kept on the same course & rate.

Req. the course the cruiser must shape & the distance she must run at 7 miles an hour to join them.



SD = 33°.45′
S d = 45..00
2 ∠C = 78..15
180..00
2)101..15
½ sum 50..37..30 mink 2d

SD = 33°.45′.00″
D d = 62..25..30
S d = 97..10..30
SW = 90..00..00
W d = 7..10..30

DS = 33°.45′.00″
SP = 45..00..00
P e = 37..49..30
2 ∠A = 110..34..30
180..00..00
62..25..30

7 miles
6 Hours
GB 25 miles

6 Hours
4 miles
AG 24 miles

GB = 25 m
AG = 24 m
sum = 49 m
diff = 1 m

2 ∠A = 110°.34′.30″
2 E = 30..44..00
141..18..30
180..00..00
∠F = 38..41..30

gd = 30°.44′.00″
W d = 7..10..30
W q = 23..33..30
90..00..00
S q = 66..26..30

To find ∠s A and B (In s. △ AGB) To find AB

As sum given sides = 49 m 8.22915	As sine B = 27°.49′.30″ . . 9.21236
To diff. of those sides = 1 m 1.04139	To side AG = 24 m 1.38021
To tabl. ½ sum mink 2d ½ = 50°.37′.30″ . . 10.08682	To sine ∠G 76°.45′ 9.99157
To tan ½ diff these ∠s = 12.58 . . . . 9.35637	To si AB 28.28 m. . . . 1.44414
50°.37′.30″	
∠A 62..25..30	
∠B 27..49..30	

To find ∠ FEi (In ▽ FAEi) To find AEi

As side FEi = 7m 9,154480	As sine ∠A = 63°.25'.36" ... 0,048946
To sine ∠A = 63°.25'.36" ... 9,995134	To side FEi = 7m 0,845510
To side AF = 4m 0,60206	To sine ∠E = 32°.41'.36" 9,732439
To si ∠ FEi = 32°.41' 9,708440	To side AEi = 4,227m ... 0,626005

To find DB (In ▽ DAB)

As side AEi = 4,227m .. 9,237391
To side AB = 38,38m .. 1,58410
To side FEi = 7m 0,84510
To side DB = 63,85m .. 1,80517

Ans { The cruiser mast stands 66.26.30 W. & 63,85m }

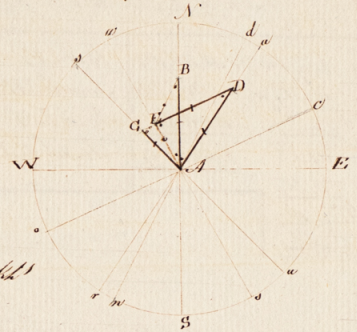
Example 6th.

A Ship after doubling a cape & sailing N.E. b. N. 45 miles, receives in the night considerable damage from a storm; she then bore directly towards a light house lying 24 m to the N.W. of the cape, & having run 40m, & the day breaking, she discovers a port 42m to the north of the cape. What was her course & distance to that port.

AD = 45 m
AG = 24 m
sum = 69 m
diff = 21 m

N.s = 45.00
N.a = 33.45
∠1 = 78.45
180.00
2 | 101.15
50.38 ½ sum unk.ls

∠DAE = 62.36
N.a = 33.45
∠A = 28.51
180.00
2 | 151.09
½ sum 75.34 unk.ls



AD = 45 m
DE = 40 m
sum = 85 m
diff = 5 m

∠D = 36.17
180.00
2 | 149.43
74.31½ sum unk.ls

AB = 42.00 m
AL = 22.71 m
sum = 64.71 m
diff = 19.29 m

To find ∠G & D (In ▽ GAD) To find GD

As sum given sides = 69 m 8.16115
To diff those sides = 21 m 1.32222
So tan ½ sum unk.ls = 50.38 .... 10.08596
To tan ½ diff those ls = 20.21 .... 9.56033
50.38
∠G = 70.59
∠D = 30.17

As sin ∠G = 70.59 .... 9.97562
To side AD = 45 m 1.65327
So sin ∠A = 78.45 .... 9.99517
To side GD 46.69 .... 1.67282

To find ∠s E & A (In ▽ ADE) To find AE

As sum given sides	85 m	8,07058	As si ∠D 87.08	0,00055
To diff. those sides	5 m	0,69897	To side AD 15 m	1,66321
To tan ½ sum unk.s ∠s	74.52	10,56762	To si ∠A 28.17	9,70266
To tan ½ diff those ∠s	12.46	9,33747	To side AE 22,7 m	1,35642

74.52
∠E 87.08
∠A 62.36

To find ∠s E & B (In ▽ BAE) To find EB

As sum given sides	64,7 m	8,18903	As si ∠E 55.14	0,08541
To diff. those sides	19,20 m	1,28533	To side AB 42 m	1,62325
To tab ½ sum unk.s ∠s	75.34	10,38943	To si ∠A 28.51	9,68334
To tan ½ diff those ∠s	49.12	10,06279	To side EB 24,67 m	1,39217

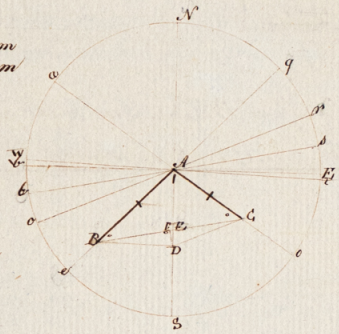
75.34
∠E 24.46
∠B 26.22

Ans.r { The course from E to B is N 26.22 E
The Distance is 24,67 m }

WILLIAMS' 33 1-2 PATENT L.B

Example 7th.

Two ships departed from a port at the same time, & after one had sailed N.E. by E. 50 miles & the other S.W. 50 miles they were equally distant from a rock which lies directly south of the port. Required the bearing & distance of each from the rock.



∠A = 101°15′
180.00
2) 78.45 sum
39.22½ sum
FC = 34.920 m
FD = 5.539 m
sum = 40.459 m
diff = 29.381 m
∠F 90.00
180.00
2) 90.00 sum
45.00 ½ sum
Nq = 45.00
qS = 34.00
NS = 79.00
rS = 9.00
Nr = 70.00

AB = 50 m
AC = 40 m
sum = 90 m
diff = 10 m
BC = 6.985 m
GF = 34.92 m
EG = 33.86 m
FE = 1.06 m
∠F = 90.00
∠E = 79.10
169.10
180.00
∠D = 10.50
Se = 45.00
ec = 34.00
bc = 9.61
Sb = 88.10

To find ∠B and C (In the △ BAC) To find BC

As sum given sides = 90 m 8.04576	As sin ∠C = 47.35 .... 0.15360
To diff those sides = 10 m .... 1.	To side AB = 50 m 1.69897
To tan ½ sum ∠s = 39.22 .... 9.91404	To sin ∠A = 78.45 .... 9.99157
To tan ½ diff those ∠s = 5.13 .... 8.95980	To side BC 6.985 m 1.84414
∠C 44.35	
∠B 34.09	

To find EC (In the △ EBC) To find DF (In the △ FED

As sin ∠E = 79.10 .... 0.00781	As sin ∠D = 10.50 .... 0.72595
To side BC = 40 m 1.60206	To sine EF = 1.06 m 0.02531
To sin ∠B = 34.09 .... 9.91985	To sin ∠F = 79.10 .... 9.99210
To side EC 33.86 m 1.52972	To side DF 5.539 m 0.74545

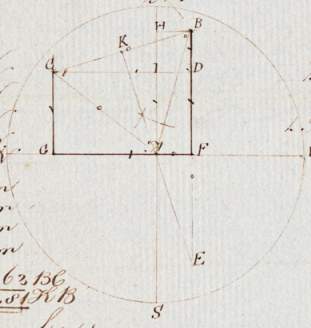
To find ∠D & G	(In ▽ ABC)	To find DC.
As sum of given sides	40,439 m . . . 8.39799	As sin ∠D 80°.59 0,00530
To diff^{ce} of those sides	29,281 m . . . 1.46606	To side BC 34,920 m . . . 1,54307
As tan ½ sum of unk^{n} ∠s 49°.00	10	As sin ∠F 90°.00 10
To tan ½ diff^{ce} of those ∠s 35°.59 . . .	9,86105	To side DC 35,35 m . . 1,54837
	49.00	
∠D	80.59	
∠G	9.01	

Ans^{r} { The Ship at B is 35,35 m to the S 80°.10' W of ▽ rock
The Ship at G is 35,35 m to the N 70°.00 E of the rock }

WILLIAMS' 83 1/2 PATENT L.B

Example 8th

Two ships A and B, both going at the rate of six knots an hour, parted compy in the lat of 45. N. A steering in y^e N E and B in the N W quarter; the next day at noon they both arrived at their posts, A in lat 45.58 N B in lat 45.36 N. and the sum of their departures was 64 m. At what hour did they part, and what was the course and distance each ship had sailed.



Lat from 45.00 N
Lat in 45.58 N
Diff lat 00.58 N

Dep from 45.00 N
Lat in 45.36 N
Diff lat 00.36 N

KB 58 m DB 22 m
HD 36 m CD 64 m
DB 22 m sum 86 m
Diff 42 m

∠D 90.00 — 26.03.36
180.00 — 35.57 KB
2) 90.00
45.00 ½ sum ank r/s

∠H 90.00
∠E 10.59
100.59
180.00
∠A 71.01

BE 103.9 m
BH 88.0 m
HE 45.9 m

G H 64.00 m
G B 15.83 m
G A 48.17 m
C I 48.17 m
I E 36.00 m
sum 84.17 m
Diff 12.17 m

60.13 ½ — ∠H 90.00
10.02 — 180.00
12.00 — 2) 90.00 sum
1.56 — 45.00 ½ sum

To find ∠s B and C (In ▽ ∆ CDB)	To find BC
As sum given sides dC m 8,06510	As sine ∠C 18.59 . . . 9,48773
To diff those sides 42 m 1,62325	To side BD 22 m . . 1,34242
To tan ½ sum ank r/s 45.00. 10.	To sine ∠D 90.00 . . 10.
To tan ½ diff those ∠s 26.01. 9,68845	To side BC 6763 m 9,83015

45.00
B 71.01
C 18.59

To find BE (In ▽ ∆ BHE)	To find HE (In ▽ ∆ EHI)
As sine ∠E 18.59 . . . 9,48773	As sine ∠I 71.01 9,97429
To side BH 33,81 m . . 1,52906	To side HE 45.9 m 1,66281
To sine ∠H 90.00 . . 10.	To sine ∠E 18.59 9,51227
To side BE 103.9 m 2,01678	To side HI 13.83 m 1,19937

To find ∠ A and C In ▽ ABC | To find AC.

As sum of given sides	8,417 m	8,07581	As sine ∠ A 53. 14	9,09632
To diff. those sides	12,17 m	1,08529	To side BC	8,17 m	4,68270
Taken ½ sum angles	45. 00. 00	10,	To sine ∠ B	90. 00.	10,
To tan ½ diff those ∠s	8. 14	9,16613	To side AC	60,13	1,77910

45,00
∠ A 53. 14
∠ C 36. 46

To find ∠ A In ▽ ABH

As side AB	60,13 m	8,22091
To sine ∠ H	90. 00	10,
To side HB	15,83 m	1,19921
To sine ∠ A	15. 15	9,42012

Ans^r { The ships parted at 1. 58 in ye morning
As course is N 15. 15 E
As course is N 53. 14 W
dist run by each is 60,13 miles }

WILLIAMS'S 35 1/2 PATENT L.B

Sailing to Windward.

When the wind is directly, or partly, against a ship's direct course to the place she is bound to, she reaches her port by a kind of zigzag, or oblique course; which is made by sailing with the wind first on one side of the ship, & then on the other.

In a ship, & looking toward the stem or head, Starboard signifies the right hand side. Larboard the left hand side.

Forward or Afore, is toward y^e^ head or stem of the ship.

Aft or Abaft, is toward y^e^ hinder part, or stern. The Beam signifies athwart, or across the middle of the ship.

When a ship sails the same way that y^e^ wind blows, she is said to sail or run before the wind; and the wind is said to be right aft, or right astern; & her course is then 16 points from the wind.

When the ship sails with the wind blowing directly across her she is said to have the wind on the beam & her course is eight points from the wind.

When the wind blows obliquely across the ship, the wind is said to be abaft the beam, or before the beam, according as her course is more or less than eight points from y^e^ wind.

When the ship endeavours to sail towards that point of the compass from which the wind blows, she is said to sail on a wind or to ply to windward. A vessel sailing as near as she can to the point from which the wind blows, is said to be close hauled. The generality of ships will lie within about six points of the wind; but sloops and some other vessels will lie much nearer. The windward, or weather side is that side of the ship on which the wind blows; and the other side is called the leeward, or lee side. Tacks and sheets are large ropes made fast to the lower corners of the fore and main sails by which either of these corners is hauled fore or aft. When the ship sails on a wind, the windward tacks are always hauled fowards, and the leeward sheets aft. The starboard tacks are aboard, when the starboard side to windward, & the starboard to leeward. And y^e^ larboard tacks are aboard, when y^e^ larboard side to windward, & the starboard to leeward.

WILLIAMS'S 35 1/2 PATENT

The most common cases in turning to windward may be constructed by the following precepts: Having drawn the meridian and parallel of lat (or east and west line) in a circle representing the horizon of the place, mark the place of the wind in the circumference, draw the rhumb passing through the place bound to, and lay on it the distance of that place from the center.

On each side of the wind, lay off in the circumference the points or degrees shewing how near the wind the ship can lie, and draw these rhumbs.

Now the first course will be on one of these rhumbs according to the tack the ship leads with. Draw a line through the place bound to, parallel to the other rhumb & meeting the first, & this will show the course and distance on the other tack.

Example 1st.

A privateer lying by with the wind East N. N. E. sees a sloop on the Starboard tacks that had just doubled a point 18 m to the Westward. The sloop ran for her but within 6 points of the wind. The privateer gave chace at the rate of eight knots an hour and in four hours came up with her.

Required the sloop's rate of sailing, and also the courses of both vessels.



OW = 67.30
WB = 67.30
E.g = 11.15
∠B 146.15
180.00
33.15

∠B 146.15
∠C 18.13
164.28 sum
180.00
∠A 15.32

NO 45.00
Oc 18.13
Nc 63.13

4/15.42m
3,85 P hour

To find ∠C (In 1st △ ABC) To find BC.

As side AC 32 m	8,49485	As sine ∠B 33.45	9,25526
To sine ∠B 33.45	9,74474	To side AC 32 m	1,50515
To side AB 18 m	1,25527	To sine ∠A 15.32	9,42781
To sine ∠C 18.13	9,49106	To side BC 15.42	1,18822

Ans. { The sloop sails N W at the rate of 3,85 m P hour
The privateer's course is N 63.13 W

Example 2nd.

Having my star board tacks on board, I met a snow that had sailed N E b. N. 28½ miles from a port I am bound to, and next day I met a brig which had sailed from thence 32 6 miles, with the wind then at S b E. two points abaft the beam. Required my course and distance bet.n the meeting these ships, and how near the wind I had lain.



l. A 22° 36
dc. 00
2)157. 30 sum
78. 45 ½ sum

l. B 97° 51
dc. 00
82. 09

rc = 39. 39
E n = 33. 45
E c = 26. 34
90. 00
S c = 64. 26

AB 28½ m
AC 32 6 m
sum 61 0 m
Diff 4 2 m

E ✱ 78° 45'
E c 25. 34
✱ c 53. 11

To find 1st B and C In the △ BAC To find BC

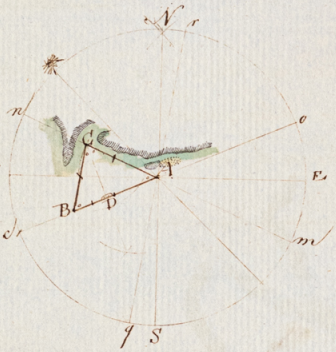
As sum given sides	61 0 m	7,21167	As sine l. B	82° 69	9,00519
To diff those sides	4 2 m	1,62325	To side AC	326	2,51322
To tan ½ sum unk.nl l.	78° 45	19,70134	To sine l. A	22° 36	9,58285
To tan ½ diff those l.s	19. 66	9,53926	To side BC	126 m	2,10025
	78. 45				
l. B	97 51				
l. C	39. 39				

Ans.r The Ships course is S 64. 26 E and 53. 11 from ye wind
And Distance 126 m.

Example 3rd

A pirate gave chace to a sloop, 15 miles ahead to the W.S.W. and both on their starboard tacks, the wind at N.W.; the sloop sending she lost way, tacked about for an island in the N.W. quarter, 11 m distant from her, and 13 miles from the pirate.

Required how near the wind the sloop must run to fetch the island, and whether the pirate can lie up for it or not.



N o = 67. 30
r o = 57. 34
N r = 9. 56

r N = 9. 56
N ✻ = 45. 00
✻ r = 54. 56

N x = 112. 30
x n = 45. 34
N n = 66. 32

A C = 13 m
B C = 11 m
sum = 24 m
diff = 2 m

2 | 15
7 | 5

N n = 66. 26
N ✻ = 45. 00
21 30

To find y diff of y segments

As the base or sum of the segments 15 m 0,82391
Is to the sum of the other two sides 24 m 1,38021
So is the diff of those sides 2 m 0,30103
To the diff of y segments of y base 2 | 3,200 0,50515

1,600
7,500
A D 9,100
D B 5,900

WILLIAMS'S 83 1-2 PATENT L.B

To find ∠C In ▽ △ ABC

As side AC = 11 m. ... 8,95861
To sine ∠D = 90.00 ... 10
So side AB = 3,90 m. 0,77085
To sine ∠C = 32.26 ... 9,72946

∠D 90.00
sum 122.26
180.00
∠B 57.34

To find ∠C In ▽ △ ADC

As side AC = 13 m. ... 8,08606
To sine ∠D = 90.00 ... 10
So side AD = 9,100 m. 0,95904
To sine ∠C = 44.26 . 9,84510

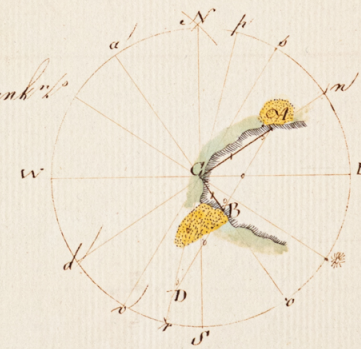
∠D 90.00
sum 134.26
180.00
∠B 45.34

Ans { The sloop's course is N 9.26 E and nearly 5 points from the wind. And the pirate cannot fetch the island

Example 4th

A ship sailing close by a point sees two light houses, one 18m to the N & E the other 8m to the S E & S.; the ship is on her larboard tacks, within 6 points of the wind, then at S E & E:

Req.d how far she must run to bring the light houses to bear in one line; and what will be the bearing and dist.ce of them from the ship at that time.



☉ r	76. 19
☽ ☍	32. 15
☽ r	44. 04

C A	18 m
C B	8 m
sum	26 m
½	13 m
C	10 m
∠ C	137. 49
	180. 00
	42. 11

n E	33. 15
E o	56. 45
∠ C	90. 00
	180. 00
2	90. 00
	45. 00 ½ sum unk.r s
n E	33. 15
E S	90. 00
S r	14. 04
∠ C	137. 49
∠ N	23. 59
	161. 48
	180. 00
∠ D	18. 12

N n	56. 45
S n	23. 59
N S	32. 46
C A	35. 70 m
A B	19. 68 m
C B	19. 02 m

To find ∠s A and B (in ꝺ A. A C B) To find A B.

As sum given sides 26 m.	8,58503
To diff those sides 10 m.	4
To tan ½ sum unk.r ∠s	45. 00 10.
To tan ½ diff those	21. 01 9,58503
½ B	45. 00
	66. 01
½ A	23. 59

As sine ∠ A 23. 59	0,39907
To side C B 8 m.	0,90309
To sine ∠ C 90. 00	10.
To side A B 19,68 m	1,29406

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To find CD (In $\triangle$ ACD)	To find AD.
As sine ½ D = 18. 12 0,50538	As sine ½ D = 18. 12 0,50538
To side C A 18m 1,25527	To side C A 18m 1,25527
So sine ½ A = 23. 59 9,60903	So sine ½ C = 42. 11 9,82705
To side CD = 23,13m 1,36968	To side AD = 38,70m 1,58770

Ans { The ship runs on her larboard Q. W ¼ W dist 23,13m
Then the lighthouses bear N 32. 16 E
The distance of A is 38,70m & of B 19,02m

Example 5th

There is a certain port, to sail into which you must bring the spot and a mill, 5 miles to y^e east thereof, to bear due west; at which time a beacon, 5 m to y^e N E b. N ½ E. of the mill, must bear N b E ½ E; you then steer directly for the beacon, & from thence to the port: Req^e the best wind, in y^e N W quarter, for a ship that goes equally well on both tacks, log^r with the distance she must run on each tack from the said situation to y^e port.



∠ 1	18° 30′ 30″
∠ D	19 51 00
	70 18 30
	180 00 00
∠ C	109 41 30
∠ N	109 41 30
	180 00 00
	70 18 30
C D	5 m
D B	5 m
sum	9 m
½	1 m
W N	90 00 00
N b	39 22 30
∠ B	129 22 30
	180 00 00
	50 37 30 sum
	25 18 45 ½ sum

sunk r^d ½ °

W b	20° 19′ 45″
	90 00 00
N	69 40 15
N b	19 51
N W	90 00
W b	20 19 45
	138 00 45
	69 00 22
	69 00 22
N b	49 11 00
N	19 49 22

To find A D in y^e △ B A D	
As sine ∠ A	70° 18′ 30″ ... 0,02617
To side B D	5 m ... 0,69897
So sine ∠ B	50 37 30 ... 9,88818
To side A D	4,105 m ... 0,61332

To find ∠D and C (In ▽ ABD) To find DC

As sum given sides 9m . . . 9.0457 | As sine ∠C 28° 19′ 45″ 9.72373
To diff those sides 1m . . . 0. | To side BD 3m 0.69897
To tan ½ sum unk.ᵗˢ ½ 25° 18′ 45″ 9.67483 | To sine ∠B 50° 37′ 30″ 9.88810
To tan ½ diff those ∠ˢ 3° 01′ 45″ 8.72059 | To side DC 8.158m 0.91150

25° 18′ 45″
∠C 28° 19′ 45″
∠D 22° 17′ 45″

Ans { She will run on her Starb.ᵈ ∡ NbE ½ E dist ½ 10.5m
And on her Starb.ᵈ ∡ SbE 5° 45′ W dist 8.158m
The best wind is that blowing from the
N W ½ W nearly }

Example 6th

A Ship that can lie on either tack within 6 points of the wind, then at E by E, having her starboard tacks aboard, sees her port direcly to windward, and a head-land that lies 6m to the N W. of her port, bore E by N½ now she wants to reach her port at two boards. Req.d her course and distance of each, and also the bearing & distance of the headland when she tacks.



C B = 6m
B D = 11.3m
sum 17.3m
d C = 5.2m
∠ B = 45.00
180.00
2)135.00 sum
67.30½ sum ∠s
N E = 90.00
E d = 45.00
N d = 45.00

∠ S = 45.00
S q = 78.45
∠ C = 33.45
q W = 11.15
q W = 22.30
∠ A = 33.45
∠ C = 123.45
∠ A = 33.45
sum 157.30
180.00
∠ B = 22.30
∠ C = 123.45
180.00
56.15

To find A B In ▽ ▵ A C B		To find A D In ▽ ▵ A B D	
As sine ∠ A 33.45	9,25526	As sine ∠ D 45.00	9,15052
To side C B 6m	0,77815	To side A B 8,980m	0,95328
So sine ∠ C 56.15	9,91985	So sine ∠ B 67.30	9,96561
To side A B 8,980m	0,95326	To side A D 11.73m	1,09611

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To find ∠s D & C (In △ C B D) To find D C

As sum given sides	17,73 m	8,75129	As sine ∠ D. 29. 32		9,20722
To diff those sides	5,73 m	0,75815	To side C B	6 m	0,77815
To tab ½ sum ∠s	75°. 30.	10,38278	To sine ∠ B	15. 30	9,49418
To tan ½ diff those ∠s	37. 30	9,89222	To side C D	8,07 m	0,93185

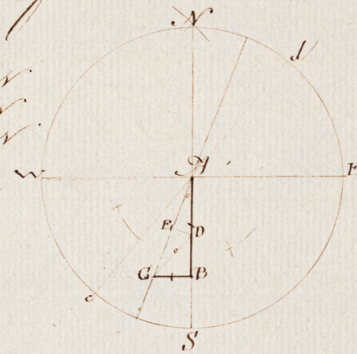
67. 30
− C 105. 28
∠ D 29. 32

Ans. { Ship runs on her starb N E dist 11,73 m
And on her larb S dist 11,73 m
The headland bears S 29. 32 W dist 8, 07 m }

Example 7th

A ship in latitude 47° 30 N and bound to a port in latitude 42° 00 N and 120 m to the west, proposes to reach her port by running equal distances on each tack, the first to be on the meridian. What must the course be on the second tack, and how far on each.

Lat in 47° 30 N
Lat bound to 42.00 N
Diff lat° 5.30 N
Co
330 m



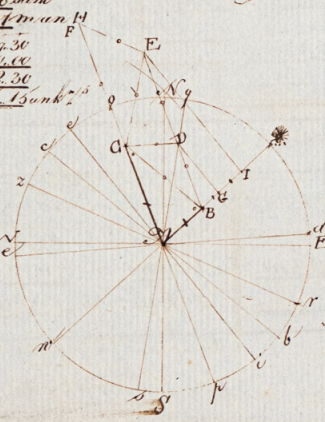
∠ BAC = 19.59
∠ ACD = 19.59
∠ BDC = 39.58

In ⅂ ABC To find AC	In ⅂ CBD To find DC
As dep 120 m 7,920 82	As rad 90.00 ..... 10, —
To rad 90.00 ..... 10, —	To dep 120 m 2,079 18
To diff lat 330 m 2,518 91	To cosec ACB 39.58 .... 10,192 23
To colat Co 19.59 .... 10,139 33	To dist 187 m 2,271 41

Ans: The first course is S, the 2nd S 33.58 W and the distance on each tack 187 m

Example 8th.

At six o'clock in the evening a privateer saw a ship directly to windward bearing N E. about 3 leagues distant, they were both close hauled, on their starboard tacks; the privateer having sailed 18m at 9 o'clock, the ship bore E. b. N ½ E. Now the privateer finding she got a-head, would speak with the ship at two o'clock in the morning. Req.d at what rate p.r hr privateer must sail & at what time tack about, keeping 6 pts from the wind, supposes the other ship does not alter her course & rate of sailing.



3 leag NE = 11,75 m
3 EC = 2,83 m
9 m S 7,98 sum
7,54 m anH

AB = 9 m ∠D 67.30
AG = 18 m 180.00
sum 27 m 2) 112.30
diff 9 m ½ sum 56.15 unk ∠
0 ✳ 67.30
d ✳ 30.22.30
∠D = 116.52.30
∠B = 29.44.00
136.36.30
180.00.00
∠G 43.23.30
∠D = 116.52.30
180.00.00
73.07.30
9.00.00
3.26.21
12.20.21

GD = 86,87 ∠D 73.07.30
DF = 26,65 180.00.00
sum = 28737 2) 106.52.30
diff = 18,36 53.26.15
0 ✳ = 67.30
0 2 = 26.22.45
9 ✳ 42.07.45
9 ✳ 42.07.45
r ✳ 67.30.00
∠E 109.37.45
∠G 25.22.15
135.00.00
180.00.00
∠F 45.00.00
∠E 109.37.45
180.00.00
70.22.15

7,51) 28,83 (3H
22,53
6,30
60
7,51) 190,00 (26 274
150,2 60
47,80 7,51) 164,40 (21
45,06 150,2
2,74 14,20
7,51
6,69

To find ∠ B and G (In ▽ ABG)	To find BG.
As sum given sides 27 m 8,36861	As sine ∠B 82.46 9,00347
To diff those sides 9 m 0,95424	To side AG 18 m 1,25527
To tan ½ sum unk ∠ 56.15 .. 10,17511	To sine ∠A 67.30 9,96561
To tan ½ diff those ∠ 26.31 .. 9,69799	To side BG 16,76 m 1,22435
∠B 82.46	
∠G 29.45	

To find BD (In △ CBD) To find DC

As sine ∠D	73°07′30″		0,01912
To side BC	16,76 m		1,22427
To sine ∠C	43°23′30″		9,83691
To side BD	12,03 m		1,08032

As sine ∠D	73°07′30″	0,01912
To side BC	16,76 m	1,22427
To sine ∠B	29°45′	9,69545
To side DC	36,07 m	1,93001

To find DE

As BG	3 m	9,52288
To GF	5 m	9,69897
So is BD	12,03 m	1,08037
To DE	20,05	1,30222

To find ∠ C and E (In the △ CDE) To find EC

As sum given sides	28,737 m		8,54156
To diff those sides	11,363 m		1,05549
So tabl ½ sum unk ∠	22°26′15″		10,12980
To tan ½ diff those ∠	28°54′		9,72605
	53°26′15″		
∠C	81°30′15″		
∠E	25°22′15″		

As sine ∠C	81°30′15″	0,00170
To side DE	20,05 m	1,30211
So sine ∠D	73°07′30″	9,98088
To side EC	19,39 m	1,28769

To find FE (In the △ CEF) To find FC

As sine ∠F	46°50′		0,15052
To side CE	19,39 m		1,28758
So sine ∠C	25°22′15″		9,63192
To side FE	11,75 m		1,07002

As sine ∠F	46°50′	0,15052
To side CE	19,39 m	1,28758
So sine ∠E	70°22′15″	9,97400
To side FE	25,73 m	1,41210

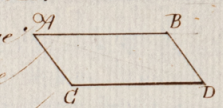
Ans { The privateer must tack at 12h 26′ 21″ And sail 7,51 m per Hour }

WILLIAMS'S 83 1/2 PATENT L.S.

Sailing in Currents.

A Current or Tide is a progressive motion of the water causing all floating bodies to move that way toward which the stream is directed. The setting of a tide or current, is that point of the compass toward which the water runs; & the drift of a current is the rate it run per hour. A Body moving in a current may be considered in three cases. Namely, Moving with, or the same way the current sets. Moving against, or the contrary way to that which the current sets. Moving obliquely to the currents motion. When a ship sails with the current, its velocity will be equal to the sum of its proper motion & the currents drift. When a ship sails against a current, its velocity will be equal to the diffce of the currents drift & its own motion. So that if the current drives stronger than the wind, the ship will drive astern, or lose way. When the course set is oblique to the motion of the current, the ships real course, or that made good, will be somewhere betn that in which the ship endeavours to go, and the track in which the current tries to drive it: that is, it will always be along the diagonal of a parallelogram, on which one side represents the distce run by the ship, & the other

adjoining side is the current's drift at the same time.
For suppose A B D C to be a parallelogram, the diagonal of which is A D.
Now if the wind alone would drive the ship from A to B, in the same time as the current alone would drive it from A to C. Then as the wind neither helps nor hinders the ship from coming toward the line C D, the current will bring it there in the same time as if the wind did not act. And as the current neither helps nor hinders the ship from coming toward the line B D, the wind will bring it there in the same time as if the current did not act. Therefore the ship must, at the end of that time, be found in both these lines, that is, in their meeting D. Consequently the ship must have passed from A to D in the diagonal A D. When the ships course and distance by account, & the setting and drift of the current, are given, the true course and distance of the ship may be found, either by working a traverse, in which the setting and drift of the current are used as the course and dist^ce, and by a trigonometrical solution of the triangles forming the figure so which is thus constructed. From the center of a circle draw the ships proper course and dist^ce, and the currents setting & drift, with these lines form a parallelogram, and from the center draw a diagonal



WILLIAMS' 83 1/2 PATENT

which will shew the ships real course and distance.

It may frequently happen, that a ship is plying to windward across a tides way. in such cases

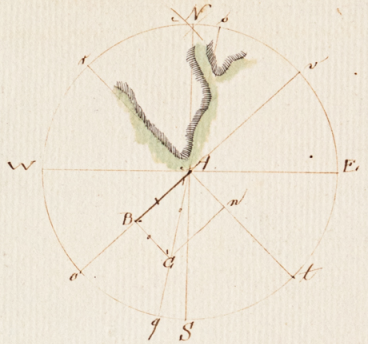
First construct the figure for the plying to windward. Then with the course on each tack, & the setting of the current, form parallelograms; on each of which one side must be the ships rate, and the other the current drift.

And lines drawn to meet one another from the center and parallel to the diagonals of those parallelograms, will shew the courses and distances which the ship must run on each tacks.

Example 1st.

A ship after doubling a cape meets a strong tide setting S.E; after running S.W. 18 m by the log, the cape bore N. $\frac{3}{4}$ E. Required the dist.ce of the cape, and the drift of the current.

∠ B = 90.00
∠ A 36.34
126.34
180.00
∠ C 53.26



To find AC (In $\frac{3}{4}$ △ ABC)	To find BC.
As sine ∠ C = 53.26 9,09529	As sine ∠ C = 53.26 9,09529
To side AB 18 m. 1,25527	To side AB 18 m. 1,25527
So sine ∠ B = 90.00 .... 10.	So sine ∠ A 36.34 9,77507
To side AC = 22,42 m 1,35056	To side BC 13,35 m 1,12562

Ans.r { The ship has run 22,42 m / The drift of the current is 13,35 m }

Example 2nd.

A ship sailing in with a current that set N.W. W.½W. discovers a riff of rocks on the larboard, the eastern point of which is nine miles to the N.E.b.N.½E.; and can be weathered by steering E.b.S. Req.d the dist.ce the ship must run by the log and also the drift of the current.



∠ A 61.62.30
∠ C 67.20.00
129.22.30
180.00.00
∠ B 50.37.30

To find A B (In the △ A B C)	To find B C
As sine ∠ B = 50.37.30 9,11182	As sine ∠ B = 50.37.30 .. 9,11182
To side A C = 9 m. 9,95424	To side A C = 9 m. 9,95424
So sine ∠ C = 67.30 .. 9,96361	So sine ∠ A = 61.62.30 .. 9,94542
To side A B = 10,73 m. 1,03167	To side B C = 10,27 m. 1,01118

Ans.r { The run by the log is 10,73 m. / The drift of the current is 10,27 m. }

Example 3rd

Passing by a cape at 10 o'clock in the evening a light-house bore S.W.b.S and a strong tide setting W.S.W½W at the rate of 3½ m an hour; we steered three Hy for the light house & at 1 in the morning we passed over a sandy point 28½ from the cape & light-house then bearing E.N.E½E. Req. the dist. bet. the light House and cape, also the course the ship has run.



35 m
2h
175 m

w E 16°.52'.30"
E S 90°.00'.00
Sh 33°.15'.00
∠B 140°.37'.30
180°.00'.00
39°.22'.30

∠B = 140°.37'.30"
∠C 22°.56'.00
163°.33'.30
180°.00'.00
∠A 16°.26'.30

Sh = 33°.15'
h w r = 22°.56
S r = 56°.11

To find ∠C (In y° △ ABC) To find BC.

As side AC = 28.5 m ... 8.545516	As sine ∠B = 39°.22'.30" ... 9.801776
To sine ∠B = 39°.22'.30" ... 9.800226	To side AC = 28.5 m ... 1.454484
To side AB = 17.5 m ... 1.243054	To sine ∠A = 16°.26'.30" 9.451179
To sine ∠C = 22°.56' ... 9.590556	To side BC 12.71 m 1.105663

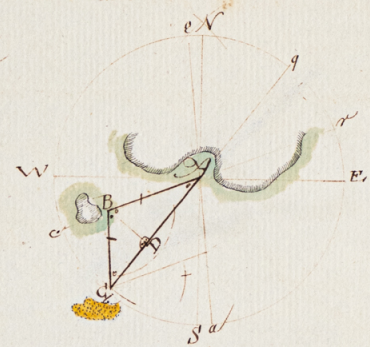
Ans. { The distance is 12.71 m
The course is S.E.½E }

WILLIAMS'S 83 1-2 PATENT L.B

Example 4th

A ship departs in the evening for an island, 16m to ye WSW. but by a strong tide running to the Southward she found herself next morning on a shoal 65m from her departed port & 35m from the island. Reqd ye bearing of her port & the setting of the tide.

AB = 16m
BC = 24m
sum = 80m
diff = 12m
½ 65m
32,5m



Nr = 67°.30
Ng = 29.34
57.36
½ a = 42.72
67 = 27.36
Ja = 15.36

To find ye diff of ye segments

As ½ ye base or sum of the segments = 65m 8,187709
To ye sum of ye 2 other sides = 80m 1,90309
So is ye diff of those sides = 12m 1,07918
To ye diff of ye segments of ye base = ½ 14,77m 1,16936
29,54m

AD 32,5
DC 25,02 m
7,52

To find ∠ B (In ye △ CDB

As side CB = 38m ... 8,46852
To sine ∠ D = 90.00 ... 10,
So side CD = 25,12m ... 1,40002
To sine ∠ B = 41.38 ... 9,86854
90.00
∠ C 48.22

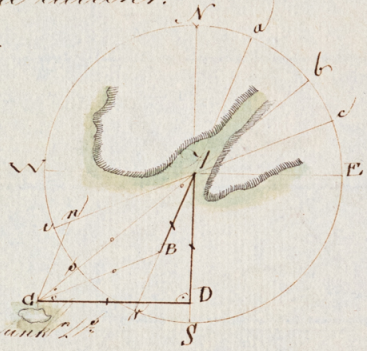
To find ∠ B (In ye △ ADB

As side AB = 16m ... 8,33724
To sine ∠ D = 90.00 ... 10,
So side DA = 30,08m ... 1,60076
To side ∠ B = 6.06 ... 9,93300
90.00
∠ A 29.54

Ans { The port bears N 37°.36 E
{ The current sets S 41.6 E

Example 5th

At eight o'clock in the evening, a Ship sailed from her port on a S.S.W. course, four miles an hour; a strong tide was then setting between the South & West; & at six o'clock in the morning she passed by a rock 70 miles to the westward and 60 to the southward of her port. Req^d the setting and drift of the current.



8 m
10 h
80 m

∠ A 89.24
90.00
∠ C 40.26
S 89.24
S 22.30
∠ 126.54
180.00
2)103.06
76.33½ sum and 2^d

A B 80 m
A C 92.19 m
sum 132.19 m
diff 32.19 m
186 9.30 m
5.93 m

S 89.24
2^d 17.46
S 67.10

To find ∠ A (In ▽ △ A D C) To find A C.

As diff lat 60 m	8.22185	As rad 90.00	10.
To radius 90.00	10.	To dep 70 m	4.84510
To dep 70 m	4.84510	To cosec H^a 59.24	10.11960
To tan C 49.24	10.06695	To dist^ce 92.19 m	1.96470

To find ∠^s B & C (In ▽ △ A B C) To find B C

As sum given sides 132.19 m	7.87870	As sine ∠ C 17.46	0.51540
To diff these sides 32.19 m	1.50759	To side A B 80 m	1.90206
To tan ½ sum unk^d 76.33	10.61132	To sine ∠ A 26.54	9.65556
To tan ½ diff these 58.47	10.21761	To side B C 29.30 m	1.47302
∠ B 185.20			
∠ C 17.46			

Ans^r { The setting of the current is S 67.10 W
The drift of the current 5.93 m an hl

WILLIAMS & 33 1/2 & 2 PATENT L^D

Example 6th.

Coming out of a port, in to which the tide of flood was then setting, I saw two headlands, the northern one of which bore E. N E. ½ E. was known to be 15 miles to the N. N E. ½ E. of the other, which then bore S by E. after running E b S ½ E 22 m by the log, the northern headland bore N W and the other W b S. Req.d the setting and drift of the tide

○E	16.. 52.. 30
E f	78.. 45.. 00
∠ACB	95.. 37.. 30
Sk	73.. 07.. 30
Sh	22.. 30.. 00
∠CBA	145.. 30.. 00
∠ACB	95.. 37.. 30
	240.. 37.. 30
	180.. 00.. 00
∠C. AB	60.. 22.. 30
∠ ACB	95.. 37.. 30
	180.. 00.. 00
	84.. 22.. 30
∠. ADB	= 56.. 15
∠. ABD	= 73.. 07.. 30
	129.. 22.. 30
	180.. 00.. 00
∠ B. AD	50.. 37.. 30



AC	10.66 m
AD	17.26 m
sum	27.92 m
diff	6.60 m

∠C. AD	= 90.. 00
	180.. 00
	2) 90.. 00
	45.. 00
p r	= 31.. 52
p s	= 16.. 52.. 30
∠C	= 14.. 49.. 30
	180.. 00.. 00
	2) 165.. 10.. 30
	82.. 55.. 15

CD	= 20, 29 m
CE	= 24.60 m
sum	= 44.. 29 m
diff	= 4.. 31 m
Sm	= 93.. 37.. 30
m G	49.. 48.. 15
i S	43.. 49.. 15

In a ▵ A B C. To find A C	To find A D. In a ▵ A B D
As sine ∠ C = 84.. 22.. 30 0,00210	As sine ∠ D = 56.15 ..... 9, 08015
To side A B = 15 m 1,17609	To side A B = 15 m 1,17609
So sine ∠ B = 45.. 00 .. 9, 84948	So sine ∠ B = 73.. 07.. 30 .. 9, 98088
To side A C = 10,66 m .. 1,02767	To side A D = 17.26 m .. 1,23712

To find ∠s C and D (In a ▵ C A D) To find C D.

As sum given sides = 27.92 m .. 9, 55408	
To diff. of base sides = 6.60 m .. 9, 81954	
So tan ½ sum unk.d ∠s = 45.. 00 .. 10,	
To tan ½ diff. of base ∠s = 12.. 18 .. 9, 37362	
∠ C = 57.. 18	
∠ D = 32.. 42	

As sine ∠ C = 88.18'	9, 07017
To side A D = 17.26 m	1, 23704
So sine ∠ A = 90.. 00.. 00	
To side D C = 20, 29 m	1, 30721

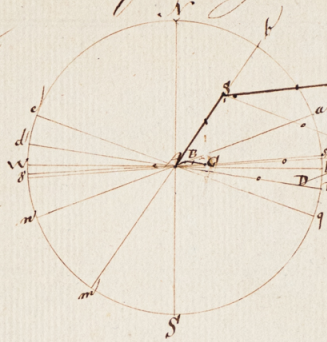
To find ∠D and E (In ▽ CDE) To find DE

As sum given sides 1,429m ... 8,35369	As sine ∠E = 49.48.15 9,11690
To diff those sides 371m ... 9,56937	To side CD = 1029m 1,30728
To tan ½ sum unk ½ 82.35.15. 10,88573	To sine ∠C = 15.49.30 9,40806
To tan ½ diff those ½ 32.47 .. 9,80879	To side DE 679m 9,83224
D 115.22.15	
E 19.48.15	

Ans. { The current sets S 15.49.15 W
And its drift is 6,796m }

Example 4th

A ship running five knots an hour, discovers Scilly lights bearing N ℄ E. ½ E. distant six leagues, the flood tide being just made, and setting E. N. E. 3 miles an hour. Required what course she must steer, & how far by the log, to bring herself five leagues to the South of the Lizard, which lies 11 leagues to the E by. N ½ E. of Scilly.



6 leag = 5 leag = 11 leag
3m 3m 2m
18m 15m 33m

LS = 33m
LE = 15m
sum = 48m
diff = 18m

L L = 84. 22. 30
180. 00. 00
2) 95. 37. 30 sum
½ sum 47. 48. 45 sum ℄'s

AS = 18m
SE = 34,87m
sum = 52,87m
diff = 16,87m

L S = 10 4. 02. 45
180. 00. 00
2) 75. 57. 15 sum
37. 58. 37 sum ℄'s
b r = 51. 57. 37
b a = 33. 45. 00
L A = 18. 12. 37
L E = 70. 12. 27
L = 84. 22. 30
75. 25. 57
180. 00. 00
L D = 104. 34. 03
b r = 51. 57. 37
b E = 36. 15. 06
r E = 14. 57. 29
r D = 18. 12. 37
E D = 13. 55. 14
E S = 90. 00. 00
S D = 76. 04. 46

To find L S and E (In the △ SLE) To find SE

As sum given sides	48m	8,31876	As sine L E 70. 17. 45	9,97610
To diff those sides	18m	1,25527	To side S L 33m	1,51851
To tan ½ sum sum ℄'s	47. 58. 45	10,04271	So sine L S 84. 22. 30	9,99790
To tan ½ diff those L's	22. 20	9,61674	To side S E 34,87m	1,54251
L B	70. 17. 45			
L S	25. 19. 45			

To find L's A and E (In ye △ ASE) To find AE

As sum given sides	52,87m	8,27679	As sine E 73. 39. 37	9,98070
To diff those sides	16,87m	1,22712	To side A S 18m	1,25527
So tan ½ sum sum ℄'s	75. 57. 15	10,89245	So sine L S 75. 57. 15	9,98682
To tan ½ diff those L's	13. 59	9,39636	To side AE 42,93m	1,63279
L E	34. 37. 37			
L A	75. 59. 37			

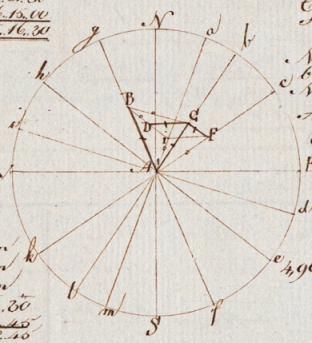
To find ∠C (In ⅀△ ABC	To find ∠D (In ⅀△ ADE
As side AB = 5m. . . . 9,30103	As sine ∠D = 26.33.37 9,36771
To sine ∠A = 18.12.37 . 9,49485	To side AE = 42,92m. . 1,63276
To side AB = 2m. . . . 0,30103	To sine ∠E = 18.12.37 . . . 9,49485
To sine ∠C = 7.11. . . . 9,09691	To side AD = 31,28m. . 1,49532

Ans { The ship must sail S.76.0.W. to D
Distance 31,28m }

WILLIAMS & 33 1/2 PATENT L.B.

Example 8th

In lat^{e} 18.50 N. we were attacked by a privateer, who upon receiving our broadside lay to, and we stood away N. NW; some time after the privateer bore away N E b N. and two hours after, he having by our estimation run ten miles, was 7 miles directly east of us. Now by our account we had sailed four leagues since the action, & the tide all the time had run two miles an hour. Req^{d} the setting of the tide, the time the privateer lay to, our rate of sailing, & each ship's direct course & dist^{ce}



AB=12m ∠A=56.15 Vi=71.31.30
AC=10m 180.00 LW=56.15.00
sum=22m 2)123.45 Wi=15.16.30
diff=2m 61.52.30

BC=10,52m ∠C 15.16.30
CD=7,00m 180.00.00
sum=17,52m 2)64.43.30
diff=3,52m 82.21.45

Cd=15.16.30 CA=10m
de=26.04.15 CF=4m
ce=41.21.15 sum=14m
180.00.00 diff=6m
∠C 138.38.45
56.15.00 BD=4,195m
∠G 82.23.45 DF=4,000m
180.00.00 BF=8,195m
2)97.36.15 ∠B=2.12.30
48.48.07 ∠B 26.04.15
∠B 26.08.45
AB=12m 180.00.00
BF=8,195m 2)153.51.15
sum=20,195m 76.55.37
diff=3,805m

Ed=15.16.30 3/4 195
de=26.04.15 bl 2,097
ce=41.21.15 60
Ef=90.00.00 + 5820
fc=48.38.45 60
+ 49200
Nb=33.45 ad=37.52.37
bc=22.43.30 gV=24.30.00
Nc=56.28.45 aV=13.22.37
Ad=5.49 ∴ 5,88m
60 60
5,88666 60 6,000000
4,969444 4000000
3882
2000000
8000000
8000000
5000000 m
4,969444 / 5,882000000 / 1,183
4969444
9123560
4969444
4156160
3970552
1856080
1490832
1477440

To find ∠s A and C (In △ AB. BC) To find AC

As sum given sides=22m ... 9,865758	As sine ∠C=71.31.30 ... 9,922908
To diff those sides = 2m ... 0,30102	To side AB=12m ... 1,079181
To ½ sum ∠s A & C=61.52.30 10,27207	To sine ∠A=56.15 ... 9,919855
To tan ½ diff those ∠s = 9.39 ... 9,23067	To side BC=10,52m 1,022201
∠C 71.31.30	
∠A 52.13.30	

To find ∠s B and D (In ▽ △ BCD) To find BD.

As sum given sides 17,52 m. . . . 8,79617	As sine ∠ B 18. 26. 0″. 45‴ . . . 9,35693
To diff those sides 3,52 m. . . . 0,54654	To side DC 7 m. 0,84510
So tan ½ sum unk ∠s 82. 34. 15. 10,87259	To sine ∠ C 15. 16. 30. . . . 9,42070
To tan ½ diff those ∠s 56. 17. . . 10,17560	To side BD 4,195 m. . 0,62273
∠ D 138. 33. 15	
∠ B 26. 08. 45	

To find ∠s A and A (In ▽ △ ACA) To find AA

As sum given sides 11 m. . . . 8,85387	As sine ∠ A 74. 53. 07″. 9,01529
To diff those sides 6 m. . . . 0,77815	To side AC 10 m. . . 1.
So tan ½ sum unk ∠s 48. 48. 07. 10,05781	So sine ∠ C 82. 33. 45. 9,99616
To tan ½ diff those ∠s 26. 05. . . 9,68982	To side AA 10,27 m. 1,01145
∠ A 74. 53. 07	
∠ A 22. 43. 07	

To find ∠s A and I (In ▽ △ ABI) To find AI.

As sum given sides 20,195 m. . . 8,69476	As sine ∠ I 6. 11. 23. 9,02626
To diff those sides 3,805 m. . . 0,58035	To side AB 12 m. . . . 1,07918
So tan ½ sum unk ∠s 76. 55. 37. 10,63442	So sine ∠ B 26. 08. 45. 9,64410
To tan ½ diff those ∠s 39. 02. . . . 9,90923	To side AI 5,882 m. 0,76954
∠ I 115. 58. 37	
∠ A 27. 52. 37	

Ans.
- The side sep's I & 8. 28. 45″ E
- The privateer lay to 2. 5. 49
- Our rate of sailing is 4,183 m an hour
- The privateer's course is N 56. 28. 07″ E dist.ce 10,27 m
- The other ship's course is N 15. 22. 37″ E dist.ce 5,882 m

WILLIAMS'S 33 1/2 PATENT L.B

WILLIAMS'S 33 1/2 PATENT L.B

Globular Sailing.

Definitions & Principles.

By Globular Sailing is meant the manner of estimating either the difference of lat^e, diff^ce of long^e, departure, course, or dist^ce from any two of them being known, upon principles deduced from the spherical figure of the Earth. As meridians are circles on the terraqueous globe, meeting in the poles, it is obvious that any two of these circles must recede more from one another at greater distances from both poles; or at the equator, the distance bet^n the meridians is the greatest.

The true place of a ship at sea depends upon its dist^ce from the equator, & from some noted meridian; & because the meridional dist^ce that is the dist^ce bet^n two

meridians, varies in every lat.e; it is therefore convenient that this distance should be reckoned in a fixed lat.e & where the degrees are of the same magnitude with those of the meridian, which can be only at the equator, where 60 nautical miles make a degree.

Now a ship sailing from any lat.e upon an obliquez humb until she has made a known departure, by the rules of plane sailing will reckon herself under the same meridian whether she sails to the northward or southward.

But from what has been said, this can not be true for the same meridional distance which in this case is taken as departure, will not serve two different lat.es on the same side of the equator therefore the common methods of plane sailing alone are not sufficient to shew the ship's true place, a necessary correction being wanted with respect to her long.e, & this correction depends on the following principles.

The distance bet.n any two meridians at the equator, is in proportion to their dist.e in any parallel of lat.e, As the radius, to the cosine of that lat.e.

Any part of a parallel of lat.e is to a like part of a meridian as radius is to the secant of the lat.e of that parallel. The distance of any parallel of lat.e from the equator is expressed by the sum of the secants of all the arcs between the equator & that parallel.

Hence it appears that by the addition of the secants of small arcs the dist^ces of the parallels of latitude from the equator are obtained. And these several dist^ces, which are called meridional parts, being disposed in a table corresponding to the degrees & minutes in a quadrant, form a table of meridional parts.

The difference betw^n the meridional parts to two given lat^des is called the Meridional Diff^ce of Lat^e, to distinguish it from the diff^ce of lat^e of those two places reckoned in miles of the equator, which is called the proper difference of lat^e.

If a ship sails on any oblique rhumb, the proper diff^ce of latitude, the departure, the meridional diff^ce of lat^e, the diff^ce of long^e, & the radius to the tan^t of the course, are all in the same ratio.

The tangents of the angles which different rhumbs make with the meridians are directly proportional to the diff^ces of long^e made on those rhumbs to equal meridional diff^ces of lat^e: Or are reciprocally proportional to unequal meridional diff^ces of lat^e on those rhumbs, when the differences of long^e are equal.

If a ship sails on several different rhumbs, the whole meridional diff^ce lat^e is equal to the sum or diff^ce of all the successive meridional diff^ces of lat^e:

And the whole diff^ce long^e is also equal to the

sum or diffᵉ of all the successive diffᵉˢ of longᵉ the sums being taken when they are of the same name, & the diffᵉ when of contrary names

If the course betᵉ any two places, outwards & homewards be on the opposite rhumbs; then in each case the diffᵉˢ of latᵉ, departures, & diffᵉˢ of longᵉ, will always be the same. The divisions of the meridian line on the Mercator's chart form a table of the diffᵉˢ of longᵉ answering to each minute of latᵉ on the rhumb line making an angle of 56 with the meridian.

On the globe, the rhumb lines obliquo to the meridians are spirals, which continually approach the poles.

The meridian line is a scale of logarithmic tangents of ½ the complement of the latᵉ beginning from the radius.

WILLIAMS'S 33 1/2 PATENT LB

Parallel Sailing.

Parallel Sailing is the art of finding what dist^ce a ship should run due east or west, in sailing from the meridian of one place to that of another place, in any parallel of lat^e.

Rule 1st

As radius ..... R
To the cosine of the lat^e of any parallel ..... S
So is the miles of long^e betw^n any two meridians ... L
To the dist^ce of those meridians in that parallel .. D

S : R :: D : L
L : D :: R : S

Rule 2nd

As the cosine of one lat^e ..... S
Is to the cosine of another lat^e ..... s
So is a given merid^l dist^ce on the first parallel ... D
To the corres^g merid^l dist^ce in the 2nd parallel ... d

D : d :: S : s.

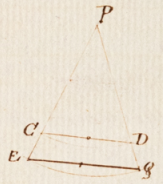
Case 1st

Given the lat^e & diff^ce of long^e;
Req^d the distance.

Example

A ship in the lat^e of 37. N sails due east till her diff^ce of long^e is 275 m^s. Req^d the distance she has run

To find the dist^ce		
As radius	90. 00	10.
To cosine lat^e 37. 00		9.77946
So miles long^e 275 m^s ...		2.43933
To miles dist^ce 165.5 m^s ...		2.21879



Ans^r Distance 165.5 m^s

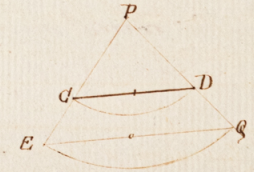
Case 2nd

Given the lat^e & meridional dist^ce;
Req^d the diff^ce of long^e

Example

A ship from the lat^e 51. 37 N long^e 5. 15 E sails due west 305 miles. Req^d her present long^e

To find the diff^ce of long^e		
As cosine lat^e 51. 37		9.79579
To radius 90. 00		10.
So mer^l dist^ce 305 m^s ...		2.48430
To diff long^e 389.1 m^s ...		2.69009



Ans^r Diff^ce long^e 389.1 m^s

WILLIAMS'S 83 LAKE PATENT

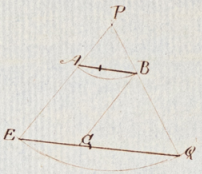
Case 3rd.

Given the diffce of longe & ps correspondeng mer. distes
Reqd the parallel of late

Example

In what late do 153m of meridional distce answer to 210m of diffce of longe.

To find the late
As diff longe 210m ... 7,38722
To mer. distce 153m ... 2,18469
So radius 90d0 ... 10,
To cosine late 68.05 ... 9,57191



Ans. late 68.05 N

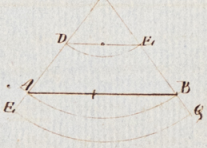
Case 4th

Given the distce of two places in a given parallel of latitude; Reqd the distance of two other places, under the same meridian with the former, in another given parallel of late

Example

From two ports in the late 31.15 N distant 138m two ships sail directly north, how far are they distant from one another when they come to a late 37. N.

To find the Distce
As cosine 1st late 31.15 ... 6,08808
To cosine 2nd late 37. ... 9,70271
So mer. distce 1st parallel 138m ... 2,58133
To mer. distce 2nd parallel 259m.1 ... 2,41512



Ans. Distce 259,1m

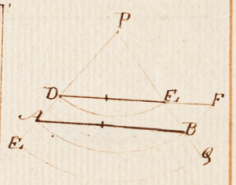
Case 5th

Given the dist.ce & l.e the same two meridians in diff.t parallels & also the lat.e of one parallel, to find the lat.e of the other.

Example

If two ships in the lat.e 54.37 N. distant from one another 260 miles should both sail directly South until her distance is 384 miles. What lat.e is she come to.

To find the lat.e		
As mer.l dist.ce 1st parallel	260 m.	7,585486
To mer.l dist 2nd parallel	384 m.	2,584133
To cosine 1st lat.e	54.37	9,762971
To cosine 2nd lat.e	31.15	9,931900



Ans.r Lat.e 31.15 N.

WILLIAMS'S 33 1/2 PATENT L.B

Middle Lat.e Sailing

Middle Latitude Sailing is a manner of solving the several cases of globular sailing, by the principles of plane & parallel sailing jointly.

This method, which is not quite accurate, is founded on the foll.g supposition.

Rule 1.st

If the places are on the same side of the equator, add the lat.e sailed from to the lat.e arrived at, & take half the sum for the middle lat.e; but if they are on different sides of the equator half their diff.ce is the middle lat.e

Rule 2.nd

As the cosine of the middle lat.e C
Is to the radius ..... R
So is the departure ..... D
To the diff.ce of long.e L

L : D :: R : C
R : C :: L : D

Rule 3.rd

As the cosine of the middle lat.e C
Is to the tan.t of the course ..... T
So is the diff.ce of lat.e ..... P
To the diff.ce of long.e L

P : L :: C : T

Mercator's Sailing

Mercator's Sailing is the art of resolving the several cases of Globular Sailing by Plane Trigonometry with the assistance of a table of meridional parts, or of logarithmic tangents. The computations herein are performed by the following rules.

Rule 1st

As merid.l diff.lat.e ..... M
To diff long.e L
So is the radius ..... R
To lang.t course ..... T

R : T :: M : L
T : R :: L : M

Rule 2nd

As the prop.l diff.lat.e ..... P
To the departure ..... D
So mer.l diff.lat.e ..... M
To diff long.e L

M : L :: P : D.

WILLIAMS'S 33 L.2 PATENT L.B

Problem 1st

Given the lat.e of two places:
Req.d the merid.l diff.ce of lat.e bet.n those places.

Case 1st

When both places are on one side of the equator.
Rule. The difference of the merid.l parts answering to each lat.e will give the meridional difference of lat.e req.d.

Example 1st

What is the merid.l diff.ce of lat.e bet.n Dundee and Bergen.

Dundee's lat.e 56° 26′ N
Bergen's lat.e 60° 10′ N
Merid. parts to 56° 26′ is 4121
Merid. parts to 60° 10′ is 4547
Merid. diff.ce of lat.e ... 426

Example 2nd

What is the diff.ce of lat.e bet.n Flamborough Head & Shields.

Flamborough Head lat.e 54° 08′ N
Shields lat.e 55° 02′ N
Merid. parts to 54° 08′ ... 3878
Merid. parts to 55° 02′ ... 3971
Merid. diff.ce lat.e ... 93

Case 2nd

When the given places are on diff.t sides of the Equator. Rule. The sum of the merid.l parts answering to each lat.e will be the meridional diff.ce of lat.e req.d.

Example 1st

What is the merid.l diff.ce of lat.e bet.n Dublin and Cape Horn.

Dublin's lat.e 53° 21′ N
C. Horn's lat.e 55° 59′ S
Merid parts to 53° 21′ is ... 3799
Merid parts to 55° 59′ is ... 4072
Merid diff.ce of lat.e 7871

Example 2nd

What is the merid diff.ce of Lat bet.n Calais & C. Farewell.

Calais's lat.e 50° 58′ N
C. Farewell 44° 35′ S
Merid parts to 50° 58′ is ... 3566
Merid parts to 44° 35′ is ... 2995
Mer Diff.ce lat.e ... 6561

Problem 2nd.

Given the lat.e of one place, & the merid.l diff.e of lat.e bet.n that & another place.

Req.d the lat.e of the other place.

Case 1st.

When the given lat.e & mer diff lat.e have like names. Rule To the merid parts of the given lat.e add the given mer diff lat.e seek the sum in y.e table of mer parts, & the corres degrees & minutes will be the lat.e sought.

Example 1st.

A ship from the lat.e 56.17 N. sails northward till she finds her mer diff lat.e 354. What lat is she come to.

Lat from 56.17 N. M P 928 N
Mer diff lat.e 354 N
Sum 1475

Which ans.r to 59.35 N lat sought

Example 2nd.

A ship from the lat.e 52.26 N sails northward till her mer.l diff lat.e is 854. What lat is she come to.

Lat from 52. 26 N. M P 1121 N
Mer diff lat.e .... 854 N
Sum 1975

Which ans.r is 67.32 N. lat sought

Case 2nd.

When the given lat.e & mer diff lat.e have unlike names Rule. Take the diff.ce bet.n the mer parts of the given lat.e & the mer diff lat.e the rem.r found in the table of mer parts, will give the lat.e sought

Example 1st

A ship from the lat.e 28.07 S sails northward till her mer diff lat.e 254. What lat.e is she come to

Lat from 28.07 S. M P 1759 S
Mer diff lat.e 254 N
Diff.ce 1505

Which ans.r is 28.19 S lat sought

Example 2nd.

A ship from the lat.e 52.20 N sails Southward till her mer diff lat is 382. What lat is she come to.

Lat from 52.20 N. M P 2809 N
Mer diff lat.e .... 384 S
Diff.ce 2425

Which ans.r is 37.26 N lat.e sought

WILLIAMS & SON PATENT

Problem 3rd

Given the lat^es & long^es of two places:
Req^d the bearing & dist^ce.

Example

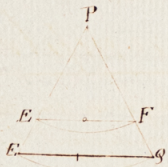
What is the course & dist^ce bet^n Cape Clear in Ireland, & the island of St. Mary one of the Azores.

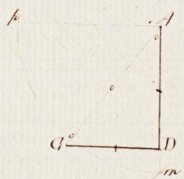
By Middle Lat^e Sailing

C. Clear's lat^e = 51. 16. N
St. Mary's lat^e = 37. 00. N
Diff lat^e = 14. 16 N = 858 m.

C. Clear's long^e = 9. 50 W
St. Mary's long^e = 25. 00 W
Diff long^e = 15. 10 W = 910 m.

Sum of the lat^es 88. 16, mid lat^e 44. 08; comid lat^e 45. 51



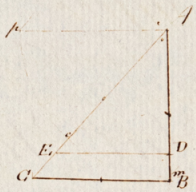


To find the Course.		To find the Dist^ce
As diff lat^e = 858 m.	7,06651	As cosi C = 37. 16 0,09918
To diff long^e = 910 m.	2,95904	To diff lat^e = 858 m. 2,93346
To cosine m. lat^e = 45. 51	9,85383	To rad 90. 00 .... 10. —
To tan^t C = 37. 16	9,88138	To dist^ce 1078 m. 3,03267

Ans^r { Course S 37. 16 W
Dist^ce 1078 m }

By Mercators Sailing.

C: Clears long^{e} 9.50 W	C: Clears lat^{e} 34.40 N. M P 3597
S^{t}. Marys long 25.00 W	S^{t}. Marys lat 37.00 N. M P 2393
Diff long^{e} 15.10 W = 910	Diff lat^{e} 2.18 = 858 M Diff 1204



To find the Course	To find the Dist^{ce}
As m d lat = 1204 . . 6,91927	As radius = 90.00 . . . . 10, —
To diff long^{e} = 910 . . 2,99094	To prop^{d} lat = 858 m . . 2,93319
To radius = 90.00 . . 10, —	To sec^{t} Co. 37.05 . . 10,09813
To tan Co. 37.05 . . 9,87841	To dist^{ce} 1076 . . . 3,02162

Ans^{r} { Course 37.05 W / Dist^{ce} 1076 m }

WILLIAMS'S 83 1-2 PATENT

Problem 7th

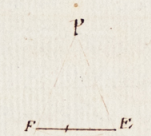
Given the bearing & ..... lat^s of two places
Req^d the dist^ce & diff^ce of long^e

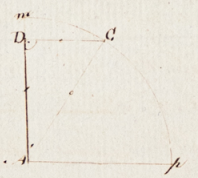
Example

A ship which had taken her departure from a place in lat^e 37.00 N long^e 22.56 W steers N 33.19 E; and cloudy weather coming on she got no obs^n till 8 days after, & then found herself in lat^e 51.18 N. Req^d the true dist^ce & present long^e.

By Middle Lat^e Sailing.

D^p last lat^e	37.00 N	Sum of 2 lat^s	88.18	Co 7.86	
Pres^t lat^e	51.18 N	mid lat^e	44.09	E 13.06 D long^e	
Diff lat^e	14.18 858^m		90.00	W 22.56 Dep long^e	
		Co mid lat^e	45.51	W 9.50 pres^t long^e	





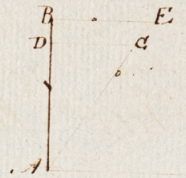
To find the Dist^ce		To find the Diff^ce long^e	
As rad	90.00 ... 10	As cosi. m lat^e	51.69 ... 9.796417
To d. lat^e	858 m ... 2.933349	To tan^t Co	33.19 ... 9.817776
To sec^t Co	33.19 ... 10.077708	To diff^ce lat^e	858 m ... 2.933349
To dist^ce	1027 m ... 3.011057	To diff^ce long^e	786 m ... 2.895542

Ans^r { Dist^ce 1027 m
Pres^t long^e 9.50 W }

By. Mercator's Sailing.

Depart lat^{e} 37.00 N. MP 2392
Present lat^{e} 51.18 N. MP 3597
Diff lat^{e} 14.18 = 858 MDL 1205

60 / 721
E 12.11 D long^{e}
W 22.56 Dep^{t} long^{e}
W 9.45 Pres^{t} long^{e}



To find the Dist^{ce}		To find the D: long^{e}	
As cosi. Co 33.19	0,07798	As rad 90.00	10.
To d. lat^{e} 858 m	2,93249	To tan Co 33.19	8,91776
To radius 90.00	10.	To m.d lat^{e} 1205 m	3,08063
To dist^{ce} 1027 m	3,01047	To d. long^{e} 741 m	2,89839

Ans^{r} { Dist^{ce} 1027 m
Pres^{t} long^{e} 9.45 W }

Problem 5th

Given one lat.e course & dist.ce
Req.r the other lat.e & diff.ce of long.e

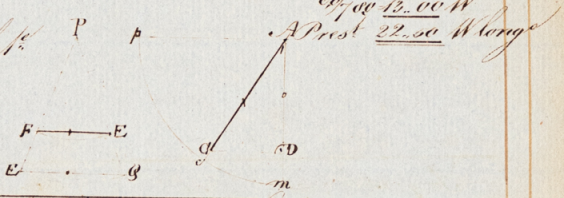
Example

A ship takes her departure from a place in lat.e 51.10 N long.e 9.50 W; & steers S 33.00 W till she has run 102 m:
Req.r her present lat.e & long.e

By Middle Lat.e Sailing

Nor.e 51.10 N Dep. lat.e
Co 85 7 11.17 E Diff lat.e
27.01 N Pres.t lat.e
2 88.19 Sum of the lat.e
44.09 Mid. lat.e
90.00
45.51 Co.m. lat.e

Nor.e 9.50 W
Co 89 13.00 W
A Pres.t 22.50 W long.e



To find the Diff lat.e	To find the Diff long
As rad 90.00 ..... 10, ——	As cos m lat 44.09 ... 9,84417
To Dist.ce 102 m ... 2,01030	To tan Co 33.00 9,81473
As cos Co 33.00 9,92293	To d. lat.e 85,5 m .. 2,93323
To d. lat.e 85,5 m .. 2,93323	To d long.e 80,1 m 2,89213

Ans { Pres.t Lat.e 27.01 N
Pres.t Long.e 22.50 W }

By Mercator's Sailing.

Now 54.18 N dep. lat.
97.57 = 43.17 S Diff lat.
27.01 N Prest lat.

A Parts to 54.18 is 3597
A Parts to 27.01 is 2394
A D Lat 1202

Now 9.50 W Dep. Long.
97.83 = 13.05 W Diff long.
22.55 W Prest long.

To find the Diff lat.		To find the diff long.	
As sec. Co 32.00	9,92293	As rad 90.00	10, —
To Dist. 102 m.	3,01030	To m. d. lat. 1203 m.	3,08027
As rad 90.00	10, —	As tan Co. 32.00	9,81473
To d. lat. 85 7/5 m.	2,92323	To diff long. 78 2/5 m.	2,89500

Ans { Prest lat 27.01 N
Prest long 22.55 W }

Problem 6th

Given the lat^es of two places & their dist^ce
Req^d their bearing & diff^ce of long^e

Example

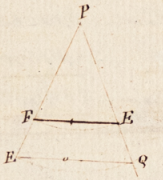
A ship in lat^e 51.18 N long^e 22.06 W, is bound to a port in the SE quarter, distant 102 m, and in lat^e 37.00 N: what is her direct course, & how much must she alter her long^e to arrive at the desired port.

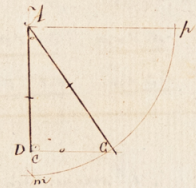
By Middle Lat^e Sailing

Lat from 51.18 N
Lat in 37.00 N
D. lat^e 14.18 N 858^m

Sum lat^es 88.18 N
Mid lat^e 44.09 N
90.00
Co-mid lat^e 45.51 N

Now 22.06 W Long from
60 779 12.19 E diff long^e
9.07 W pres long^e



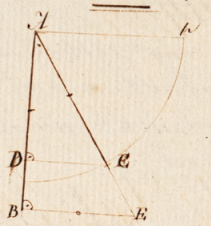


To find the Course	To find the diff long^e
As dist^ce 102 m ... 6,98960	As cos^n mid lat^e 44.09 .. 9,84417
To rad 90.00 .... 10, —	To Plan Co 33.06 .. 9,81418
Sub. lat^e 858 m ... 2,93349	Sub. lat^e 858 m .. 2,93349
To cos Co 33.06 9,92309	To d. long^e 779 m .. 2,89184

Ans^r { Course 33.06 E or SE by S nearly
Present long^e 9.07 W }

By Mercators Sailing.

Lat from 51.18 N | M.P.... 3597 | Nau 22.10 W long.e from
Lat.e to 27.00 N | M.P.... 2393 | 97.43.0 E diff long.e
Diff lat.e 24.18 N 858 m. M.D.L. 1204 | 19.12 W pres.t long.e



To find the Course	To find the diff long.e
As diff lat.e 858 m. .. 7,06651	As rad 90.00 .... 10.
To radius 90.00 .. 10.	To m.d lat.e 102.4 m. 3,08063
To dist.ce 102.4 m. .. 3,01030	To tan C 33.05 9,81390
To sec C 33.05 .. 10,07681	To d. long.e 78.4 m. .. 2,89453

Ans { Course S 33.05 E or S E by S nearly
Present long.e 19.12 W }

WILLIAMS'S 33 1/2 PATENT L.D

Problem 7th

Given the lat^es of two places, & the departure.
Req^r the course dist^ce & diff of long^e

Example

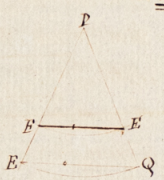
A ship from the lat^e 27.00 N long^e 18.20 W, sails 6 d^s the N and E & being come to the lat^e 32.18 N, finds she has made 564 m of departure: what was her direct course, dist^ce run, & pres^t long^e

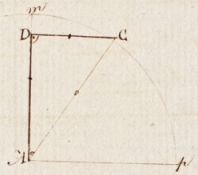
By Middle Lat^e Sailing.

Prest lat^e 32.18 N
Dep lat^e 37.00 N
Diff lat^e 14.18 N 858

Sum lat^s 58.18 N
Mid lat 41.09 N
90.00
C^e mid lat^e 75.51

Now 18.20 W dep long^e
69.26 = 12.06 E diff long^e
25.14 W prest long^e





To find the course	To find the Dist^ce	To find the diff long^e
As d. lat 858 m ... 9.06651	As sine C^o 32.19 ... 9.26022	As cos m. l 41.09 ... 9.87417
To rad 90.00 ... 10.	To dep 564 m ... 2.75128	To tan C^o 32.19 ... 9.81776
To dep 564 m ... 2.75128	To rad 90.00 ... 10.	To d. lat 858 m 2.93319
To tan C^o 32.19 ... 9.81779	To dist^ce 1027 m 3.01150	To long^e 706.0 m 2.89512

Ans: { Course N 32.19 Cor. NE. N nearly
Dist^ce run 1027 m
Present long^e 25.14 W }

By Mercators Sailing

Pres^t lat^e 51. 18 N | M P .... 3697 | Note 58.20 W dep^t lat^e
Dep^t lat^e 37. 00 N | M P .... 2393 | 6798 13.11 E diff^e long^e
Diff lat^e 14. 18 N = 858^m. M D L 1204 | 35. 09 pres^t long^e



To find the Course	To find the Dist^e	To find the diff^e long^e
As dep 858^m 7,241879	As lan C 33. 19 .. 9,818224	As rad 90. 00 10,
To rad 90. 00 10,	To dep 858^m .. 2,933649	To m. d. l 120^m .. 2,079181
To p. d lat 858^m 2,933649	To sec C 33. 19 . 10,093798	To lan C 33. 19 9,818776
To cot C 33. 19 .. 10,181221	To dist^e 1027^m 3,011830	To diff^e long^e 791^m 2,898139

Ans^r { Course N 33. 19 E or N E b N nearly
Dist^e run 1027^m
Present lat^e 35. 09 N }

WILLIAMS'S 83 1-2 PATENT L.S

Problem 8th

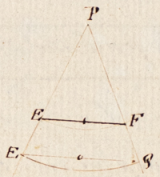
Given one lat.e the course, and departure
Req.d the other lat.e dist.ce & diff.ce of long.e

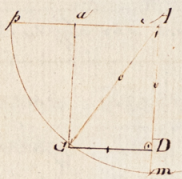
Example

A ship from the lat.e 51. 18 N sails S 33. 19 W until her departure is 56 miles required her present latitude long.e & dist.ce

By Middle Lat.e Sailing

Now 51.18 N dep lat.e
69.8 38 1/2 .. 18 S diff lat.e
37. 00 N pres lat.e
Sum lat.es 88. 18 .. N
Mid. lat.e 44. 09 .. N
90. 00
Co-m lat.e 45. 51





Now 9. 50 W dep long.e
69.7 06 13. 06 W diff long.e
22. 56 W pres long.e

To find the diff lat.e	To find the Dist.ce	To find the diff long.e
As rad 90. 00 .. 10.	As Sec Co 33. 19 .. 0,26022	As cos m. lat 44. 09 .. 0,14417
To dep 56 m. .. 2,75128	To dep 56 m. .. 2,75128	To tan Co 33. 19 .. 9,81776
So cal Co 33. 19 .. 10,18224	So rad 90. 00 .. 10.	So d. lat 58 81 m. .. 2,93255
So d. lat 58 81 .. 293352	So dist 1027 m. .. 3,01150	So d long 7 01 0 m. 2,09547

Ans: { Pres lat 37. 00 N
Pres long 22. 56 W
Dist 1027 m }

By Mercator's Sailing.

Now 51.18 N dep lat^{e}
69.28 = 14.18 S diff lat^{e}
37.00 N pres lat^{e}

M P to 51.18 . . 3507
M P to 37.00 . . 2393
M D 1 1114

Now 9.50 W dep long^{e}
69.92 = 13.11 W diff long^{e}
23.01 W pres long^{e}



To find the diff lat^{e}	To find the Dist^{e}	To find the d. long^{e}
As tan Co 33.19 . . . 9.81224	As rad 90.00 . . . 10.	As rad 90.00 . . . 10.
To dep 564 m . . 2.75128	To dep 564 m . . 2.75128	To m d l 1204 m . . 3.08062
So rad 90.00 . . 10.	So co-sec Lo 33.19 10.26022	So tan Co 33.19 . . 9.81776
To d. lat 1858.1 m 3.26952	To dist^{ce} 1027 m 3.01150	To d. long^{e} 791.4 m 2.89838

Ans^{r} { Present lat^{e} 37.00 N
Present long^{e} 23.01 W
Dist^{ce} 1027 m }

WILLIAMS'S 83 1/2 PATENT L.S.

Problem 9th

Given one lat^e, the dist^e, & departure
Req^d the other lat^e, course, & diff^e long^e

Example

A ship from the lat^e 27.00 N, long^e 9.02 W, having sailed betw^t the N and W 107 miles, reckons she has made 56^m of departure; what was her direct course, & the pres^t lat^e & long^e

By Middle Lat^e Sailing

Now 27.00 N Dep. lat^e
9.838 4.18 N Diff lat^e
51.18 N Pres lat^e

Sum lat^s 78.18
Mid lat^e 45.09
90.00
Co-m. lat^e 45.51

Now 9.02 W dep long^e
9.838 13.06 W diff long^e
22.08 W Pres long^e



To find the Course	To find the diff lat^e	To find the d. long^e
As dist^e 107 m. 9,9881	As tan Co 33.49 ... 9,8224	As cos m lat 45.09 ... 9,8417
To rad 90.00 ... 10,	To dep 56 m ... 2,7528	To tan Co 33.49 ... 9,8177
To dep 56 m ... 2,7528	To rad 90.00 ... 10,	To d. lat^e 858.1m 2,9335
To sine Co 33.49 ... 9,7397	To d. lat^e 858.1m 2,9335	To d long^e 786.0m 2,8954

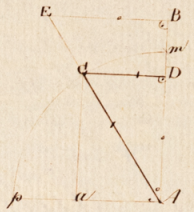
Ans^r { Course S 33.49 W
Pres^t lat 51.18 N
Pres long^e 22.08 W }

By Mercator's Sailing.

Now 37.00 N dep Lat
69838 - 14.18 N diff Lat
51.18 N Pres Lat

M Parts to 37.00 is 2293
M Parts to 51.18 is 2597
M D L 304

Now 9.02 W dep Long
69791 - 13.11 W diff Long
22.13 W Pres Long



To find the course	To find the d. Lat	To find the d. Long
As dep 36 m. 1.24872	As rad 33.19 . . . 9.26022	As rad 90.00 . . . 10.
To rad 90.00 . . . 10.	To dep 36 m. 2.75528	To m. d. Lat 120 m. 3.08402
Sub dist 102 m. 2.01157	Sum Co 33.19 . . 9.92202	To tan Co 33.19 . . . 9.81776
To cos Co 33.19 . 9.26029	To d. Lat 858 m 2.93352	To d long 794 m 2.89839

Ans. { Course N 33.19 W
Prest Lat 51.18 N
Prest long 22.13 W }

WILLIAMS'S 83 1/2 PATENT L.B

Problem 10th

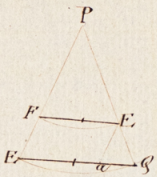
Given one lat.e departure & diff.ce of long.e
Req.d the other lat.e course & dist.ce

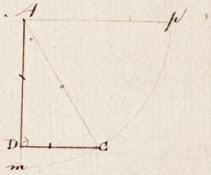
Example

A ship from the lat.e 51.10 N, long.e 22.00 W, having sailed bet.n the S and E. for several days, reckons she has made 36 m.s of departure, & 78 m.s of long.e: what is the lat.e & long.e of the place arrived at, & also her direct course & dist.ce

By Middle Lat.e Sailing.

Acco Comid lat.e	51. 51	
	90. 00	
Mid lat.e	141.09	
	2	
Sum of 2 lat.e	80. 18	
Dep.t lat.e	51. 10 N	
Prest lat.e	27. 08	
Diff lat.e	1. 10 = 838 m	
Dep.t long.e	22.06 W	
78 E ... 2 ...	13.06 E	
Prest long.e	9.00 W	





To find ½ co-mid lat.e	To find the Course	To find the Dist.ce
As d long.e 78 m 7.10 58	As dlat.e 838 m. 7.06 51	As sin Co 33.19 9.26022
To dep.t 36 m. 2.75128	To rad 90.00 10.	To dep.t 36 m. 2.75128
To rad 90.00 10.	To dep.t 36 m. 2.75128	To rad 90.00 10.
To cosin lat.e 27.09 9.95506	To tan Co 33.19 9.81779	To dist.ce 102.7 m. 3.01150

Ans.r { Present lat.e 27.00 N; Present long.e 9.00 W; Course 33.19 E; Dist.ce 102.7 m }

Problem 11th

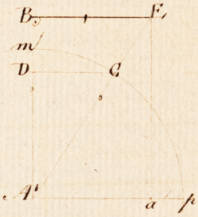
Given one Lat.e course, & diff.ce of long.e:
Req.e the other Lat.e & distance.

Example

A ship from the Lat.e 37.00 N. long.e 22.56 W. runs on a course S 32.19 E. until she finds her diff.ce of long.e is 786 m.s what is her present Lat.e & distance sailed.

By Mercators Sailing.

Dep.t Lat.e 37.00 N. M.l P. 2393
M.l D.L 1196
Pres.t Lat.e 54.13 N. M.l P 3589
Diff.ce Lat.e 17.13 N. 853 m.s



To find the mer.l diff.ce Lat.e	To find the distance
As tan.t 32.19 . . . 9.80224	As cos.e 32.19 . . . . 9.97798
To d.f long.e 786 m.s . . . 2.89542	To d.f Lat.e 853 m.s . . . 2.93095
To rad 90.00 . . . . 10.	To rad 90.00 . . . . . 10.
To mer.l d.L 1196 3.07766	To dist.ce 1021 m.s . . . 3.00893

Ans.r { Present Lat.e 54.13 N
Dist.ce 1021 m.s }

WILLIAMS'S 83 1/2 PATENT L.B

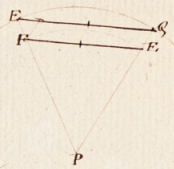
Problem 12th

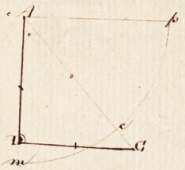
A ship from Cape Comorin having sailed 68 leagues the Sea Wd and East reckons she has made 286 miles of Dep. & 3.249m of long.e: Required her course & distance & present lat.e & long.e

By Middle Lat.e Sailing.

Dep 56.52 Co-mid lat.e
90.00
32.08 Mid lat.e
2
66.16 Sum lat.e
7.55 Dep lat.e
58.21 Prest lat.e
30.26 Diff lat.e

90)3249 (36.07 E dep. long.e
270
549
450
990
990
50.59 E dep long.e
135.00 E Prest long.e





To find the mid lat.e	To find the Course	To find the dist
As d. long.e 3.249m 6.511800 To dep. 286 m 2.456307 To rad 90.00 .. 10. To cos. m.l 23.00 .. 9.962207	As sd. lat.e 302.6m 6.519153 To rad 90.00 .. 10. To dep. 286 m 2.456307 To tan Co 13.25 .. 9.976610	As rad Co 13.25 .. 9.962385 To dep 286 m 2.456307 To rad 90.00 .. 10. To dist.e 276.7m 2.418692

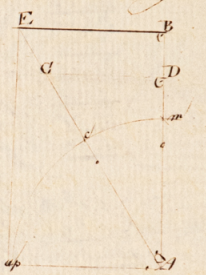
Ans.r { Course S 13.25 E; Dist.ce 276.7m; Prest lat.e 58.21 N.; Prest long.e 135.00 E }

Problem 13th

A ship from Lima runs on a course N 31.16 W and her diff.e of long.e is 3786 m: What is her present lat.e & distance?

By Mercator's Sailing

Lat from 12.01 S. M Parts 726
M D Lat 6235
Prest lat.e 67.14 N. M P 5509
Diff lat.e 79.15 N. = 4755 m.



To find the mer.d diff lat.e	To find the Dist.ce
As tan C 31.16 9,21660	As cosi C 31.16 0,06816
To d. long.e 3786 m . . . 3,57818	To d. lat.e 4755 m . . . 3,67715
To rad 90.00 . . . . . 10.	To rad 90.00 . . . . 10.
To merd. lat.e 6235 m . . 3,79484	To dist 5563 m . . . 3,74534

Ans.r { Prest lat.e 67.14 N / Dist.ce 5563 m }

Compound Courses corrected by Longitude.

How to solve compound courses or traverses, on the principles of plane sailing, has been already shewn; but it is necessary also to shew how the proper allowances for longitude are to be introduced into such accounts; which is easily done by any of the following methods.

Rule 1st Complete the traverse table to each course & dist.ce, & find the whole difference of latitude, departure, & present latitude.

2nd With the difference of latitude & departure find the direct course & distance.

3rd If you work by middle latitude, with the departed latitude & present lat.e find the co-mid. latitude with which & the departure find the difference of longitude.

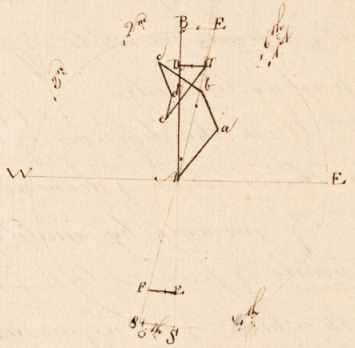
Or 3rd If by Mercators, with the departed lat.e & present lat.e find the meridional diff.ce of lat.e with which & the direct course, find the diff.ce of long.e

Example 1st

Yesterday noon we were in Lat. by obs. 28° 14' N, Long. by Chron. 25° 56' W, & have since run the follg courses & distances.
1st. N E b. N ½ E. 56 m; 2d. N. N W. 38 m; 3d. N W b. W. 46 m;
4th. S S E. 30 m. 5th. S b. W. 20 m; 6th. N E b. N. 60 m.
Req. The ships place with her direct course & dist.

Traverse Table

Courses	D.	N.	S.	E.	W.
N E b. N ½ E.	56	45,0		33,4	
N. N W.	38	35,1			14,5
N W b. W.	46	25,6			38,2
S S E.	30		27,7	11,5	
S b. W.	20		19,1		5,9
N E b. N.	60	49,9		33,3	
		155,4	47,2	78,2	56,6
		47,2		56,6	
	Diff. Lat.	108,2	Dep.	21,6	



Dep. & Lat. 28° 14' N
Diff. Lat. & Log. 1,48 N
Mer. Lat. 30,02 N

To find the course	To find the Dist.
As dif. Lat. 108,3 m. ... 7,96537	As rad. 90.00. ... 10,
To rad. 90.00. ... 10,	To d. Lat. 108,3 m. ... 2,03463
To dep. 21,6 m. ... 1,334445	To sec. C. 11.29. ... 1,00078
To tan. C. 11.29. ... 9,29982	To dist. 110,5 m. ... 2,04341

WILLIAMS'S 33 1-2 PATENT L.B

By Middle Lat.e Sailing

Dep.t lat = 38. 4 N	Sum lat.s = 78. 16	Dep.t long.e = 25. 56 W	
Prest lat = 40. 52 N	Mid lat = 39. 08	Diff long.e $\frac{28}{20}$ = 0. 28 E	
Sum lat.s = 78. 16 N		Prest long.e = 25. 28 W	
	90. 00		
	Co.m lat.e = 50. 52		

To find the diff long.e	
As co.m lat.e 39. 08	0. 11032
To radius	10.
To dep.t	24. 6 m.l
To d. long.e	178 m.l

By Mercators Sailing

Dep.t lat 38. 4 N	M Parts 2486	Dep.t long.e 25. 56 W	
Prest lat 40. 52 N	M Parts 2625	Diff long.e $\frac{28}{20}$ = 0. 28 E	
M D Lat.e = 139		Prest long.e 24. 28 W	

To find the diff long.e	
As radius = 90. 00	10.
To m. d lat.e = 139 m.l	2. 14310
To tan Co = 41. 29	9. 30782
To d. long.e = 28. 24 m.l	1. 45092

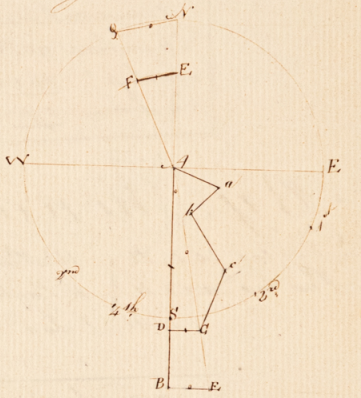
Ans.r { Present lat.e 40. 52 N; Present long.e 24. 28 W; Course N 41. 29 E; Dist.ce 110. 5 m.l }

Example 2nd

Suppose a ship from the latitude 2° 20′ N & long. 14° 28′ W has sailed the following courses, viz.

1st E.S.E. 43m 2nd S.W. 22m 3rd S.E.b.S. 58m 4th S.S.W. 60m

Required her present lat. & long. with her direct course & dist.



Traverse Table

Courses	D	N	S	E	W
E.S.E.	43		16,4	39,7	
S.W.	22		22,0		22,6
S.E.b.S.	58		48,2	32,2	
S.S.W.	60		55,4		23,0
		DLat	126	71,9	45,6
				45,6	
				26,3	Dep.

Dep. Lat. = 52,20 N
Diff. Lat. 43 = 2,22 S
Arres Lat. = 49,97 N

To find the Course		To find the Dist.	
As dep. 26,3 m.	8,58001	As cosec. 10,27	0,00726
To rad. 90.00	10, —	To d. lat. 126 m.	2,10112
To d. lat. 126 m.	2,10112	To rad. 90.00	10, —
To cotan. 10,27	10,73116	To dist. 145,0 m.	2,16138

WILLIAMS'S 83 1-2 PATENT L.B

By Middle Lat.e Sailing.

Dep.t Lat.e = 52.20 N	Sum of the Lat.s	102.17		Dep.t long.e = 14.38 W	
Prest Lat.e = 49.57 N	Mid Lat.e	51.08		D. long.e Co = 0.42 E	
Sum Lat.e 102.17		90.00		Prest long.e 13.56 W	
	Co mid Lat.e	38.52			

To find the diff long.e	
As cos.e m.l = 51.08	0,20238
To rad = 90.00	10,
So dep.t = 262 m.	2,41996
To d. long.e = 491 m.	2,62234

By Mercators Sailing.

Dep.t Lat.e = 52.20 N	M.t Parts = 3698		Dep.t long.e = 14.38 W	
Prest Lat.e = 49.57 N	M.t Parts = 3470		D: long.e = 0.42 E	
Mer diff Lat.e =	228		Prest long.e 13.56 W	

To find the diff long.e	
As rad = 90.00	10,
To mer.d.l = 228 m.	2,35793
So tan Co = 10.27	9,26585
To d. long.e = 420 m.	2,62378

Ans.r
- Present Lat.e 49.57 N
- Present long.e 13.56 W
- Course S 10.27 E
- Dist 425,0 m.

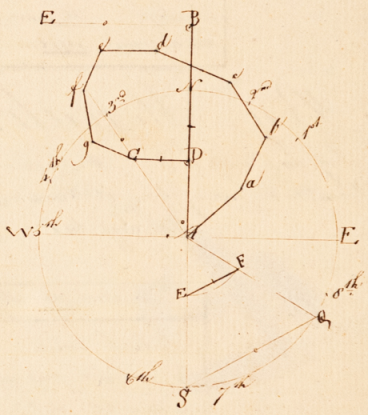
Example 3rd

Suppose a Ship from the latitude 67.30. N, long.e 2.46 W, has sailed the following courses viz.

N E. 64 m. N. N E. 50 m. N N W. N. 38 m. W. N W. 72 m.
West, 18 m. S. S W. 38 m. S E. 45 m. E S E. 40 m.
Req.d her present Lat.e & long.e

Traverse Table.

Courses	D	N	S	E	W
N E	64	45.3		45.3	
N. N E	50	46.2		19.1	
N W. N	38	38.2			32.2
W N W	72	27.6			66.5
West	18				18.0
S S W	38		35.1		14.5
S E	45		31.1	31.8	
E S E	40		15.3	36.9	
		167.3	94.5	110.1	161.2
		94.5			110.1
	Dlat	72.8		Dep	51.1



Dep.t lat 67.30 N
D. lat.e 72 1.13 N
Pres.t lat.e 68.43 N

To find the Course	To find the Dist.
As rad 90.00 ... 10.	As rad 90.00 ... 10.
To dep.t 51.1 m. ... 1.70842	To dep.t 51.1 m. ... 1.70842
To tan.t 35.0 ... 9.84629	To sec.t 35.0 ... 10.08699
	To dist 88.4 m. 1.95912

WILLIAMS'S 83 1/2 PATENT

By Middle Lat.e Sailing

Dep lat.e = 67.50 N | Sum lat.es = 2.136.12 | Dep long.e = 8.16 W
Pres lat.e = 68.42 N | Mid lat.e = 68.06 | D: long.e 137 / 60 = 2.17 W
Sum lat.e 2 = 136.12 | 90.00 | Pres long.e = 11.02 W
Comid lat.e = 21.54

To find the d: long.e	
As cos m.l = 68.06	6,42831
To rad = 90.00	10,
So dep = 54.1 m	4,70842
To d: long.e = 137.0 m	2,13673

By Mercators Sailing

Dep lat.e = 67.50 N. Mer Parts 5552 | Dep long.e = 8.16 W
Pres lat.e = 68.42 N. Mer Parts 5747 | D: long.e 120 / 60 = 2.18 W
Mer diff lat.e = 195 | Pres long.e = 11.01 W

To find the diff long.e	
As radius = 90.00	10,
To m. d. lat = 195 m.	2,29002
So tan Co = 25.01	9,84620
To d: long.e = 137.8 m.	2,13622

Ans.r { Pres lat.e 68.42 N / Pres long.e 11.01 W }

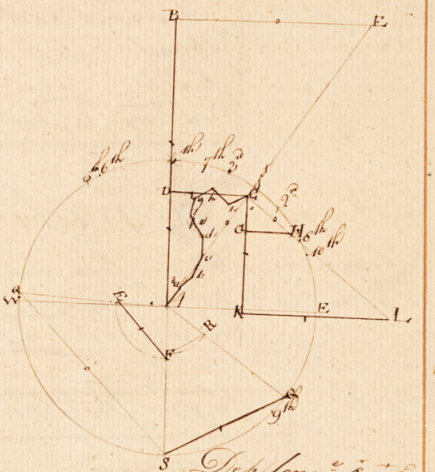
Example

Suppose a ship from the Lat^{e} of 68.28 N, long^{e} 8.40 E has sailed the following courses viz. N E b. N 62 m. N E 28 N. N E 56 m. North 20 m. N W b. N 25 m. N N W ½ W 26 m. N b E 40 m. N E b E ½ E 72 m. S E 50 m. E S E 65 m.

Req^{r} her present Lat^{e} & long^{e} with her direct course & distance to the North Cape in Lat^{e} 71.27 N long^{e} 26.20 E.

Traverse Table

Courses	D.	N	S	E	W
N E b N.	62	58.4		23.0	
N E	28	26.9		26.9	
N N E	56	51.7		21.1	
North	20	20.0			
N W b N	25	24.8			12.9
N N W ½ W	26	24.8			17.0
N b E	40	39.2		7.8	
N E b E	72	66.9		62.5	
S E	50		25.2	25.3	
E S E	65	21.9		60.0	
		346	253	249	29.9
		253		209	
Diff		276.3	Sal	210.0	Dep



Dep lat^{e} = 68.28 N. MP = 5724
Dif lat^{e} = 26.25 N.
Pres lat^{e} = 73.14 N. MP = 6582
Sum lat^{e} = 141.57. M P L = 649.
Mid lat^{e} = 71.06

Co mid lat^{e} = 90.00 − 19.04

Prest lat^{e} = 73.14 N. MP = 6583
N Cape lat^{e} = 71.27 N. MP = 6230
Diff lat^{e} = 1.47 = 107. M D L = 353
Sum lat^{e} = 144.41
Mid lat^{e} = 72.20

Co mid lat^{e} = 90.00 − 17.40

Dep long^{e} = 8.40 E
D long^{e} = 11.10 E
Pres long^{e} = 19.50 E

Dep long^{e} = 8.40 E
D long^{e} = 11.12 E
Pres long^{e} = 19.52 E

Pres long^{e} = 19.50 E
N Cape long^{e} = 26.20 E
Diff long^{e} = 6.30 = 400 m

To find the Course Distance & diff long^{e} betw^{n} the place sailed from & the place come to

By Middle Lat^{e} Sailing

To find the course	To find the Dist	To find the d. long^{e}
As d. lat = 276 .. 7,55862	As cos Co 38 24 .. 9,10585	As cos m.l 70,56 .. 0,48589
To rad = 90.00 .. 10,	To d. lat = 276 m. 2,44128	To rad 90.00 .. 10,
So dep = 219.0 .. 2,34044	So rad = 90.00 .. 10	So dep 219.0 .. 2,34044
To tan Co = 38.24 .. 9,89906	To dist 352,6 m. 2,54723	To d. long^{e} 670 m. 2,82633

By Mercators Sailing

To find the Course	To find the Dist	To find the d. long^{e}
As dep 219 m. 7,65956	As rad Co 38 24 .. 9,20681	As rad 90.00 .. 10.
To rad 90.00 .. 10	To dep 219.0 .. 2,34044	To m.d.l. 849 m. 2,92891
To d. lat 276 m. 2,44128	So rad 90.00 .. 10	To tan Co 38 24 .. 9,89905
To co Co 38 24 .. 10,10094	To dist 352 m. 2,54725	To d. long^{e} 672 .. 2,82796

To find the Course & Distance betw^{n} the Place come to

S. N. Cape.

By Middle Lat^{e} Sailing

To find the Course	To find the Dist
As d. lat = 107 m. .. 7,97062	As cos Co 48 36 .. 9,17959
To d. long^{e} = 400 m. .. 2,60206	To d. lat = 107 m. .. 2,02928
As cos m. lat = 72 20 .. 9,48212	So rad = 90.00 .. 10,
To tan Co = 48.36 .. 10,05481	To dist 161.8 .. 2,20887

By Mercator's Sailing

To find the Course		To find the Dist	
As m.d. Lat = 383 m.	7,15222	As cos Co 48.24	0,17931
To rad 90.00	10,	To d. Lat 107	2,02928
To d. Long = 400 m.	2,60206	To rad 90.00	10.
To tang 48.24	10,05429	To dist 161,7	2,20869

Ans { Present Lat 7.24 N
Pres Long 19.50 E
Course to N Cape S 48.24 E
Dist to N Cape 161,7 m }

WILLIAMS' 3 1 2 PATENT

WILLIAMS'S 83 1/2 PATENT L.S

Spherics

Definitions & Principles

Spherics is that part of the Mathematics which treats of the position & magnitude of arcs of circles described on the surface of the Sphere. A Sphere is a solid contained under one uniform round surface such as would be formed by the revolution of a circle about its diameter that diameter being immoveable during the motion of the circle. The Center & Axis of a sphere are the same as the center & diameter of a generating circle and as a circle has an undefinite number of diameters so a sphere may be considered as also having an undefinite number of axes round any one of which

the sphere may be conceived to be generated.

Circles of the Sphere are those circles described on its surface by the motion of the extremities of such chords in the generating circle as are at right angles to the diameter, or to the axis of the sphere.

The Poles of a circle on a sphere, are those points on its surface equally distant from the circumference of that circle.

A Great Circle of the sphere, is that circle which is equally distant from both its Poles.

Lesser Circles of the sphere, or small circles, are those circles which are unequally distant from both their Poles.

Parallel circles of the sphere, are those circles, the planes of which are considered as parallel to the plane of some great circle. A Spheric Angle is the inclination of two great circles of the sphere meeting one another.

A Spheric Triangle is formed on the surface of a sphere by the mutual intersections of three great circles.

The Stereographic Projection of the sphere, is such a representation of its circles, upon the plane of one of them passing through the centre, & called the plane of projection, as would appear to an eye placed in one of the poles of that great circle, & thence viewing the circles on the sphere.

WILLIAMS'S 83 1/2 PATENT L.

The Orthographic Projection of the sphere is the method of representing its circles on the plane of any one of them as they would appear to an eye placed in the axis of that circle at an infinite dist from it.

The place of the eye is called the projecting point, or lower pole & the point diametrically opposite is called the remotest, or opposite, or upper pole. Also the projection of any point on the sphere, is that point in the plane of projection through which the visual ray passes to the eye. The Primitive Circle is that great circle on the plane of which the representation of all the other circles are supposed to be drawn.

An Oblique Circle is one which has its plane oblique to the eye.

A Great Circle is that which is perpendicular to the plane of the primitive circle & if it be a great circle its plane passes through the eye, & is seen edgewise; consequently it is represented by a straight Line drawn through the center of the primitive circle.

Axioms

The diameter of every great circle passes through the center of the sphere but the diameters of small circles do not pass through the same center. Also the center of the sphere is the common center to all its great circles.

Every section of a sphere, by a plane passing through its circumference, is a circle. A sphere is divided into two equal parts by the plane of every great circle, & into two unequal parts by the plane of every small circle.

The poles of every great circle is a 90 distance from it on the surface of the sphere. And no two great circles can have a common pole.

The poles of a great circle are the extremities of that diameter or axis, of the sphere, which is perpendicular to the plane of that circle.

If one great circle meet another, the angles on either side are supplements to one another & every spheric angle is less than 180°.

A spheric angle is measured by an arc of a great circle intercepted betⁿ the legs of that circle, 90 distant from the angular point.

If two circles intersect one another the opposite angles are equal. Two spheric triangles are congruous, if two sides & their contained angle in one, are equal to two sides & their contained angle in the other, each to each: Or if two angles & the contained side in the one, are equal to two angles & the contained side in the other each to each. Or if three sides in the one are respectively equal to the three sides in the other.

WILLIAMS' 83 1-2 PATENT L.P.

All parallel circles have the same pole, and may be conceived to be concentric to the great circle which they are parallel to. All parallel circles on the sphere, having the same pole, are cut into similar arcs by two great circles passing through that pole.

Prop. 1st. Great circles of a sphere mutually cut one another into two equal parts. Corol. 1st. The circumferences of any two circles intersecting one another twice, make the angles at both sections equal.

Corol 2nd. Two great circles of the sphere will cut each other twice at the distance of 180°, or in opposite points of the sphere.

Prop. 2nd. The Distance of the poles of two great circles, is equal to the angle formed by the circumferences of those circles.

Corol. 1st. Two great circles are at right angles to one another, when they pass through each others poles.

Corol 2nd. The pole of a great circle is 90° distant from it, taken in any other great circle, or is an arc of it, drawn perpendicular to the former circle.

Corol. 3rd. Two or more great circles, at right angles to another great circle, intersect one another 90° distant from it, [illegible] in the pole of the latter circle.

And the like of arcs of great circles. Corol 4th. If several great circles intersect one another, in the pole of another

great circle; then are the former circles perpendicular to the latter.

Prop. 3d. In the Stereographic projection of the sphere, the representations of all circles, not passing through the projecting point, will be circles.

Corol 1st. Hence the middle of the projected diameter is the center of the projected circle, whether it be a great circle or a small one. Corol 2nd. Hence in all circles parallel to the plane of projection, their centers & poles will fall in the center of the projection.

Corol 3d. The centers & poles of circles, inclined to the plane of projection, fall in that diameter of the primitive circle which is at right angles to the diameter drawn through the projecting point.

Corol 4th. All oblique circles cut the primitive circle in two points diametrically opposite.

Prop. 4th. The measure of the angle which the projected diameter of any circle subtends to the eye, is equal to the distance of that circle from its pole, which is most remote from the projecting point, taken on the surface of the sphere. And that angle is bisected by a right line joining the projecting point & that pole. Corol 1st. Hence every projected oblique great circle cuts the primitive circle in an angle equal to the inclination of the plane of that oblique circle to

the plane of projection

Corol 2nd The distance betn the projections of a great circle & any of its parallels is equivalent to their distance on the sphere. Prop 5th Any point of a sphere stereographically projected, is distant from the centre of projection, by the tangent of half the arc intercepted between that point & the pole opposite to the eye. The semidiameter of the sphere being made radius.

Prop 6th The angle made by the intersection of the circumferences of two circles in the same plane, is equal to the angle made by tangents to those circles in the point of section; & also is equal to the angle made by their radii drawn to that point.

Prop 7th The angle which any two circles make, when stereographically projected, is equal to the angle which those circles make on the sphere.

Prop 8th The distance betn the poles of the primitive circle & an obligue great circle, in stereographic projections is equal to the tangent of half the inclination of those circles; and the distance of their centres is equal to the tangent of their inclination. The semidiameter of the primitive circle being made radius.

Prop 9th If through any given point in the primitive circle an obligue circle be described; then the

centers of all other obliquer circles passing through that point, will be in a right line drawn through the center of the least obliquer circle at right angles to a line passing through the given point & the center of the primitive.

Prop. 10th Equal arcs of any two greater circles of the sphere, will be intercepted between two other circles drawn on the sphere through the remotest poles of those great circles.

Prop. 11th If lines be drawn from the projected pole of any great circle, cutting the peripheries of the projected circle & plane of projection, the intercepted arcs of those circumferences are equal.

Prop. 12th The radius of any small circle, the plane of which is perpendicular to that of the primitive circle, is equal to the tangent of that lesser circle's distance from its pole; & the secant of that distance, is equal to the distance of the centers of the primitive & lesser circle.

Spherical Geometry.

Spheric Geometry, or spheric projection, is the art of describing or representing such circles, or arcs of circles as are usually drawn upon a sphere on the plane of any one of them; & of measuring such arcs, & their position to one another, when projected.

WILLIAMS'S 83 1/2 PATENT

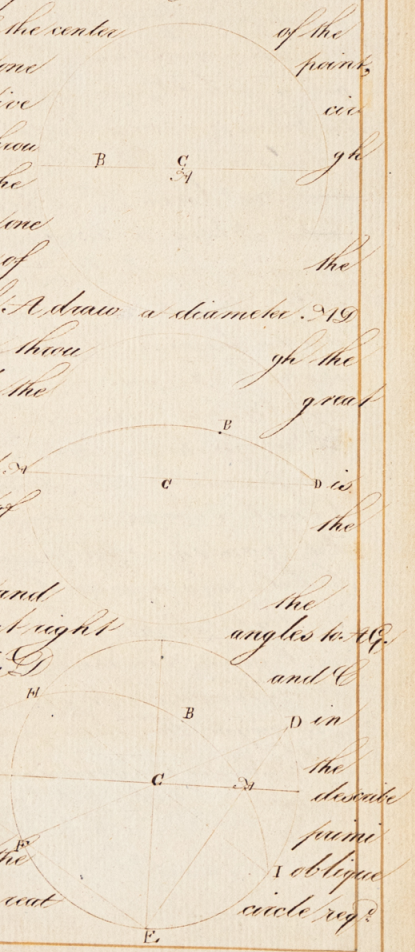
Problem 1st

To describe a great circle that shall pass through two given points in the primitive circle or plane of projection.

Let the given points be A, B, & C the center of the primitive circle. Case 1st When one of the points, A, is the center of the primitive circle. Const. A diameter drawn through the given points A, B, will be the great circle req.d Case 2nd When one point, A, is in the circumference of the primitive circle. Const. Through A draw a diameter A D through the center C, then an oblique circle described through three points A, B, D will be the great circle req.d

Case 3rd When neither point A, B, nor C is at the center or circumference of the primitive circle.

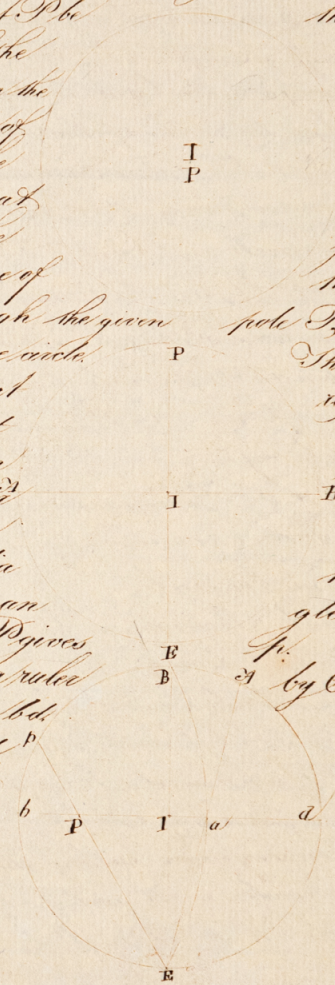
Const. Through one point A, and center C draw A G, & C E at right angles to A C, & C A, & by E and G gives D, & by D and G gives F, & by E and F gives G, A C E continued. Through three points G, B, A, describe a circumference cutting the primitive circle in H and I. Then the oblique circle H B. A I will be the great circle req.d



Problem 2nd

About any given point as a pole, to describe a great circle in a given primitive circle. Let P be the given point, & I the centre of the primitive circle. Case 1st. When the given pole, P, is in the centre of the primitive circle. Const. The primitive circle will be the great circle req.d. Case 2nd. When the pole P is in the circumference of primitive circle. Const. Through the given pole P, & the centre I, draw the diameter D E, & another diameter A B, drawn at right angles to D E, will be the great circle req.d. Case 3rd. When the given pole P is neither in the centre nor circumference of the primitive circle. Const. Through P draw a diameter b d, & another B E at right angles to b d, then a radius by E & P gives the great circle req.d.

Make the arc P A = 90°, & radius A P gives a in the diameter b d. Then a circle described through the three points B, a, E, is the great circle req.d



WILLIAMS'S 83 1/2 PATENT L.B

Problem 3d

A Projected circle being given, to find it's poles.

Case 1st When the given circle A E B is the primitive

Const. Find the center C, it is the pole sought.

Case 2d When the given circle A C B is a right circle.

Const. Draw a diameter D E at right angles to A B, & the ends or points D, E, of that diameter are the poles sought.

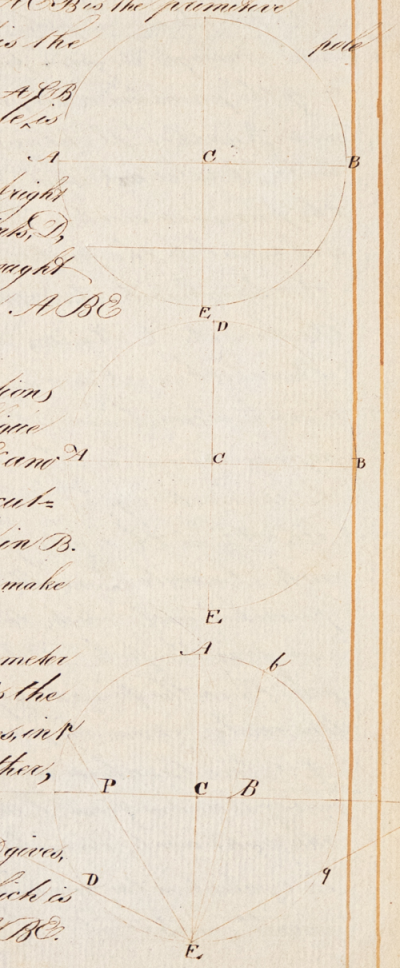
Case 3d When the given circle A B E is obliquo.

Const. Through the intersections of the primitive circle & obliquo circle draw a diameter A C, & another at right angles to A C, cutting the given obliquo circle in B.

Make b c equal to B C, & make b p, b q, each equal an arc of 90°.

A radius b c and p gives, in the diameter through B, the point P, which is the pole required. And a radius b C q gives, in P C B continued, the point Q, for the other, or opposite or inclusive pole.

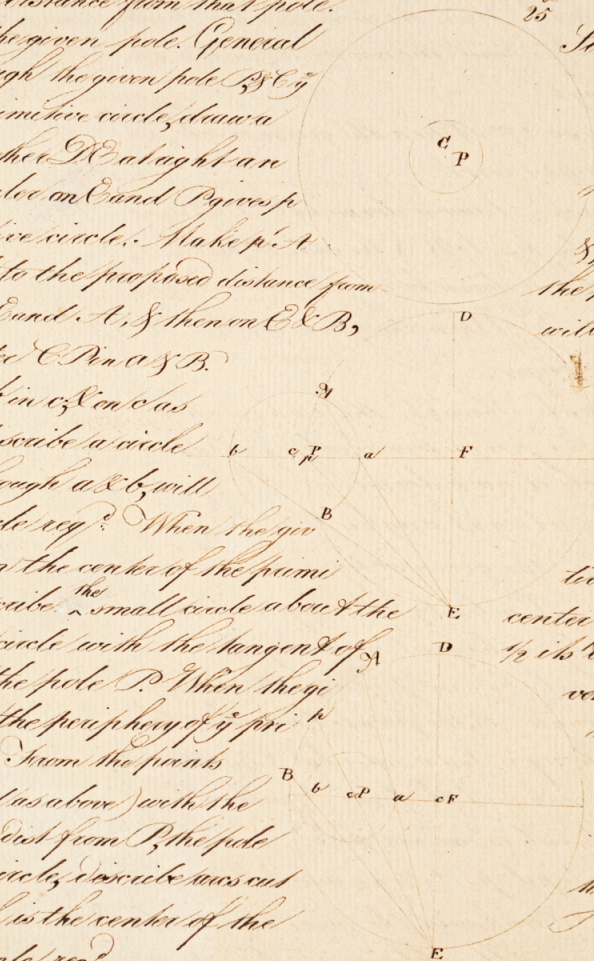
Make p d = p A, then a radius C D gives, in B C continued, the point R, which is center of the obliquo circle A B E.



Problem 25th

About any given point as a pole, to describe a small circle at a given distance from that pole.

Let P be the given pole. General tion. Through the given pole P draw any two of the primitive circles, as a meter, & another D at right an wise. Rule on C and P given to the primitive circle. Make P A each equal to the proposed distance from. Rule on C and A, & then on C & B, the diameter C D in a & B. Bisect a b in c, & on c as a center describe a circle passing through a & b, will be the circle req.d When the gi Pole P is in the center of the primi circle. Describe the small circle about the primitive circle with the tangent of ance from the pole P. When the gi pole P is in the periphery of the pri tive circle. From the points A, B, found as above with the gent of their dist from P, the pole the right circle, describe two arcs on c, which is the center of the small circle req.d



Solu con dia gle in & P, B, the pole will cut

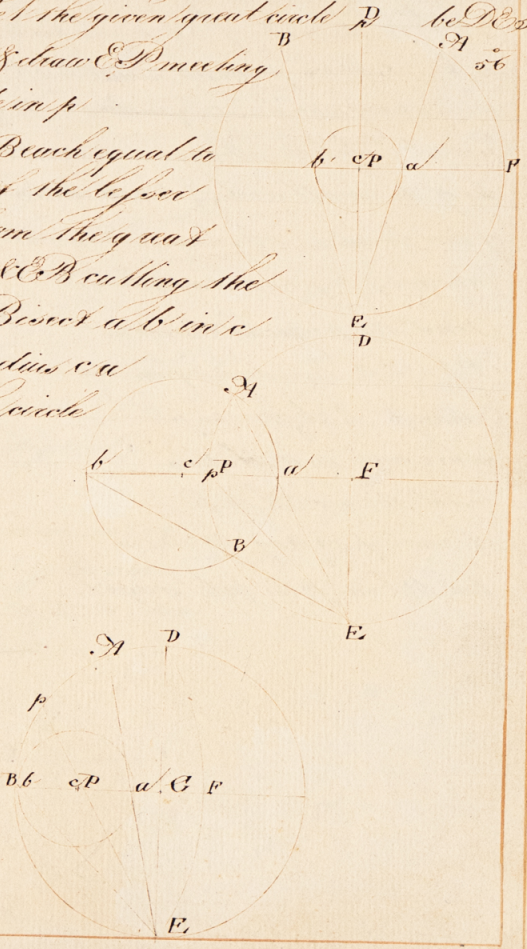
con tive center of & its dist con mi tan of ting

WILLIAMS' 83 1-2 PATENT L.B

Problem 5th

To a proposed distance from a great circle to describe a parallel circle. Let the given great circle be D D & the proposed distance B b. Find P the pole & draw E P meeting the primitive circle in p.

Make p a, p B each equal to the complement of the proposed circles distance from the great circle. Draw C A & C B cutting the line c F in a b. Bisect a b in c & on c with the radius c a or c B describe the circle required.



27th Apr. 1827.

Problem 6th

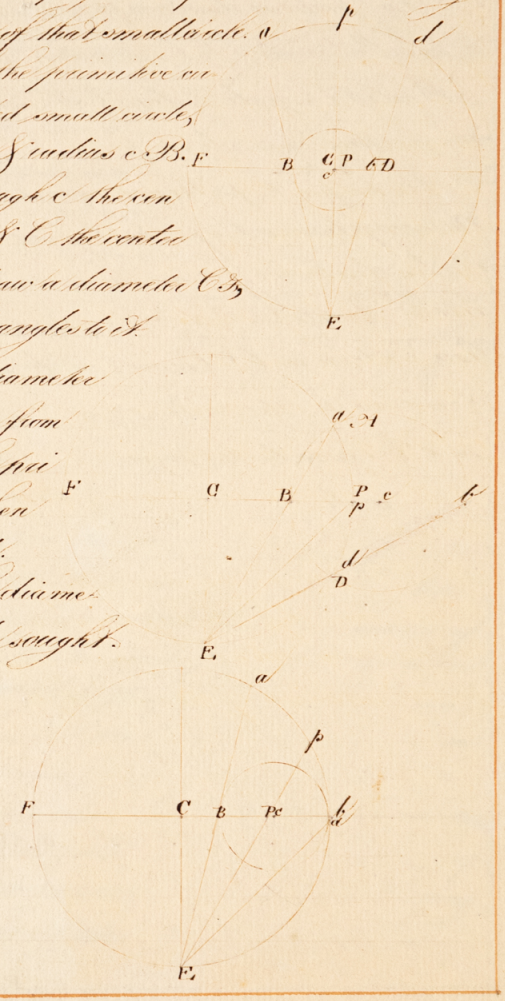
The primitive circle, & the projection of a small circle, being given; to find the pole of that small circle.

Let C be the centre of the primitive circle, & A B D a projected small circle, the centre of which is c, & radius c B.

General Solution. Through c & the centre of the small circle, & C the centre of the primitive circle, draw a diameter C b & another, C E, at right angles to it.

Find the projected diameter B b = 2 c B. Lines drawn from E through B & b cut the primitive circle in a, d; then bisect the arc a d in p.

Draw C p, cutting the diameter B b in P, the pole sought.



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Problem 17th

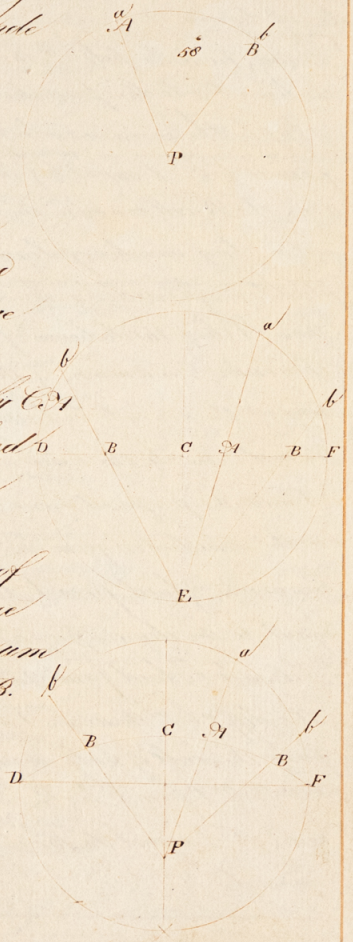
To measure any arc of a projected great circle.

General Solution. Find the pole of the given circle.

From that pole draw lines through the ends of the proposed arc cutting the primitive circle. Then the intercepted arc of the primitive circle applied to the scale of chords will give the measure sought.

By the semi tangents apply CA CB to the semi tangents & find by that means the degrees &c intended.

If A & B are on the same side of C take their diff.ce but if they are on contrary sides take their sum for the measure of the arc AB.



Problem 8th

To a given projected great circle to take an arc of a given number of degrees.

Draw, from the pole P, through A the line Pa to the primitive circle. Apply the given number of degrees from A to C. Draw PC & the intersection arc A B will contain the degrees required. Any number of degrees is readily laid to a right circle by the scale of half tangents. When the distance of the point the center C is known, & the given number of the arc is to be laid from toward D. To the known distance A C add the proposed arc A B taken on the same taken from the scale of tangents & laid from C to B will be equal to the degrees proposed. But A B is to be laid from A toward D. the difference of the arcs A B & A C from the scale of half tangents toward D from C to B, will make arc A B equal to the degrees proposed.



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WILLIAMS'S 33 1/2 PATENT LD

Problem 9

(projected)

To measure any spherical angle.

General Solution. Find the poles of ỹ two circles which form ỹ angle, & from the angular point draw lines through those poles to cut ỹ primitive circle. Then the measure of that angle, if acute, will be ỹ intercepted arc of the primitive circle; or the supplement of that arc, will be the measure of the angle when obtuse. Let ỹ proposed angle D. A B, formed by the great circles D. A, B. A, the poles of which are C & P, & lines drawn from the angular point A thro the poles C & P cut the primitive circle in E & F.

1st When ỹ angle is formed by the primitive & oblique circles. Then the arc F E measures the acute angle D. A B. But the obtuse angle B. A Y is measured by the supplement of F E.

2nd When the angle is formed by right & oblique circles meeting in the primitive circumference. Then the arc F E measures angle D. A B.

3rd When the angle formed by right & oblique circles meeting within the primitive circle. Then the arc F E measures the acute angle D. A B. But ỹ obtuse angle is measured by the supplemt of F E.

4th When the angle formed by oblique circles meeting within ỹ primitive circle. Then ỹ acute angle D. A B is measured by ỹ supplement of F E. But ỹ sup obtuse ∠ D. A Y.

Problem 10th

Through a given point in any projected great circle, to describe another great at right angles to the given one.

General Solution. Find the pole of the given circle. Then a great circle described through that pole & the given point will be at right angles to the given circle. Let the given projected great be B A D & A the given point.

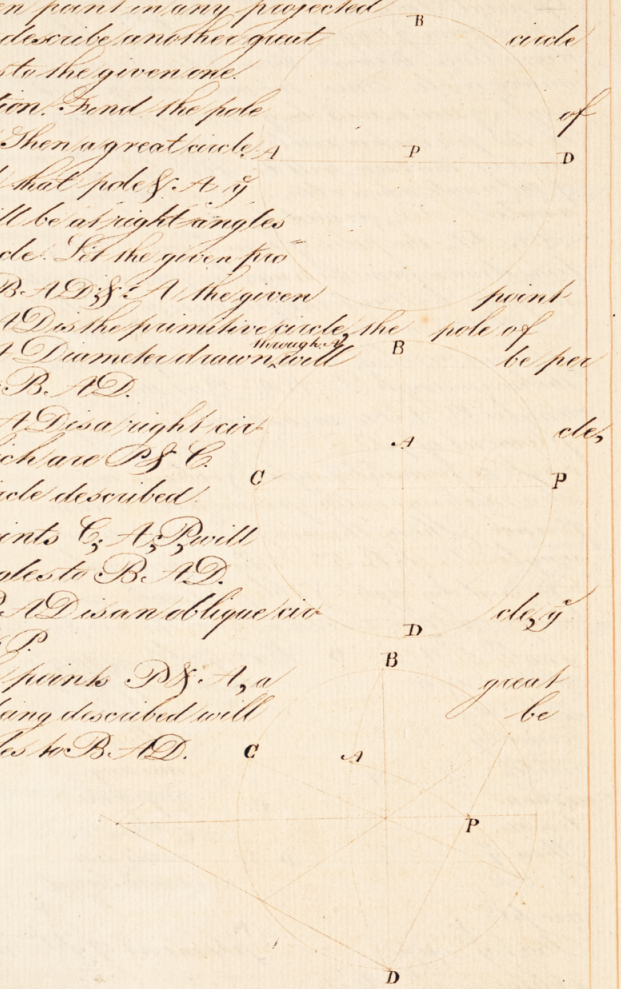
1st When B A D is the primitive circle, the pole of which is P. A Diameter drawn through A will be perpendicular to B A D.

2nd When B A D is a right circles the poles of which are P & C.

An oblique circle described through the points C, A, P will be at right angles to B A D.

3rd When B A D is an oblique circle of which P is pole.

Through the points D & A, a great circle P A C being described will be at right angles to B A D.



Problem 11th

Through any assigned point in a projection to describe another great circle cutting in an ∠ of a given No. of Deg.s

Pbe a given point on any great circle A P B. 1st When A P B is y proj.n of a circle. Through y given point draw a diam.r P E & draw y diam.r at right ∠s to P E. Draw P D cutting in D, so that y ∠ C P D is equal to y given ∠. On D with the radius D P describe y great circle P F E. Then will the ∠ A P F contain y given deg.s

C D equal to y tang.t of y given angle to y rad.s C P. C z. Make P D equal to the chord of that ∠. 2nd When A P B is a circle. Draw a diam.r G H at right ∠s to A P B. Then as rad.s is to G P so is the sine of y given angle to the sine of y ∠ required. Make H M = A z. As rad.s is to G P so is sine of y ∠ A P B to H M. Draw C D at right ∠s to A B. Draw C D cutting in D, so that y ∠ C P D = comp.t of y ∠ required. 3rd When A P B is an oblique circle. From y given point P draw y lines P C, P D through y centres of y primitive & given oblique circles, & through C y centre of A P B draw C D at right ∠s to P C. Draw P D making C P D = given deg.s & cutting C D in D. From D with y radius D P a circle F P E being described will be a great circle cutting A P B in an angle required.



Problem 12th

Through any point in ye plane of projection, to describe a great circle that shall cut a given great circle in any given angle. Provided ye angle that is proposed is not less than ye distance ye given point & circle. Let ye given point through which a circle is to be described, cut a given great circle BDC, ye pole of which is P, in an angle equal to a given number of degrees. General Solution.

About the given point as a pole, describe a great circle EGF. About B ye pole of ye given circle BDC describe a small circle at a distance equal to ye given angle, cutting the great circle EGF in G.

About the point G as a pole, describe a great circle cutting ye given circle BDC in D. Then will the angle CGD be equal to the angle required. When it is to make with E. Then about G as a pole describe a great circle cutting ye given circle BDC in D. Then will the angle CGD be equal to the angle required. When it is to make with E. Then about G as a pole,

Problem 13th

Any great circle on the plane of projection being given; to describe another great circle that shall make given ∠s with the primitive & given circle.

Let the given great circle be ADC, & its pole D. Solution. About P, the pole of the primitive circle, describe an arc m n, at the distance of as many deg.s as are in the angle which the req.d circle is to make with the primitive. Suppose 72°.

About D, the pole of the other given circle, & at a distance equal to the measure of the angle which the req.d circle is to make with the given circle ADC. Suppose 50°, describe an arc o n, cutting m n in n.

About n, as a pole describe a great circle E D F cutting the given circles in C & D.

Then the angle ADE = 72°, ADC = 50°.



Sphere Trigonometry.

Definitions

Sphere Trigonometry is the art of computing the measures of the sides & angles of such triangles as are formed on the surface of a sphere, by the mutual intersections of three great circles described thereon. A Sphere Triangle consists of three sides & three angles.

A Right Angled Sphere Triangle has one right angle: The sides about the right angle are called Legs, & the side opposite to the right angle is called the Hypothenuse. A Quadrantal Sphere Triangle has one side equal to ninety deg^s. An Obleque Sphere Triangle has all it's angles obliqne. The Circular Parts of a triangle, are the arcs which measure it's sides & angles.

Two sphere triangles are said to be supplements to one another when the sides & angles of the one are respective supplements of the angles & sides of the other: And one in regard to the other is called the supplemental triangle.

Two arcs or angles, when compared together are said to be alike or of the same kind when both are acute or less than ninety degrees, or when both are obtuse or greater than ninety degrees:

But when one is greater & the other less than 90 they are said to be unlike

Spherical Theorems.

Theorem 1st. In every spheric triangle, equal angles, are opposite to equal sides; And equal sides, are opposite to equal angles. Theorem 2nd. Either side of a spheric triangle is less than the sum of the other two sides. Theorem 3rd. Each side of a spheric triangle is less than a semicircle or 180°. Theorem 4th. In every spheric triangle, the greatest side is opposite the greatest angle. Theorem 5th. If from the three angles of a spheric triangle as poles be described three arcs of great circles, forming another spheric triangle then will the sides of the latter & the opposite angles of the former, be supplements to one another. Also the angles in the latter & their opposite sides in the former are the supplements of one another. Theorem 6th. The sum of the three sides of every triangle is less than a circle or 360°. Theorem 7th. The sum of three angles of a spheric triangle is greater than two right angles & less than six & will always fall between 180° & 540°. Theorem 8th. If one side of a spheric triangle be produced then the outward angle is either equal to, less or greater than the inward opposite angle adjacent to that side according as the sum of the other two sides is equal to, greater or, or less, than 180 degrees. Theorem 9th. In right angled spheric triangles the obligue angles & their opposite sides are of the same kind: That is, if a leg

is less or greater than 90° its opposite angle is also less or greater than 90° Theorem 10th In right angled spherical triangles the hypothenuse is less than 90° when the legs are of a like kind: But the hypothenuse is greater than 90° when the legs are of different kinds Corol 1st The hypothenuse is less or greater than 90° according as the oblique angles are of a like or different kinds.

Corol 2nd The legs & their adjacent angles are like or unlike, as the hypothenuse is less, or greater than 90°

Corol 3rd A leg & its opposite angle are both acute or both obtuse, according as the Hypothenuse & other leg are like, or unlike

Corol 4th Either angle is acute or obtuse as the hypothenuse & the other angle are like or unlike.

Theorem 11th In every spherical triangle if the angles adjacent to either side be alike then a perpendicular drawn to that side from the other angle, will fall betn the triangle: But the perpendicular falls without the triangle when the angles adjacent to the side it falls on are unlike.

Theorem 12th If the two lesser sides of a spherical triangle are of the same kind; then an arc drawn from their included angle perpendicular to the opposite side will fall within the triangle

Rules for resolving the several cases in right angled spherical triangles.

In every right angled spherical triangle it will be

1st. As the sine of a given side
To the sine of its opposite angle.
So is the sine of another given side.
To the sine of its opposite angle.

2nd. As the sine of a given angle
To is the sine of its opposite side
So is the sine of another given angle
To the sine of its opposite side.

But in such cases of right angled spherical triangles as are not resolvable by the sines of opposite sides & angles it will be best to follow the general rule given by Lord Napier which is as follows.

The five parts of a right angled spherical triangle are the two legs; the complement of the hypothenuse, and the complements of the two obligue angles.

Now in all proportions concerning right angled spherical triangles there will be three things concerned beside the radius one of which may be called the middle term in respect to y^e other two; & those two in respect to y^e middle term may be called extremes.

When the extremes are joined to the middle term they are called extremes conjunct, but when each of them is disjoined from the middle term by an intermediate one not concerned they are called extremes disjunct

The Right Angle is not supposed to disjoin the legs

If the three parts under consideration all join the middle term is readily seen & the other two are extremes conjunct, but if only two of these three parts join those two are extremes disjunct & the other term is the middle term.

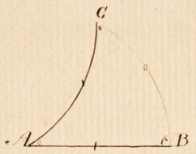
If these things be understood we shall have the general rule radius multiplied by the sine of the middle part is equal to product of the tang^s of the extremes conjunct or to the product of the cosines of the extremes disjunct.

And this rule is formed into proportions by beginning with the known extreme if an extreme be wanted but if the middle part be req^d begin with ^y radius & the two extremes will be the second & third terms of ^y proportion the thing wanted makes the fourth term

To make out the Proportions

When ^y extremes are conjunct. Use cotan^s to the angles & hypothenuse & tang^s to sides. When ^y extremes are disjunct. Use sines to ^y angles & hypothenuse & cosines to the sides But if either angle or hypothenuse be the middle term take ^y cosine of that angle or hypothenuse but take the sine of a side

Examples 1st & 2nd



To find ∠ A
As radius To cotang. AC To tang. AB To cosine ∠ A
To find ∠ C
As sine AC To radius To sine AB To sine ∠ C
To find BC
As cosi AB To radius To cosi AC To cosi BC

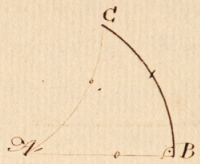


To find ∠ C
As cotan ½ ∠ A To radius To cosine AC To cotan ½ ∠ C
To find AB
As cotan ½ C To radius To cosi ∠ A To tang ½ AB
To find BC
As radius To si AC To si ∠ A To sine BC

Examples 3rd & 4th

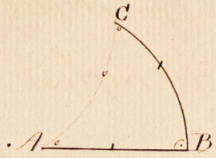


To find ∠ A
As cos. AB To radius So cos. ∠ C To sine ∠ A
To find AC
As sine ∠ C To radius So sine AB To sine AC
To find BC
As radius To tan. AB So cotan ∠ C To sine BC

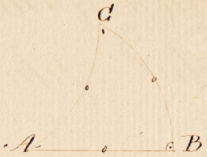


To find ∠ C
As cos. CB To radius So cos ∠ A To sine ∠ C
To find AB
As radius To tan. BC So cot ∠ A To sine AB
To find AC
As sine ∠ A To radius So sine CB To sine AC

Examples 5th & 6th



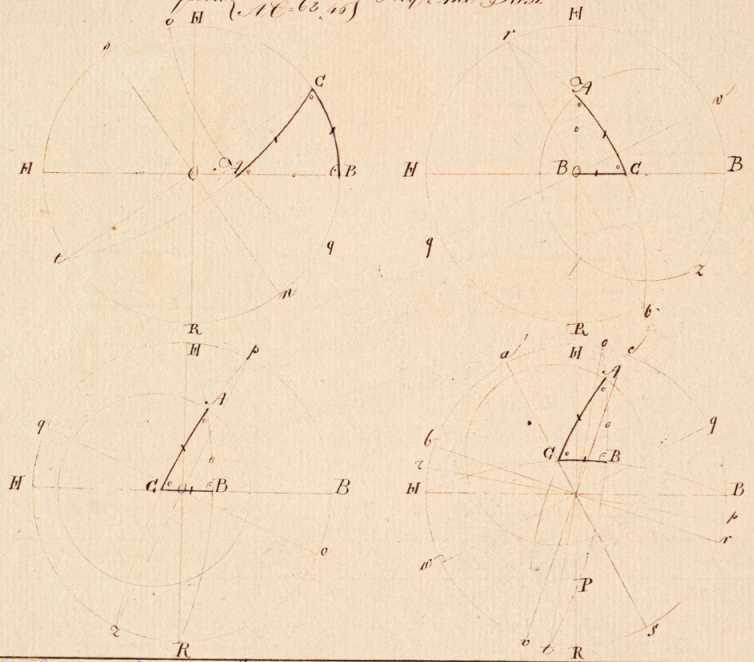
To find ∠C
As tan AB To radius So sine BC To cotan ∠C
To find ∠A
As tan BC To rad So sine AB To cot ∠A
To find AC
As rad To cosi AB So cosi BC To cosi AC



To find AB
As sin ∠C To radius So cos ∠C To cosi AB
To find BC
As sin ∠C To rad So cosi ∠A To cosi BC
To find AC
As rad To cot ∠A So cot ∠C To cosi AC

The Construction and numerical Solution of the several cases of right angled spherical Triangles.

Given { BC = 34.30 / AC = 63.45 } Req.ᵈ the Rest



To find ∠ C.	To find ∠ A	To find AB
As sine AC = 63.45 ... 9.94727	As rad = 90.00 ... R	As cosi BC = 34.30 ... 9.91501
To rad = 90.00 ... R	To tan BC = 34.30 ... 9.83713	To radius = 90.00 ... R
To sine BC = 34.30 ... 9.75313	To cot AC = 63.45 ... 9.69298	As cosi AC = 63.45 ... 9.64271
To sine ∠ A = 39.19 ... 9.80010	To cosi ∠ C = 70.11 ... 9.53011	To cosi AB = 57.32 ... 9.72972

Ans { ∠ A = 39.19 / ∠ C = 70.11 / AB = 57.32 }

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Example 2nd

Given { AC = 62° 50′ ; ∠C = 47° 45′ } Req^d the rest

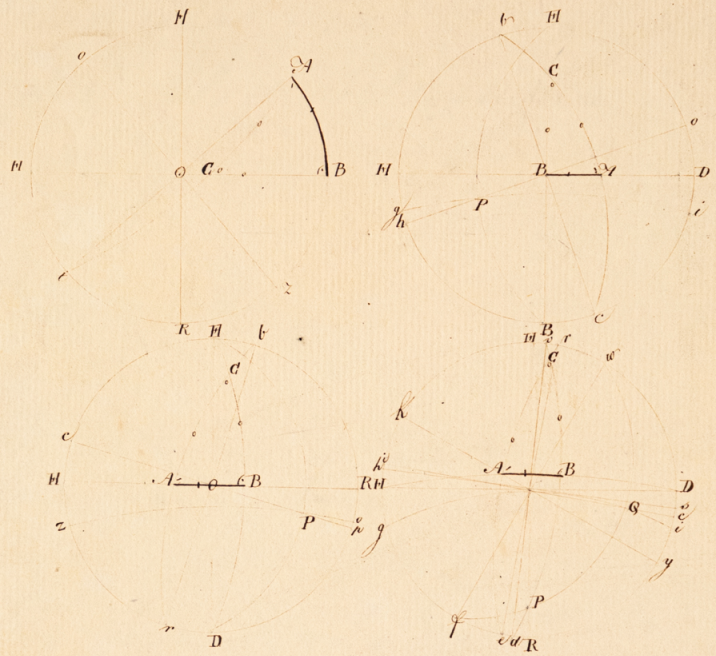


To find ∠ A	To find AB	To find BC
As cot ∠C = 47° 45′ ... 0,04175	As rad = 90° 00′ ... 10,	As cot AC = 62° 50′ ... 1,33067
To rad = 90° 00′ ... 10,	To sin AC = 62° 50′ ... 9,95746	To rad = 90° 00′ ... 10,
To cos AC = 62° 50′ ... 9,62689	To sin ∠C = 47° 45′ ... 9,86936	To cos ∠C = 47° 45′ ... 9,82761
To cot ∠A = 65° 01′ ... 9,66024	To sin AB = 42° 07′ ... 9,82682	To tan CB = 54° 36′ ... 10,15028

Ans^r { ∠A = 65° 01′ ; AB = 42° 07′ ; CB = 54° 36′ }

Example 3rd

Given { A B 40.57 / ∠ A 75.42 } Req^d the rest



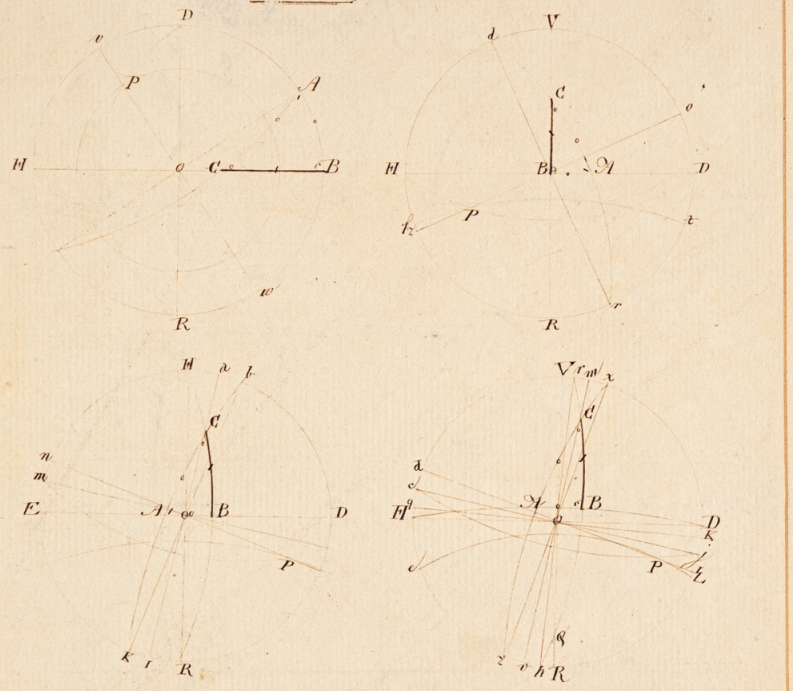
To find ∠ C	To find A C	To find B C
As rad = 90.00 ... 10.	As co. A B = 40.57 ... 9.98451	As co. ∠ A 75.42 ... 9.59264
To co. A B = 40.57 ... 9.87811	To rad = 90.00 ... 10.	To rad = 90.00 ... 10.
To sine ∠ A = 75.42 ... 9.98632	To co. ∠ A = 75.42 ... 9.39269	To sine A B = 40.57 ... 9.81651
To co. ∠ C = 42.57 ... 9.86443	To cot. A C = 74.07 ... 9.43719	To tan B C = 60.45 ... 10.41015

Ans^r { ∠ C 42.57 / A C 74.07 / B C 60.45 }

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Example 4th

Given { BC = 57.25 ; ∠A = 68.54 } Req.d the Rest

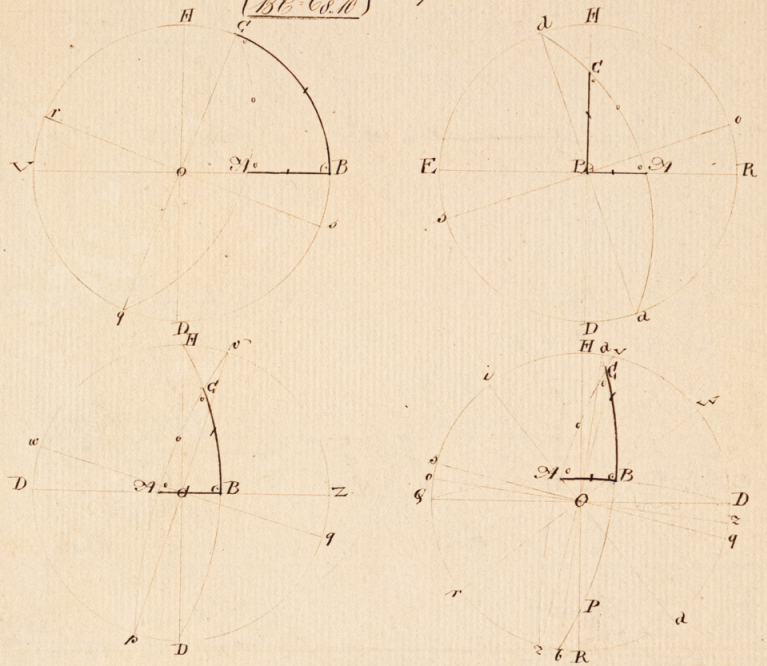


To find ∠C	To find AB	To find AC
As cosi BC 57.25 ... 9.97070	As rad 90.00 ... 10	As sin ∠A 68.54 ... 9.96814
To rad = 90.00 = 10	To cot ∠A 68.54 ... 9.58644	To rad = 90.00 ... 10
As cosi ∠A = 68.54 ... 9.55850	To tan BC 57.25 ... 10.19721	To sin BC 57.25 ... 9.92642
To sin ∠C = 42.11 ... 9.81708	As sin AB 37.25 ... 9.78265	To sin AC 62.48 ... 9.95567

Ans^r { ∠C = 42.11 ; AC = 62.48 ; AB = 37.25 }

Example 5th.

Given { AB 42.07 / BC 68.10 } Req.e the rest



To find ∠ C	To find ∠ A	To find AC
As sin AB 42.07 ... 9.826378	As sin BC 68.10 ... 9.967276	As rad 90.00 ... 10.
To rad 90.00 ... 10.	To rad 90.00 ... 10.	To sin BC 68.10 ... 9.967276
To sin BC 68.10 ... 9.967767	To sin AB 42.07 ... 9.826649	To sin AB 42.07 ... 9.826928
To cot ∠C 45.15 ... 10.041389	To cot ∠A 74.58 ... 9.859627	To cos H 72.59 ... 9.959204

Ans.r { ∠C 45.15 / ∠A 74.58 / AC 151.36 }

Example 6th

Given $\left\{\begin{array}{l} \angle A = 58.20 \\ \angle C = 67.15 \end{array}\right\}$ Req^{d} the rest.



To find AB	To find BC	To find AC
As sin^{d} A 58.20 ... 9.92961	As sin^{d} C 67.15 ... 9.96357	As rad 90.00 ... 10.
To rad 90.00 ... 10.	To rad 90.00 ... 10.	To cot^{d} A 58.20 ... 9.79015
To cos^{d} C 67.15 ... 9.58729	To sin^{d} A 58.20 ... 9.92014	To cot^{d} C 67.15 ... 9.63760
To cos^{d} AB 62.58 ... 9.63740	To cos^{d} BC 55.18 ... 9.75531	To cos^{d} AC 75.01 ... 9.41271

Ans^{rs} $\left\{\begin{array}{l} AB = 62.58 \\ BC = 55.18 \\ AC = 75.01 \end{array}\right\}$

Rules for resolving the several cases of oblique angled spherical triangles.

1st In all spheric triangles, the sines of their sides are in direct proportion to the sines of their opposite angles.

As the sine of a side
To the sine of its opposite angle
So is the sine of any other side
To the sine of its opposite angle

And

As the sine of an angle
To the sine of its opposite side
So is the sine of any other angle
To the sine of its opposite side.

2nd

As the sine of ½ the sum of two sides
To the sine of ½ their diffce
So is the cotangt of ½ the contain'd angle
To the tangt of ½ the diffce of the other 2 angles

And

As the cosine of ½ the sum of two sides
To the cosine of ½ their diffce
So is the cotangt of ½ the cont'd angle
To the tangt of ½ the sum of the other two angles

3rd

As the sine of ½ the sum of two angles
To the sine of ½ their diffce
So is the tan.t of ½ the side betw.n them
To the tan.t of ½ the diffce of the other two sides

And

As the cosine of ½ the sum of two angles
To cosine of ½ their diffce
So is the tan.t of ½ the side betw.n them
To tan.t of ½ sum of the other two sides

4th When two sides & the angle between them are given to find the third side. Add the tan.t of of less side to the cosine of the given angle their sum is the tan of an arc A

Add together the tangent of A, the cosecant of A, the cosine of the less side, & the cosine of the sum or difference of A & the greater side, according as the given angle is greater or less than 90° the sum of these four logarithms, rejecting twice the radius, will be the cosine of the third side; which is less or greater than 90°, according as the sum or difference of A & the greater side is less or greater than 90°.

5th When the three sides of a spherical triangle are given to find an angle. Add the three sides together, take half the sum, & subtract the side which is opposite the req.d angle from that half sum.

Add together the log cosecants of the two sides which contain the required angle, & the log sines of the ½ sum, and the difference betw.n the half sum & third side, half the sum of these four logarithms, rejecting twice the radius, will be the cosine of half the req.d angle.

6th When the three angles of a spherical triangle are given to find a side. Take the supplement of either of the angles

which are adjacent to the req^d side, & add this supplement & the other two angles together, take half the sum, & subtract the angle, which is opposite the side sought from this half sum.

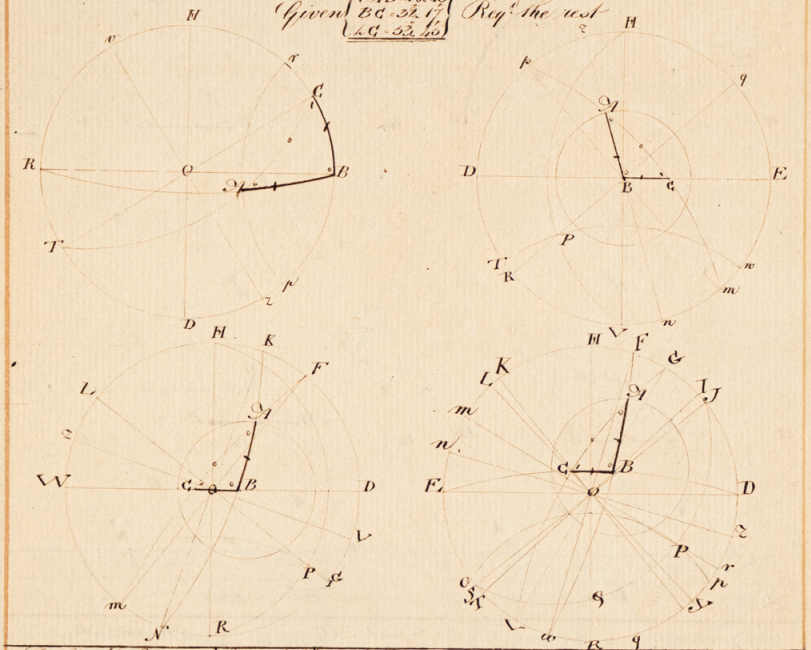
Add together the cosecants of the angles which are adjacent to the side req^d the sine of the ½ sum & diff bet^n the ½ sum & 3^d angle, ½ the sum of these four log^ms, rejecting twice the radius, will be the sine of ½ the supplement of the side sought

WILLIAMS' 837 PATENT

The Construction and Numerical solution of the several cases of Oblique angled spherical Triangles

Example 1st

Given $\begin{cases} AB = 48.42 \\ BC = 32.17 \\ \angle C = 53.45 \end{cases}$ Req^{d} the rest

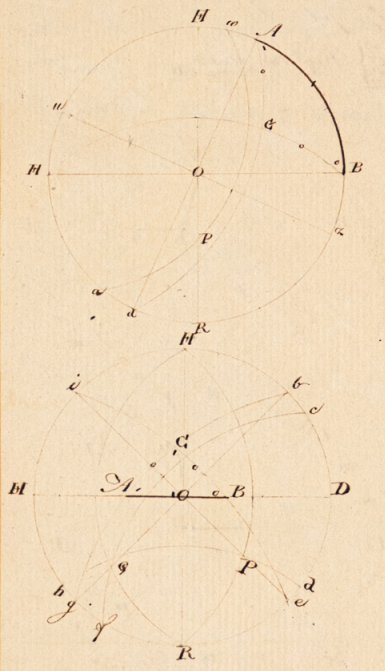


To find ∠ A		To find ∠ B	To find AC
As sin AB 48.42 ... 9.87400	AB = 48.42	As cos ½ Diff 2.8.43 ... 9.00448	As sin ∠C 53.45 ... 9.09342
To sin ∠C 53.45 ... 9.90657	BC = 32.17	To cos ½ Sum 40.30 ... 9.88106	To sin AB 48.43 ... 9.87090
To sin BC 32.17 ... 9.72762	sum 81.00	To tan ½ sum ∠ 45.24 ... 9.99016	To sin ∠B 106.10 ... 9.98248
To sin ∠A 34.58 ... 9.75870	½ sum 40.30	To tan ½ sum ∠ 32.03 ... 9.80768	To sin AC 63.31 ... 9.95181
	Diff 8.26		
	½ Diff 4.13		
	∠C 53.45		
	∠A 34.58		
	sum 88.43		
	½ sum 44.21		
		∠ 2	
		∠B 106.10	

Ans $\begin{cases} \angle A = 34.58 \\ \angle B = 106.10 \\ AC = 63.31 \end{cases}$

Example 2nd

Given { ∠A = 52.19, ∠C = 97.14, AB 65.09 } Req^d the Rest



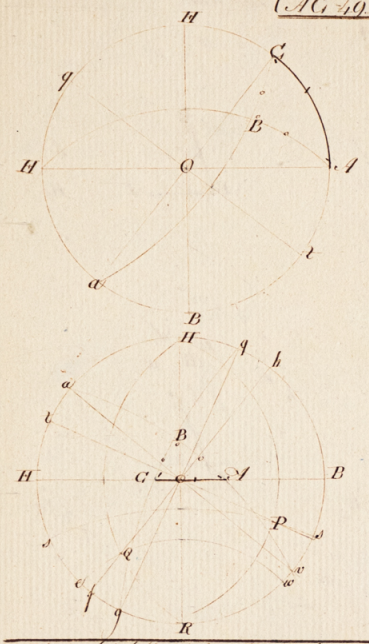
To find BC.		To find AC	To find ∠B
As si ∠C 97.14 ... 9.996367	∠C = 97.14	As si 2d ∠A 52.19 ... 8.101808	As si AB 65.09 ... 9.961210
To si AB 65.09 ... 9.963780	∠A = 52.19		
	sum = 149.33	To si ½ sum 74.46 ... 9.984147	To si ∠C 97.14 ... 9.996633
	½ sum 74.46		
	diff = 44.87		
	½ diff 22.27		
To si ∠A 52.19 ... 9.898430	AB = 65.09	To ta ½ d ∠A 9.22 ... 9.211730	To si AC 56.16 ... 9.919430
	BC = 56.24		
	diff ½ diff 16.93		
To si BC 56.24 ... 9.850987	½ d = 9.22	To ta ½ reg ∠A 52.30 ... 9.911991	To si ∠B 50.59 ... 9.881552
		× 2	
		AC 45.11	

Ans^r { BC 56.24, AC 45.11, ∠B 50.59 }

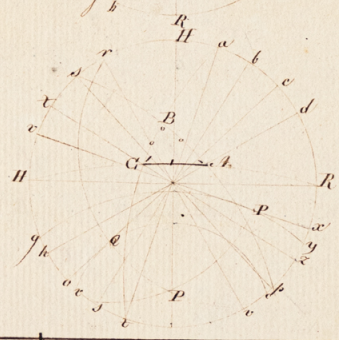
WILLIAMS'S 33 1-2 PATENT LB

Example 3rd

(Given { 1st = 12.17 / ∠C = 65.53 / AC = 49.14 } Reqd the rest







To find ½ diff 2 Sides AB, BC

As sin ½ sum 2d° = 54.35 ... 1.88680
To sin ½ diff = 11.18 ... 9.29213
To tan ½ Ind Ext Them 21.37 ... 9.66104
To tan ½ diff other 2 Sides 37.17 ... 9.87317

To find ∠B

As sine AB = 44.04 ..... 0.15771
To sine ∠C = 65.53 ..... 9.96633
To sine AC = 49.14 ..... 9.87034
To sine ∠B = 83.41 ..... 9.99438

∠C = 65.53
1st = 12.17
sum = 109.10
½ sum = 54.35

diff = 32.36
½ diff = 11.18

AC = 49.14
½ AC = 24.37

To find ½ sum 2 Sides AB, BC

As cosin ½ sum 2d° = 54.35 ... 0.22693
To cosin ½ diff = 11.18 ... 9.99130
To tan ½ Ind Ext Them = 21.37 ... 9.66104
To tan ½ sum other 2 = 37.17 ... 9.88047

AB 44.04
BC 31.20

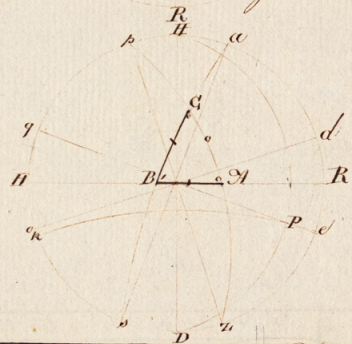
Ans { AB = 44.04 / C = 31.20 / ∠B = 83.41 }

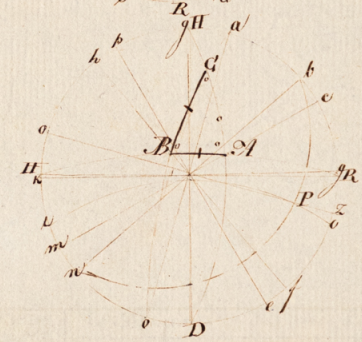
Example 4th

Given $\left\{\begin{array}{l} AB = 40^{\circ}\ 15' \\ BC = 54^{\circ}\ 57' \\ \angle B = 70^{\circ}\ 15' \end{array}\right\}$ Req^{d} the rest









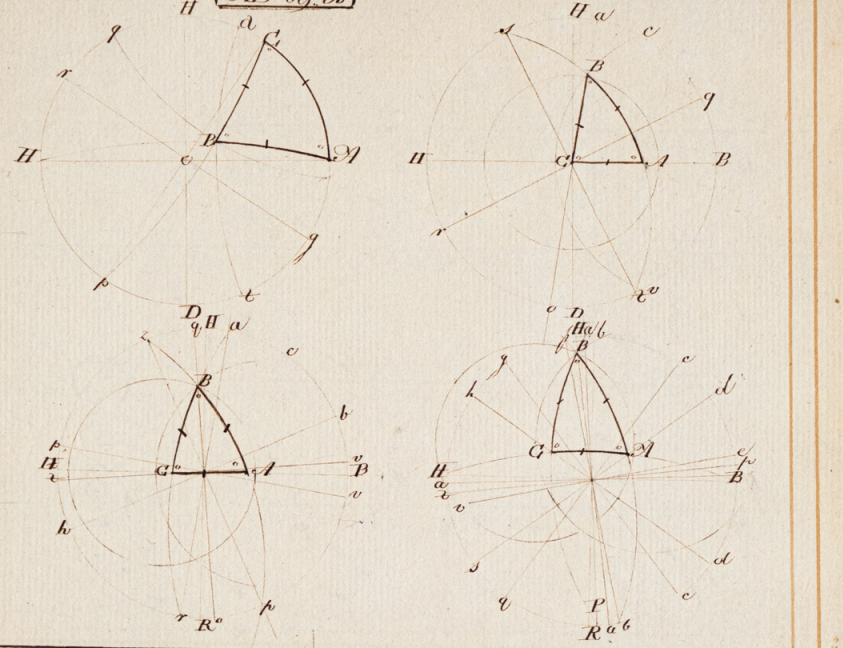
To find AC	To find ∠A	To find ∠C
AB = 40° 15′ ... tal ... 9,93533	As si. AC = 52° 00′ ... 0,10347	As si. AC = 52° 00′ ... 0,10347
∠B = 70° 15′ ... cosi ... 9,52881	To si ∠B = 70° 15′ ... 9,97267	To si ∠B = 70° 15′ ... 9,97267
Arc A = 40° 14′ ... tal ... 9,46414	To si BC = 54° 57′ ... 9,91310	To si AB = 40° 15′ ... 9,81475
Arc A = 40° 14′ ... cosec ... 10,35354	To si ∠A = 77° 34′ ... 9,90024	To si AC = 51° 44′ ... 9,89189
AB = 40° 15′ ... cosi ... 9,87942		
diff = 28° 43′ ... cosi ... 9,87723		
½AC = 52° 00′ ... cosi ... 9,78933		

Ans^{r} $\left\{\begin{array}{l} AC = 52^{\circ}\ 00' \\ \angle A = 77^{\circ}\ 34' \\ \angle C = 51^{\circ}\ 44' \end{array}\right\}$

WILLIAMS'S 83 & 2 PATENT L.S

Example 5th

Given $\left\{\begin{array}{l} AC = 55.30 \\ BC = 62.13 \\ AB = 69.35 \end{array}\right\}$ Req^e the rest

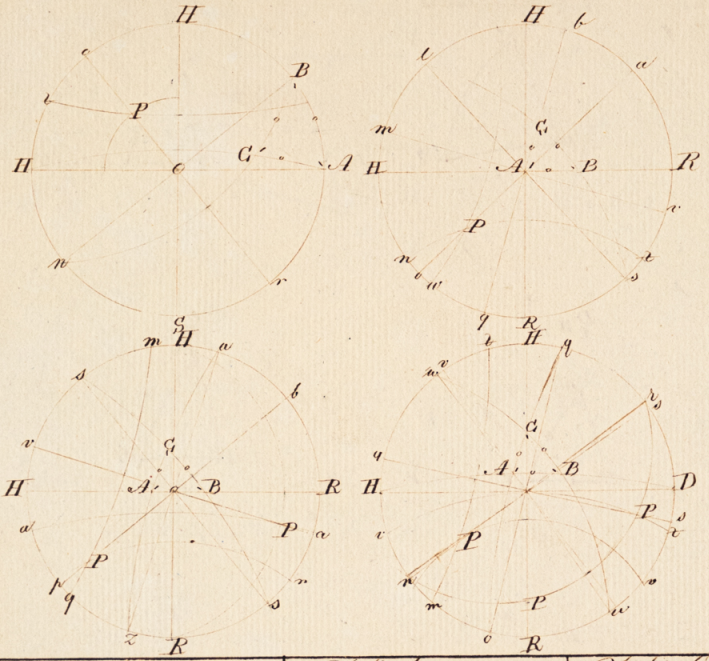


To find ∠B			To find ∠C	To find ∠A
BC = 62.13	cosec 4	10,05320	As si C.A = 55.30 ... 0,08401	As si C.A = 55.30 ... 0,08401
AB = 69.35		10,02818	To si ∠B = 60.52 ... 9,94126	To si ∠B = 60.52 ... 9,94126
AC = 55.30			To si AB = 69.35 ... 9,97102	To si BC 62.13 ... 9,94680
sum 187.18			To si ∠C = 83.23 ... 9,99709	To si ∠A = 76.24 ... 9,97407
½ sum 93.39	sine	9,99912		
AC 55.30				
diff 38.09	sine	9,79079		
		19,87129		
½ ∠B = 30.26	cosec	9,93560		
		2		
∠B 60.52				

Ans^r $\left\{\begin{array}{l} \angle B = 60.52 \\ \angle C = 83.23 \\ \angle A = 76.24 \end{array}\right\}$

Example 6th

Given $\left\{\begin{matrix} \angle A = 74.\,56 \\ \angle B = 53.\,14 \\ \angle C = 65.\,53 \end{matrix}\right\}$ Req the rest.



To find AB	To find BC	To find AC
180.00	As sin ∠C = 65.53 ... 9,96391	As sin ∠C = 65.53 ... 9,96391
∠A = 74.56 ... cosec^{n} 10,01510	To sin AB = 42.10 ... 9,83513	To sin AB = 42.10 ... 9,83513
sup = 105.04	To sin ∠A = 74.56 ... 9,98481	To sin ∠B = 53.14 ... 9,90368
∠C = 65.53	To sin BC = 46.22 ... 9,85961	To sin AC = 36.54 ... 9,77848
∠B = 53.14 cosec^{t} 10,09632		
sum 224.11		
½ sum 112.05 sine 9,96691		
∠C = 65.53		
diff = 46.12 sine 9,85830		
19,93684		
68.25 sine 9,96847		
2		
136.50		
180.00		
AB 43.10		

Ans^{r} $\left\{\begin{matrix} AB = 42.10 \\ BG = 46.22 \\ AC = 36.54 \end{matrix}\right\}$

WILLIAMS'S 83 1/2 PATENT

Astronomy.

Astronomy is a science which treats of the motions and distances of the Heavenly bodies, & of the appearances thence arising. There have been great variety of opinions among the philosophers of preceding ages concerning the situation of the great bodies in the universe, or of the position of the bodies which appear in the Heavens. But the notion now embraced by the most judicious Astronomers is, that the universe is composed of an infinite number of systems, or worlds; in every system there are certain bodies moving in free space, & revolving at different dist. around a Sun, placed in, or near, the center of the system; & that these suns & other bodies are

the Stars which are seen in the Heavens.

The Solar System, so called by Astronomers, is that in which our Earth is placed; and in which the Sun is supposed to be fixed in or near the centre, with several bodies similar to our Earth revolving round him at different distances.

The Hypothesis, which is confirmed by all the observations hitherto made, is called the Copernican System, from Nicholas Copernicus, a Polish Philosopher, who about the year 1500 revived this notion from the oblivion it had been buried in for so many ages.

Stars are distinguished into two kinds, namely, fixed, & wandering.

The Fixed Stars, are the suns, in the centers of their systems, shining by their own light; & are observed to preserve always the same situation in respect to one another.

The Fixed Stars appear of various sizes, which is doubtless occasioned by their different distances; these sizes are usually distinguished into six, or seven Classes, called Magnitudes.

The Largest or Brightest, are said to be of the 1st. Magnitude; those in the next Class or Degree of brightness, are said to be of the second Magnitude & so on to the least, or those just discernable to the naked eye.

A Constellation, is a number of stars lying in the neighbourhood of one another, on the surface of the celestial sphere which Astronomers for the sake of remembering them with greater ease ... suppose to be circumscribed by the outlines of some animal, or other figure.

The Wandering Stars, are those bodies within our system, or celestial sphere, which revolve round the Sun; they appear luminous or bright only by reflecting the light they receive from the Sun; & are of three kinds, namely, primary planets, secondary planets, & comets.

The Primary Planets are those bodies, which in revolving round the Sun respect him only as the centre of their courses; the motions of which are regularly performed in tracks, or paths, which astronomical observations do be nearly circular & concentric to one another.

A Secondary Planet, commonly called a Satellite or Moon, is a body, which while it is carried round the Sun, does also revolve round a primary planet, which it respects as its centre.

Comets, are bodies which also revolve round the Sun; probably in as regular order as the planets, but in much longer periods of time.

The Orbit of a planet or comet is that track or path along which it moves. There are seven Primary Planets; & reckoned in order from the Sun, their names & marks are, Mercury ☿, Venus ♀, the Earth ♁, or ⊙, Mars ♂, Jupiter ♃, Saturn ♄, & Georgium Sidus ♅,

Mars, Jupiter, Saturn, & Georgium Sidus, are called Superior Planets, as their orbits include that of the Earth: but Venus & Mercury, the orbits of which are contained within the Earths, are called Inferior Planets.

It has been discovered by the help of telescopes, that there are 18 secondary planets; the Earth being attended by one, called the Moon, Jupiter by four, Saturn by seven, Georgium Sidus by six.

Every primary planet is supposed to have two motions, namely, annual & diurnal.

The Annual Motion of a planet, is that whereby the planet is carried in its orbit round the Sun; which in every one is found by observation to be from West to East.

The Diurnal Motion of a planet is that by which it turns about its axis, & is also from West to East.

Each planet is observed always to pass through the constellations, Aries, Taurus, Gemini, Cancer, Leo, Virgo, Libra, Scorpio, Sagittarius, Capricornus, Aquarius, Pisces; & it also appears that every one has a track peculiar to itself.

The Equinoctial Points are those two points of the Ecliptic opposite to one another through which the Earth passes in its annual motion when the length of the day & night is equal in all parts on the Earth.

One of these points called the Vernal Equinox, answers nearly to the 20th of March; & the other called the Autumnal Equinox nearly to the twenty second of September.

The plane of the Ecliptic is supposed to divide the celestial sphere into two equal parts, called the northern & southern celestial hemispheres; & any body in either of these hemispheres is said to have north or south Latitude, according to the hemisphere it is in: So that the Lat^d of a celestial object is its nearest distance from the ecliptic, taken on the sphere.

The plane of the other five orbits are observed to be partly in the northern, & partly in the southern hemisphere; so that every one cuts the ecliptic in two opposite points called Nodes.

One called the Ascending Node, marked thus ☊, is that through which the planet passes when it moves out of the southern into the northern hemisphere; & the other called the Descending Node, marked thus ☋ is that through which the planet passes in going out of the northern into the Southern hemisphere.

The right line joining the two Nodes of any planet, is called the Line of the Nodes.

The Aphelion, or Superior Apsis, is that point of the orbit where the planet is farthest from the Sun: The Perihelion, or Inferior Apsis is that point where it is nearest to the Sun: And the transverse diameter of the orbit, or the line joining the two apses is called the Line of the Apses or

Apsides

Terrestial Astronomy

Terrestial Astronomy is that which considers the motions of the celestial bodies as seen from the Earth, which is also in motion. The motions described in the preceding section were such, as would appear to an observer viewing the heavens from the Sun. But were he placed in one of the planets, suppose the Earth & there observed the motions of the rest, the Sun & other planets would appear to him to revolve round the Earth as a center; But the Sun would be the only one that moved uniformly the same way:

For the other planets would seem sometimes to move from West to East & then to stand still, then they would seem to run from East to West; & after standing some time they would again move from West to East & so on continually.

The place of the celestial sphere that any planet appears in when seen from the center of the Earth, is called its Geocentric Place. The Direct Motion of a planet is that by which it appears to move from West to East, & this motion is said to be according to the order of the signs, or in consequentia.

When the planet appears to stand still, it is said to be Stationary, & when its motion is apparently from East to West it is then called Retrograde, or has a motion in antecedentia, or contrary to the order of the signs.

WILLIAMS' 83 1/2 PATENT LE

These different appearances follow partly, in consequence of the observer being himself in motion while he is viewing the motions of the planets, & partly because he is not in the center of the motions which he observes

Spherical Astronomy

Definitions & Principles.

Spherical Astronomy is the art of finding, the measure of certain arcs & angles formed on the surfaces of the celestial & terrestial spheres, by the apparent motions of the bodies which are seen in the heavens. The surfaces of those spheres are supposed to be concentric to the center of the Earth, and to have correspondent circles described on both spheres. Great Circles are those which divide either sphere into two equal parts. Lesser Circles, are those which divide the sphere into unequal parts.

The Poles of a circle are the points on the sphere equally distant from that circle. An Axis is a right line supposed to connect the poles. The Celestial Axis is that right line about which the heavens seem to revolve. The North and South Poles of the world, are those two points where the axis cuts the celestial sphere.

The Equinoctial or Equator, is a great circle of the sphere equally distant from the poles of the world. Meridians or Hour Circles, or Circles of Right Ascension, or Circles of Terrestrial Longitude, are great circles perpendicular to the Equator, & passing through the poles of the world. The Ecliptic is a great circle inclined to the equator in an angle of about 23½, & cutting it in two points

diametrically opposite. The ecliptic is supposed to be divided into 12 equal parts, called Signs, beginning from one of its intersections with the equator; each sign containing 30 degrees, named & noted thus:

Aries	Taurus	Gemini	Cancer	Leo	Virgo
♈	♉	♊	♋	♌	♍
Libra	**Scorpio**	**Sagittarius**	**Capricornus**	**Aquarius**	**Pisces**
♎	♏	♐	♑	♒	♓

The first six are called northern, & the latter six southern signs. The Cardinall Points of the ecliptic are the four first points of the signs ♈, ♋, ♎, ♑; those of ♈ & ♎ are called Equinoctial Points, and those of ♋ & ♑ are called Solstitial Points. The Equinoctial Colure is a meridian passing through the equinoctial points; & the Solstitial Colure is another meridian passes through the solstitial points. The colures cut one another at right angles in the poles of the world.

Circles of Celestial Longitude are great circles perpendicular to the ecliptic. The Latitude of any point in the heavens is an arc of a circle of longitude intercepted betw^n^ that point & the ecliptic & is called north or south latitude, as the point is on the north or south side of the ecliptic.

Parallels of Celestial Lat.e are small circles parallel to the ecliptic. The Longitude of any object in the heavens is an arc of the ecliptic, intercepted betw.n the first point of Aries, and a circle of longitude passing through that point. The Right Ascension of any object is an arc of the equator, contained betw.n the first point of Aries & a meridian passing through that point: Or, it is the angle formed by the equinoctial colure, & the meridian passing over that point. The Declination of any object is an arc of a meridian contained between that point & the equinoctial: If the point is on the north side of the equinoctial, it is called north declination; but if on the south side, it is called south declination. The Obliquity of the Ecliptic is the angle made by the intersection of the equator & ecliptic, & is measured by the sun's greatest declination; which according to modern observations, is about 23.° 28.'

Parallels of Declination are small circles parallel to the Equinoctial. The Tropic of Cancer is a parallel of Decl.n of 23.° 28.' distant from the equinoctial in the northern hemisphere; & the Tropic of Capricorn is a parallel of Decl.n as far distant in the Southern hemisphere.

The Arctic Polar Circle is a parallel of Decl^n at 23.28 distant from the north pole; & the Antarctic Polar Circle is a parallel of declenation as far distant from the south pole. The Zenith is a point in the heavens directly over a place; & the Nadir is the point directly underneath.

The Horizon is that great circle of the sphere which is equally distant from the zenith & nadir of any place, & divides the sphere into the upper & lower hemispheres.

The Rising of a celestial object is when its center appears in the eastern part of the horizon; & its Setting is when its center disappears in the western part of the horizon.

Azimuth or Vertical Circles, are great circles perpendicular to the Horizon, passing through its poles, which are the zenith & nadir.

The Prime Vertical is that vertical circle which passes through the east & west points of the horizon.

The meridian of a place is a vertical circle passing through the north & south points of the horizon.

The hour circle at right angles to the meridian is called the six o'clock hour circle. The Azimuth of a celestial object is an angle at the zenith formed by the meridian of any place & a vertical circle passing through that object when it is above or below the horizon. And it is measured by the arc of the Horizon intercepted betⁿ these vertical circles.

The Amplitude of any object in the heavens is usually taken as an arc of the Horizon contained betw the eastern point of it, & the center of the object at its rising, or betw the western point of it, & the center of the object at its setting; or it may be taken as an angle at the Zenith, included betw the meridian of a place & a vertical circle passing through the object at its rising or setting.

The Altitude of any object in the heavens is an arc of a vertical circle intercepted betw the center of that object & the Horizon. The Meridian Altitude, or Meridian Zenith Distance, is the altitude or zenith dist when the object is on the meridian of the place.

The Culminating of any celestial object, is the time it hath it, or comes to the meridian. And the Medium Coeli, or Mid-Heaven, to any place, is that degree of the ecliptic, or part of the heavens, over the meridian of that place, at any time. Or the Mid-Heaven is the distance of the meridian from the first point of Aries, reckoned on the equinoctial.

Almicanthers, or Parallels of Altitude, are small circles parallel to the horizon. A Parallel Sphere is that position of the sphere in which the circles apparently described by the diurnal rotation, are parallel to the horizon; which can happen only at the poles.

A Right Sphere is that in which the diurnal motions are at right angles to the horizon: Thus it appears in all places under the equator. An Oblique Sphere has all the diurnal motions oblique to the horizon: And thus the motions appear to all parts of the Earth, except under the poles & equator. Diurnal Arcs are those parts of the parallels of declination of celestial objects which are apparently described between the times of rising & setting of those objects: And Nocturnal Arcs are the parts of those parallels apparently described from the time of setting to the time of rising.

Semi-Diurnal & Semi-nocturnal Arcs, or the halves of diurnal & nocturnal arcs, are the parts of the parallels intercepted betw. the meridian & horizon.

The correspondent part of the equator answering to the semi-diurnal arcs, gives the time betw. noon & the rising or setting; & the equatorial part answering to the semi-nocturnal arc, shows the time between midnight & the time of setting or rising. The Oblique Ascension of any object in the heavens, is an arc of the equinoctial intercepted betw. the 1st. point of Aries & the eastern part of the Horizon when that object is rising; & the Oblique Descension is an arc of the equinoctial intercepted between the first point of Aries & the western part of the horizon at its setting. The Ascensional Difference belonging to any celestial object is an arc of

the equinoctial intercepted between the Horizon & the hour circle which the object is on when it rises or sets; or it is the difference betw the right & oblique ascension of that object.

In the Sun, it is the time that he rises or sets before or after the hour of six. The Latitude of any place on the Earth is an arc of a terrestial meridian contained between that place and the equator; or it is an arc of a celestial meridian intercepted betw the zenith of the place & the equinoctial; being north or south, according to the side of the equator it is on. The Longitude of any place on the Earth is an arc of the equator contained between the meridian of that place & the meridian which is chosen for the first, where the reckoning of longitude begins.

Viz it is the angle at the pole formed by the first meridian & that of the place. Refraction, in an astronomical sense, is the difference between the true and & apparent altitudes of celestial objects; they appearing more elevated above the horizon than they really are, on account of the density of the Earth's atmosphere, or air or vapours surrounding it. The Parallax of a Celestial Object is the distance between its place in the heavens, as seen from the Earth, & it place when seen from

the surface of it. The Twilight is that medium between light & darkness which happens in the morning before sunrise, & in the evening after sun-set.

The Crepusculum is a small circle parallel to the Horizon at 18° below it, where the twilight begins & ends. The Latitude of a place is expressed by an arc of a meridian, shewing the distance between the Zenith of that place & the Equinoctial, or, by an arc of the meridian shewing the height of the pole above the horizon. As the Earth makes one revolution on its axis in a common day of 24^{H}, therefore every point of the equator will describe a circle of 360° in 24^{H} & consequently, if 360° gives 24^{H}, any other number of degrees will give its proportional hours. And if 24^{H} gives 360°, any other number of hours will give its proportional number of degrees.

And hence are derived methods for converting arcs of circles into measures of time, & measures of time into arcs of circles.

Problem 1st

To reduce Degrees. Minutes &c into time.

Rule.

Divide the given Degrees. Minutes &c by 3 & the Quotient by 5 the last Quotient will be the Hours Minutes & Seconds which are equivalent to them.

Reduce 36°. 55'. 50" into time

3) 36°. 55'. 50" (2
5) 12. 18. 26 (1
2. 27. 43 . Ans^{r} ^{H}2. $^{'}$27. $^{''}$43

Reduce 74°. 55'. 49" into time.

3) 74°. 55'. 49" (1
5) 24. 56. 36 (1
4. 59. 33 . Ans^{r} ^{H}4. $^{'}$59. $^{''}$33

Reduce 55°. 03'. 44" into time.

3) 55°. 03'. 44" (2
5) 18. 21. 04 (4
3. 40. 12 . Ans^{r} ^{H}3. $^{'}$40. $^{''}$12

WILLIAMS'S 33 1-2 PATENT L.B

Problem 2.nd

To reduce time into Degrees Minutes &c.

Rule.

Multiply the given time by 5 & the product by 3, the last product will be Degrees Minutes & Seconds which are equivalent to them.

Reduce $^{H}12^{'}17^{''}15$ into Degrees Minutes &c.

$^{H}12^{'}17^{''}15$

5

$61^{'}28^{''}15$

3

$184^{'}26^{''}15$ Ans^{r} $184^{'}26^{''}15$

Reduce $^{H}3^{'}04^{''}30$ into Degrees Minutes &c.

$^{H}3^{'}04^{''}30$

5

$15^{'}22^{''}30$

3

$46^{'}07^{''}30$ Ans^{r} $46^{'}07^{''}30$

Reduce $^{H}6^{'}05^{''}49$ into Degrees Minutes &c.

$^{H}6^{'}05^{''}49$

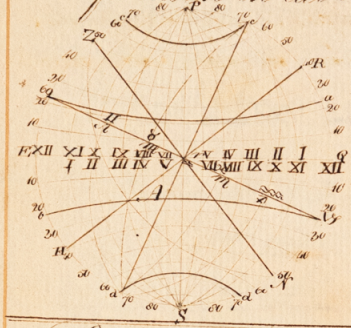
5

$30^{'}29^{''}05$

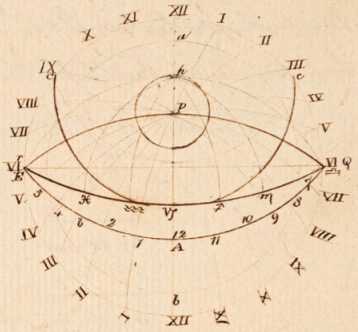
3

$91^{'}27^{''}15$ Ans^{r} $91^{'}27^{''}15$

Projection of the Sphere upon the Plane of the Solstitial Colure or Meridian of any Place suppose that of London in lat.ᵉ 51. 32



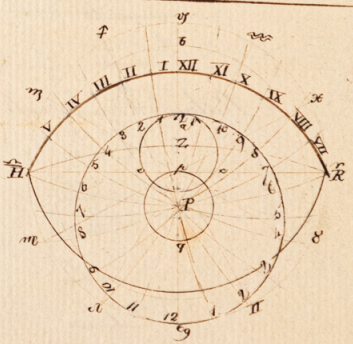
Projection of the Sphere upon the Plane of the Horizon.



Projection of the Sphere upon the Plane of the Equator.



Projection of the Sphere upon the Plane of the Ecliptic.



WILLIAMS'S 83 L.2 PATENT LB

Problems of the Sphere

Problem 1st

Given the Sun's long^{e}, & the obliquity of the ecliptic
Req^{d} the Sun's right ascension & decl^{n}.

Exam. Let the obliquity of the ecliptic, or the Sun's greatest decl^{n}, be 23.$^{\circ}$28', & the Sun's place 13.$^{\circ}$16 in Taurus:

Required the rest

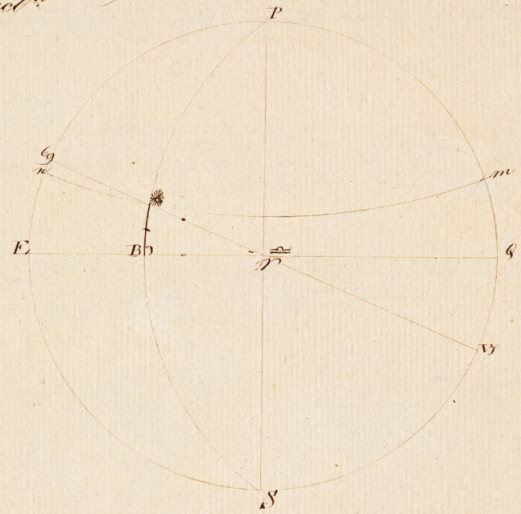


To find ♈ right ascension		*To find* ♈ Decl^{n}	
As cola ♈	= 42.$^{\circ}$44	As rad	= 90.$^{\circ}$00 ... 10
To radius	= 90.$^{\circ}$00 ... 10	To si ♈	= 42.$^{\circ}$16 ... 9.83594
So cosi 𝐿 ♈	= 23.$^{\circ}$28 ... 9.96251	So si 𝐿 ♈	= 23.$^{\circ}$28 ... 9.60012
To tan ♈B	= 46.$^{\circ}$48 ... 9.93622	So si B ♉	= 15.$^{\circ}$50 ... 9.43606

Ans^{r} { ♉ Right Ascension is 46.$^{\circ}$48
♉ Declination is 15.$^{\circ}$50. N }

Problem 2nd

Given the obliquity of the ecliptic & the suns decl^{n}
Req^{d} the suns long^{e} & right Ascension
Exam^{e}. The obliquity of the ecliptic being 23°.28' what is the suns long^{e} & right ascension when he has 20°.43' of north decl^{n}



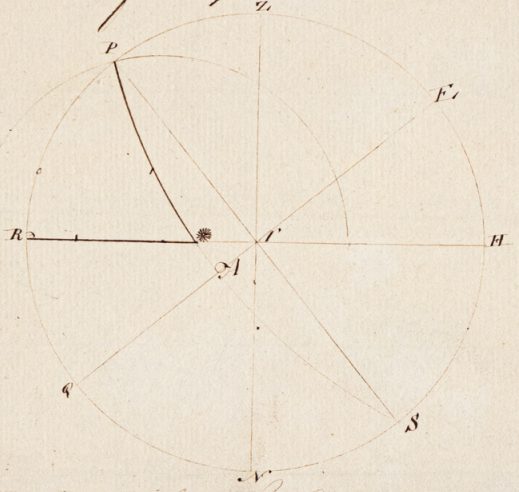
To find the suns long^{e}		To find the suns right Ascen^{n}	
As si ♈	= 23°.28'	As rad	90°.00' 10
To rad	= 90°.00' 10	To cot ♈	23°.28' 10.36239
To si B ☉	= 20°.43' 9.55869	To tan B ☉	20°.43' 9.57772
To si ♈ ☉	= 62°.50' 9.94857	To cosi ♈B	69°.20' 9.94011

Ans^{r} { ☉ Long^{e} 62°.50' or 2°.40' in ♊ ; ☉ Right Ascension 69°.30' }

WILLIAMS'S 83 L. 2 PATENT L.B

Problem 3rd

Given the Sun's Declᵗⁿ & his amplitude
Reqᵈ the Latᵈ of the place.
Exam. Being in a place where the compass had no variation, on a day when the sun's declᵗⁿ was 15.12. N, I observed him to rise 62.30 from the north towards the East
Reqᵈ the Latᵈ of the place.



To find the Latᵈ	
As cosi R☉ = 62.30	0,33557
To radius = 90.00	10,
So cosi P☉ = 74.48	9,41862
To cosi RP = 55.24	9,75421

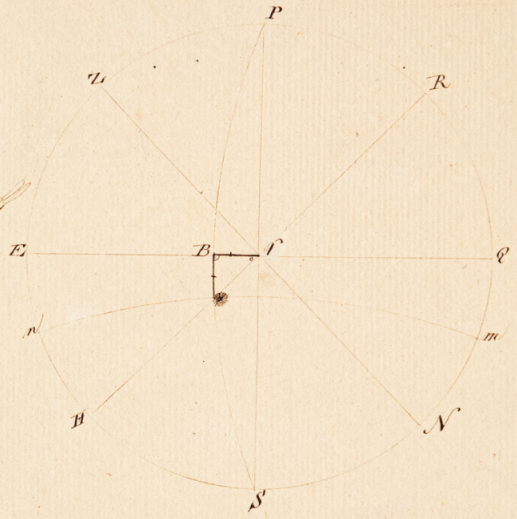
Ans { Latᵈ 55.24 N. }

Problem 4th

Given the Sun's declx, & his ascensional difference
Reqd the Latitude of the place.

Exam. When the Sun has 20.01 of declx south, he was observed to set at 4h 30m. Reqd the Latitude of the place.

	h 6.00
	4.30
	1.30
	×5
	7.30
	×3
Asc'n	22.30 Diff



To find the Latitude		
As tan B	= 20.01	9.58854
To radius	= 90.00	10.
So sine PB	= 22.30	9.58284
To cotan P	= 43.25	10.02138
	90.00	
Lat	46.25	

Ans { Latitude 46.25. N. }

Problem 5th.

Given the Sun's decl.n & altitude at six o'clock
Req.d the Lat.e of the place.

Exam. Being at Sea, on a day when the Sun's decl.n was 20°.0′ N his alt.e at six o'clock in the evening was 18°.45′. What was the Lat.e of y.e place of Obs.n



To find the Lat.e	
As si C ☼	= 20°.0′ 9.54256
To radius	= 90°.00′ ..... 10.
So si B ☼	= 18°.45′ 9.50710
To si ZC	= 69°.32′ 9.97166

Ans.r { Lat.e 69°.32′ N }

Problem 6th.

Given the Suns decl.n & his alt.e when due E or W
Req.d the Latitude of the place.

Exam. In a place where the Compass had no variation the Sun was observed to be due East when his decl.n was 16.28 N & his alt.e 20.12. What is the Lat.e of the place.



To find the Latitude	
As si C ☼ 20.12	.... 9.46181
To radius 90.00	.... 10.
So si B ☼ 16.28	.... 9.45674
To si ZC 56.00	.... 9.91855

Ans.r { Latitude 56.00 N }

WILLIAMS'S SELF-PRONOUNCING

Problem 7th

Given the Latd of the place & the Suns decl.n
Req.d the time when the twilight begins & ends.
Exam. At what time does the twilight begin & end at London when the Sun's decl.n is 15.° 12′ N the Lat.d of the place being 51.° 32′ N

24	48.° 58.′ 00″
2	59. 29. 30
	9. 55. 54
	12. 00. 00
	2. 04. 06



To find ♄ P.

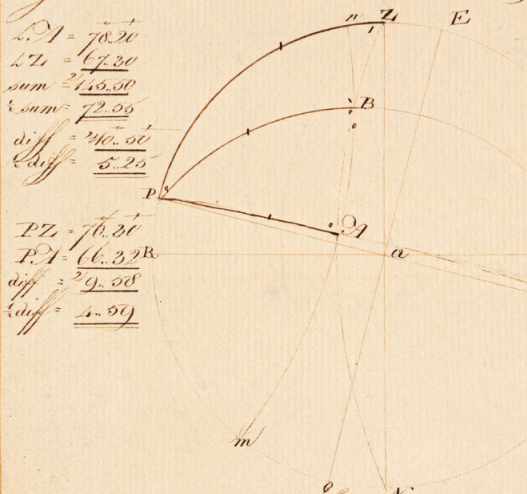
Z P	= 38.° 28′	cosec	10, 20617
P ☉	= 71. 48		10, 01357
Z ☉	= 108. 00		
sum	= 221. 16		
½ sum	= 110. 38	sine	9, 97121
Z ☉	= 108. 00		
diff	= 2. 38	sine	8, 66223
			48.° 85508
	7. 29	cos.	9, 42754
	29		
♄ P	48. 58		

Ans.r { The Twilight begins at 2h 04.′ 06″ A. M.
& ends at 9. 55. 54 P. M }

Problem 8th

Given the Lat.e of the place, the Suns decl.n & azimuth & Req.d his Altitude & time of the Observation

Exam.e In the Lat.e of 13.30. N & when the Sun has 23.28 of north decl.n What is the Suns alt.e & time of the day when she is seen on the E. N. E. azimuth circle.



∠ P = 78.20		PZ = 76.30
∠ Z = 67.30		P A = 66.32
sum = 146.50		sum = 143.02
½ sum = 72.55		½ sum = 71.31
diff = 10.50		90.00
½ diff = 5.25		Z A = 82.54
		A D = 7.06
PZ = 76.30		∠ Z = 67.20
P A = 66.32 R		∠ B = 101.40
diff = 9.58		sum = 169.00
½ diff = 4.59		½ sum = 84.35
		90.00
		B Z = 32.54
		B D = 57.06

In ∠d ∆ P Z A

To find ∠ A	To find Z A	To find ∠ P
As sin P A 66.32 ... 9.962749	As sin ½ diff 2∠ = 5.25 ... 10.2304	As sin P A 66.32 ... 9.962749
To sin ∠ Z 67.20 ... 9.996361	To sin ½ sum = 72.55 ... 9.980840	To sin ∠ Z 67.20 ... 9.996361
To sin P Z 76.30 ... 9.987782	To tan ½ diff 2 = 4.59 ... 8.910459	To sin Z A 82.54 ... 9.999666
To sin ∠ A 78.20 ... 9.999093	To tan ½ dif. sum = 81.57 ... 9.949393	To sin ∠ P 78.06 ... 9.999776
	∠ 2	29.33
	Z A 82.54	6.54.36
		12.00.00
		6.05.24

WILLIAMS'S 83 & 2 PATENT L.S

To find ∠P (In ⊿ PZB) To find BZ

As cosi ½ diff 2 ∠ P	= 4.59	... 0,00165	
To cosi ½ sum	= 71.31	... 9,50110	
So ha ½ sum 2 ∠	= 0.35	... 4,02209	
To col ½ cont ∠	= 16.35	... 10,57584	
			As si ∠ Z = 67.30 0,03439
			To si PB = 60.32 9,96251
			To si ∠P = 32.10 9,73805
			To si BZ = 32.57 9,73495

×2

3 33.10 ∠P

3 11.03.20

2.12.40

12.00.00

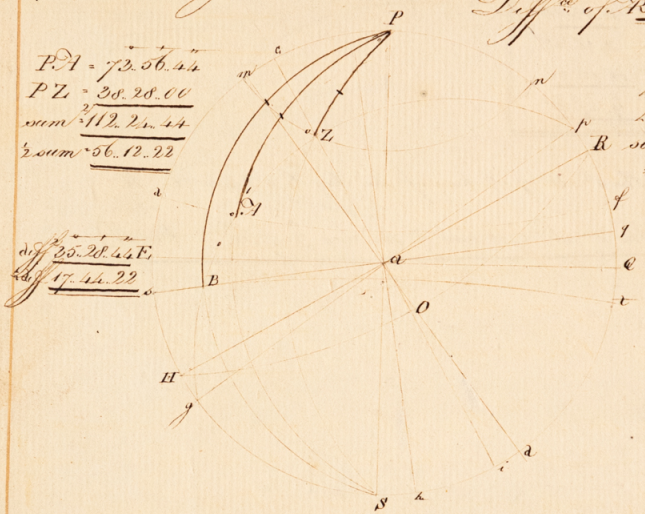
9.47.20

Ans { The 1st alt 7.06 time of the day 6.5.24 A.M.
The 2nd alt 7.06 time of the day 9.47.20 A.M. }

Problem 9th

At London, on the 10th of December 1780, at what time of the night will the stars Aldebaran & Rigel be on the same azimuth circle.

Aldebaran's Decl^{n} = 16°.02'.16" N. AR = 4h.23'.19" = 65°.49'.45"
Rigel's Decl^{n} = 8°.28'.11" S. R = 5h.03'.28" = 75°.50'.30"
Diff^{ce} of R 40.39 = 10°.09'.45"



To find AB	(In the △. APB) To find ∠A
PA = 73°.56'.44" 10,54095	As si AB = 26°.30'.00" ... 0,35047
∠P = 10.09.45 ..... 9.99314	To si ∠P = 10.09.45 ... 9.24659
Arc A = 73.42.13 10,53409	So si PB = 98.20.11 ... 9.99524
Arc A = 73.42.13 10,01781	To si ∠A = 23.41.26 ... 9.59230
PA = 73.56.44 9,44178	180.00.00
Diff = 24.45.58 9,95011	∠A 155.58.34
AB = 26.30.00 9,95179	

To find ∠ Z. (In the △ APZ.) To find ∠P

As si ZP = 38°.28′ 0, 20617
To si ZA = 23.0.26 ..... 9, 59231
So si P.A = 72.36.45 ..... 9, 98272
To si = 37.0.24 ..... 9, 78120

As cosi ½ diff 2 Alt^s = 17.45.22 ..... 0, 02115
To cosi ½ sum = 56.12.22 ..... 9, 74523
So tai ½ sum 2 L^s = 82.55.31 ..... 10, 90620
To ecka ½ reg ∠ = 11.59.37 ..... 10, 67258

180.00.00
∠ Z. 132.59.26

2
∠P 73.59.54

Aldebaran R ..... 28°.23′.19″
Sun's R 17.06.53
Time of Culminating nearly 11.16.26
As 24ʰ : 4′.24″ :: 11ʰ.16′.26″ ..... −2.04
True time of Culminating 11.14.22
Star's Dist^ce from the Merid^n 135.59
Time of Observation ..... 9.38.23 P. M.

Ans^r { On the 10th Dec^r 1780 the Stars Aldebaran & Rigel were observed to be on the same Azimuth circle at 9ʰ.38′.23″ P. M }

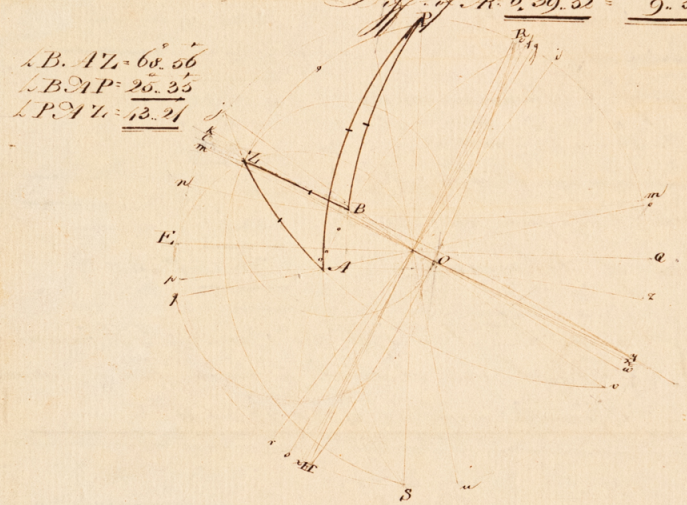
Problem 10th

Given the altitudes of two known Stars

Req.e the Latitude of the Place.

Exam.e The Altitude of the Hydra's heart was observed to be 48°.54' & of the Lion's heart 16°.00'. What is the Lat.e of the place of Obser.n?

Hydra's heart Decl.n	7°.43'.00" S.	R. 9h.16'.47"	= 139°.11'.45"
Lion's heart Decl.n	12°.02'.36" N.	R. 9.56.39	= 149.09.45
Diff.ce	19°.46'.36"	R.s 0.39.52	= 9.58.00



∠B.AZ = 68°.56'
∠B.AP = 25.35
∠PAZ = 43.21

90°.00
PZ 62.43
ZE 27.17

To find AB	(In the △ APB)	To find ∠A
PB = 76°.57'.54" tan ... 10,635412		As si AB = 22.59 0,40843
∠P = 9°.58'.00" cosi ... 9,99340		To si ∠P = 9.58 9,23824
Arc A = 76.46.20 tan ... 10,62882		To si PB = 76.57.54 9,98866
Arc A = 76.46.20 cosec.t 10,01170		To si ∠A = 25.35 9,63332
PB = 76.57.54 cosi. 9,33324		
Diff = 20.56.34 cosi ... 9,97032		
AB = 22.59.00 cosi ... 9,96,08		

To find ∠ P1 In 1st △ A Z1 B

A Z1 = 49.16	cosec	10,12047	
A B = 22.59		10,10842	
B Z1 = 45.00	..		
sum 2)117.15			
½ sum = 58.37.30	sine	9,93134	
B Z1 = 45.00.00			
diff = 13.37.30	sine	9,37211	
		2)19,83234	
34.28	cosi	9,91617	
×2			
∠ P1 68.56			

To find P Z1 In the △ A P Z1

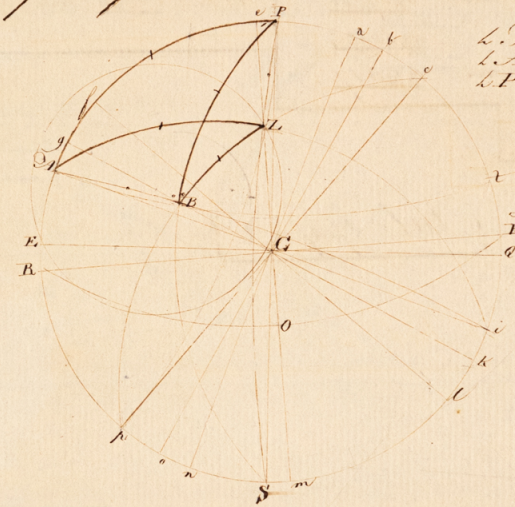
A Z1 = 49.16	tal	10,06492
∠ P1 = 43.21	cosi	9,86164
Arc P1 = 40.11	tal	9,92656
Arc A = 46.11	cosec	10,19038
A Z1 = 49.16	cosi	9,81461
Diff = 57.39	cosi	9,72982
P Z1 = 62.43	cosi	9,66127

Ans: { Sal° 27.17 N }

Problem 11th

Given the Suns Declr, two altes & the time betn ye Obsrs
Reqd the Latitude of the place.

Exam. On a day when the Suns declr was 20.00 N, in the forenoon, the Suns altitude was observed to be 18.30, & three Hours after his altitude was 44.00. What was the Latitude of the place.



∠ ABZ = 114.10
∠ ABP = 81.57
∠ PBZ = 32.13

90.00
∠P 35.58
Lat. 54.02

To find AB (In ye △ APB) To find ∠ B.

AP = 70.00	tab	10,43893	As si. AB = 42.09	0,17323
∠P = 45.00	cosi	9,84948	To si ∠P = 45.00	9,84948
Arc A = 62.46	tab	10,28841	To si. AP = 70.00	9,97299
Arc A = 62.46	cosec	10,05102	To si ∠B = 81.57	9,99370
P.A = 70.00	cosi	9,53405		
Diff = 7.14	cosi	9,99652		
AB 42.09	cosi	9,87001		

To find ∠B In the △ ABL

AB = 42.09 } cosec = 10.17323
BL = 46.00 } 10.11307
AL = 71.30
sum = 159.39
½ sum = 79.49.30 sine 9.99311
AL = 71.30.00
Diff = 8.19.30 sine 9.16079
29.47014
57.05 cosi 9.73307
×2
114.10 ∠B

To find ∠P In ▽ △ PBL.

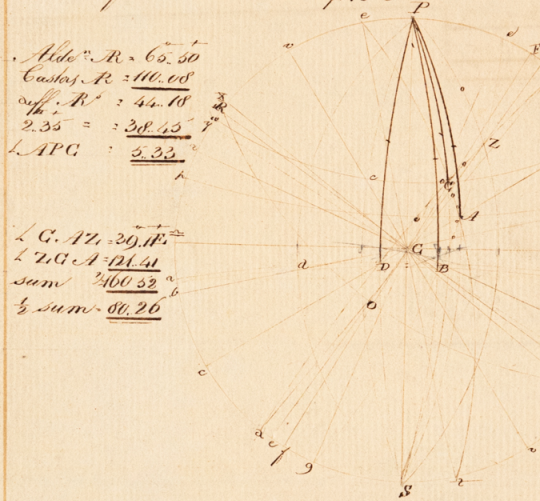
BL = 46.00 ta 10.01516
∠B = 32.13 cosi 9.92739
Arc A = 41.44 ta 9.94255
Arc A = 41.44 cosec . . . 10.18103
BL = 46.00 cosi 9.84177
Diff = 28.46 cosi 9.94279
∠LP 28.38 cosi 9.78814

Ans: { Sat^e 54.02 N }

Problem 12th

Two known stars being observed on ye same azimuth & two other known stars being observed on another azimuth & ye time betw the Observations being known to find ye Latitude of the place. Exam. The Stars Aldebaran in Taurus & Rigel in Orion were observed on the same azimuth & 2h 35m after, the Stars Castor in Gemini & the Hydra's heart were also observed on another azimuth. What was the Latde of ye place of Obs.

Aldebaran's Decl. = 16° 03′ N = right ascen. 4h 23m 19″ = 65° 50′ 00″
Rigel = 8.28 S = 5 03 08 = 75 59 30
Castor = 32.21 N = 7 20 32 = 110 08 00
Hydra's Heart = 7.42 S = 9 16 47 = 139 12 00



Alde. R. = 65° 50′
Castor R. = 110.08
diff R. = 44.18
2h 35m = 38.45
∠APG = 5.33

∠PAG = 15.0
∠PAZ = 23.01
∠GAZ = 39.11

∠PGA = 151.32
∠PGZ = 29.21
∠ZGA = 121.41

∠GAZ = 29.11
∠ZGA = 121.41
sum = 160 32
½ sum = 80.26

∠GAZ = 29.11
∠ZGA = 121.41
diff = 82.30
½ diff = 41.15

90.00
Co-lat 38.47
Lat 51.13

To find AB (In ye △ APB) To find ∠A

PA	= 72° 57′ 00″	hav	10,534108
∠P	= 10.09.30	cosi	9,993183
Arc A	= 73.42.31	hav	10,53423
Arc A	= 73.42.31	cosec	10,01780
PA	= 72.57.00	cosi	9,464166
Diff	= 24.45.29	cosi	9,92813
AB	= 25.29.30	cosi	9,95182

As si AB	= 25.29.30	9,35050
To si ∠P	= 10.09.30	9,24642
To si PB	= 98.28	9,99524
To si ∠A	= 23.01	9,59216
	180.00	
∠A	159.59	

WILLIAMS'S 3 YEAR PATENT

To find CD (In ▽ PCD) To find ∠ C

PC = 57.39	tan	10,19832	As si CD = 48.43	0, 12400
∠P = 29.04	cosi	9,94154	To si ∠P = 29.04	9,68648
Arc A = 54.04	tan	10,13986	To si PD = 97.43	9,99605
Arc A = 54.04	cosec^t	10,09168	To si = 39.51	9,80653
PC = 57.39	cosi	9,72843		180.00
Diff = 43.39	cosi	9,85948	∠C 140.09	
CD = 48.43	cosi	9,81945		

To find AC (In ▽ APC) To find ∠A To find ∠C

PC = 57.39	ha	10,19832	As si AC = 47.04	0,53242	As si AC = 47.04	0,53242
∠P = 5.23	cosi	9,99779	To si ∠P = 5.23	8,98549	To si ∠P = 5.23	8,98549
Arc A = 57.32	ha	10,19628	To si PC = 57.39	9,99675	To si P+A = 72.57	9,97622
Arc A = 57.32	cosec^t	10,07381	To si ∠P = 16.10	9,44466	To si = 18.28	9,83172
PC = 57.39	cosi	9,72843				180.00
diff = 15.28	cosi	9,98192			∠C 161.32	
∠C = 47.04	cosi	9,98044				

To find ½ diff 2 Sides (In ▽ ACZ) To find ½ sum 2 Sides

As si ½ sum 2 ∠s = 81.26		0,00608	As cosi ½ sum 2 ∠s = 81.26		9,77938
To si ½ diff = 41.13		9,81911	To cosi ½ diff = 41.13		9,87612
To ha ½ Side betⁿ them 8.32		9,47822	To ha ½ Side betⁿ them 8.32		9,47822
To ha ½ diff other 2 Sides 5.45		9,00141	To ha ½ sum other 2 Sides 34.10		9,83372
				5.44	
			∠C 28.26		

To find PZ (In ▽ CPZ

CZ = 28.26	ha	9,73356
∠C = 39.51	cosi	9,88520
Arc A = 22.34	ha	9,61076
Arc A = 22.34	cosec^t	10,41574
CZ = 28.26	cosi	9,94457
Diff = 35.05	cosi	9,91262
PZ = 28.47	cosi	9,89179

Ans^r { La hitude 51.13 N. }

WILLIAMS'S 33 1/2 PATENT L.B

WILLIAMS'S 83 1/2 PATENT L.B

1654

Days Works.

By Days Works is meant certain practical methods deduced from the foregoing Elements of finding a ships place every day at noon & also the course & distance it has then to sail; the performance of which requires a knowledge of the following Articles. 1st The means of measuring a ships run.

2nd The nature & use of the steering & azimuth compasses

3rd The nature of Leeway & the method of correcting the Courses steered on that account.

4th Methods of finding the Variation of the Compass & correcting the courses steered by it.

6th The nature & use of Instruments proper for

observing the Altitude & Angular Distances of Celestial Objects &c. Methods of finding the Latitude & Longitude of a Ship at sea by Celestial Observations by which the error in the daily account of Latitude & Longitude is discovered & the true place of the ship determined independant of it

Of measuring a Ships Run.

Many methods have been thought for measuring the rate which a ship sails at but that which is mostly in use is by the Log & half minute glass.

The Log is a piece of thin board of a quadrantal form loaded on the circular edge with as much lead as will just make it swim upright without sinking.

To this a line is fashioned of about 150 fathoms in length called a log line & divided into certain spaces called knots & this is wound on a reel. The half minute glass is composed of two hollow Cones of glass, joined together at their points forming by that means a small neck through which there is a hole & as much fine sand is put into one of the Cavities as will run through the hole in the neck into the other in half a minute.

On board every well conducted ship the rate which it sails at is determined every hour by throwing the log into the sea, & observing how many knots & decimals of a knot run off the reel while the sand runs out of one part into the other

This way of measuring a Ships rate of sailing is founded on a supposition that the length of each knot is the same part of a sea mile, that half a minute is of an hour.

Now, taking the mean between the measurements which have been made, it will be found that a degree of a great circle of the Earth contains 364800 English feet and $\frac{1}{60}$ of this or 6080 feet make one nautical mile, the $\frac{1}{120}$ part of which or $50\frac{2}{3}$ feet should be the length of each knot, but because in the act of throwing the log, it cannot but be in some small degree drawn after the ship, & also because it is safer to have the reckoning rather before the ship than after it, 50 feet may be taken for the length of a Knot & each knot is divided into ten equal parts called tenths.

Problem 1st

To correct the distance shewn by the Log.

This Problem admits of three cases, namely, when the knots on the line are not of a proper length, when the time which the Glass runs is more or less than ½ a minute, or when both are erroneous.

Case 1st

When the glass is true but the log line faulty.

Rule.

As 30 feet are to the feet betn knot & knot, so is the distance run by the log, to the true distance run by the log.

Example.

Suppose a ship sails at the rate of 4½ miles an hour by a log line which has only 38 feet to a knot. What is the ships true rate of sailg?

As 30 feet ... : ... 38 feet ... : ... 4.5 knots

4.5
190
152.
30) 171.0 (5.7
150
210
200
10

Ans. 5.7 miles an hour.

Case 2nd

When the line is truly divided, but the glass faulty.

Rule.

As the time which the Glass runs is to 30" so is the distance run by the log to the true distance run by the ship.

Example.

Suppose a ship sails 5 knots by a Glass which runs out in 17 seconds. What is the true rate of sailing.

As 17 ... : ... 30" ... : ... 5 knots

5
7) 150 (8
126
24

Ans. 8 miles an hour.

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Case 3rd

When both log line & glass are faulty

Rule.

As 5 times the number of seconds which the Glass runs is to 3 times the measured length of a knot in fact, so is the distance run by the log to the true distance run by the Ship.

Example

A Ship runs 3½ knots an hour by a log line which has only 33 feet to a knot & a Glass which runs out in 22 seconds What was the true rate of Sailg

	22	33 feet	
	5	3	
As	110	99	2.85 knots

215
495
99
297
110) 31185 (283
220
918
880
385
330
55

Ans: 2,83 miles an hour.

Of the Compass.

The Steering Compass is the Instrument which points out the way or course which the Ship sails. It consists principally of three parts, viz, the Box, the Card, & the Needle.

The Box contains the Card & the Needle.

It is generally of brass & circular, hung within a square wooden one. The circular brass box is suspended within a ring which is concentric to it, by means of an axis on which it is at liberty to vebrate freely, & this ring is connected to the outside box by means of an axis placed at right angles to the former, so that the inner brass box is at liberty to retain always a horizontal position notwithstanding the various motions of the ship.

The Card is a circular piece of stiff paper which has its circumference divided into points, & quarter points of the Compass or Horizon. The Needle is a straight piece of steel, equal in length to the diameter of the Card & made magnetical by which means it acquires the property of pointing with one of its ends towards the North, & with the other towards the South. This piece of steel is fixed to the Card with that end of it which points to the North directly under the North point of the Card; & the other directly under the South point of it, & in the center of the Card, there is a conical hole or socket which being put on an upright pin fixed in the center of the bottom of the Box the Card will be at liberty to turn

freely on its center & consequently will rest always with its North point towards the North point of the Horizon by the property of the Needle. A perpendicular line is drawn down the inside of the circular box, & the Compass is placed in a frame called the Binnacle immediately before the person who steers the ship, & with the above mentioned line directly between the center of the Compass & the stern of the ship, so that the business of the Helmsman is to keep that point of the Compass which the ship is to sail on directed to the line on the side of the Compass box.

When the wind blows from a point of the Compass which is so near the ships intended course as to render it necessary to keep the ships head as near the wind as possible, the action of the wind on its broad side will drive it to leeward of the course shewn by the Compass & this deviation is generally called a ships leeway.

Hence, Leeway is the difference betw a ships true & apparent course by the Compass.

The quantity of Leeway which different ships make under the same circumstances of wind, sea, or sail, is very different, & even the same ship when differently laden will move more or less though these circumstances be exactly the same, so that it is impossible to give rules which will serve for all ships & all circumstances & it is generally left on board most ships for every one to allow what he in his own private judgement may

think proper. But on board all well conducted ships the Leeway is determinated by actual obs^ns every time the log is thrown & occasionally in the intermediate intervals by means of a semi circular piece of lead or other substance divided into degrees & nailed on the Taffarel or rail across the stern of the ship, the diameter being placed at right angles to the keel of the ship & the bearing of the wake, & take y^e exact bearing of the center of the mast, the difference bet^n which & the opposite point to the bearing of the wake will shew the quantity of Leeway very accurately. But in circumstances where neither of those methods are made use of, the following rules may be attended to.

1^st. All sails set & the water smooth, allow a ¼ of a point.

2^nd. If all sails be set, & there be any sea, allow ½ a point.

3^rd. Allow 1 point when it blows so fresh that the small sails are taken in.

4^th. Allow 1 point & a ½ when one reef of each topsail are taken in.

5^th. Allow 2 points when the topsail are close reefed.

6^th. Allow 2 points & ½ when 1 topsail is handed.

7^th. Allow 3 points when both topsails are handed.

8^th. Allow 4 points when the foresail is handed.

9^th. Allow 5 points when under the Main sail only.

10^th. Allow 6 points when both fore & main sails are in.

11^th. Allow 7 points when under bare poles.

But in all these cases great regard must be had to the roughness of the sea & also the trim of the ship.

Problem 2nd

Having the place of the wind given, the course steered & the Leeway, it is required to find the true course of the Ship.

Rule.

Count the nearest way of the Compass from the wind to the course steered & as points & parts of a point farther as the Leeway amounts to, & you will have the true course which the ship makes.

Nb When the ship has her starboard tacks aboard alter the course steered as many points & parts of a point as leeway amounts to from the left hand, but if the larboard tacks are aboard it must be altered the like quantity toward the right.

Example.

Courses by Comp.	Winds	Leeway	True Course
NW by N	W SW	1 1/4	N by E 1/2 E
N NE 1/2 E	E by S	1/2	N N E
N W by W	N N E	2	W by N
S SE	S W 1/2 W	2 1/2	S E 1/2 E
S by W 3/4 W	S E by E	3/4	S S W 1/2 W
S E by E 3/4 E	N E by N 1/2 E	1 1/4	S E 1/2 E

Of Hadleys Quadrant.

The following are the principal parts of this Instrument

1st The Arch. Which is generally an Octant or the eighth part of a circle, but sometimes, is made a sextant; or the sixth part of a circle; On this arch, there are described three concentric circles their common centers being the center of the Instrument. When the arch is an octant, it is divided into 90 equal parts, by lines drawn across the three circles; & when it is extended to a sextant it is divided into 120 parts. Each of those equal parts is divided into three others by lines drawn across the space which is included between the two exterior circles. The former divisions are used as degrees &, consequently each of the latter will contain twenty minutes

2nd The Index: Which is a straight bar of brass, or wood that turn on the center of the Quadrant & points out the degrees & minutes which are the measure of the observed angle, on the arch of the quadrant. That part of the Index which slides over the graduated arch is broad enough to admit of a slit being cut across it suffi-

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ciently long to see, at least seven degrees of the arch through it the lower edge being circular & to coincide perfectly, through out its whole length with the outermost of the concentric circles which are described on the arch.

This side of the slit is also worked off to a very fine edge on which are cut 21 short lines, numbered from 0, to 20, & forming 20 divisions or spaces, which are together exactly equal to 19 divisions or spaces on the arch. This is usually called a Vernier.

3d. The great Speculum, or Index Glass which is fixed on the index, directly over the center of the Instrument & with its reflecting plane exactly perpendicular to the plane of the quadrant. This Glass is fixed in a brass frame which has a base, bent backward, in a right angle to the surface of the glass & the base is attached to the index by means of three screws, two of which being in a right line parallel to the surface of the glass draw it to the index, & the third which is beyond these bears it from it.

4th. The two Horizon Glasses or small Specula which are placed on that radius of the Instrument which is farthest from 0, or the beginning of the divisions of the arch.

That, nearest to the center of the Instrument is used in making the fore observation; & its plane when properly

adjusted is parallel to the plane of the index glass, when the Index stands at 0; and perpendicular to the plane of the quadrant That which stands farthest from the center of the Instrument is used in making the back observ^n, & has its plane when properly adjusted, perpendicular to the plane of the quadrant & to the plane of the Index Glass when the Index stands at 0. Each of these Glasses stand on a circular base which has two steel points in its under side; & those points rest in two dimples made in the upper surface of another base, of the same form; & the two bases are attached to each other by means of two screws, one in the front & the other at the back of the glass.

The lower base ends in a cylindric axis, which passes through the frame of the quadrant, & has a lever, or arm, fastened fast on the back of the frame, by means of which the glass may be turned round.

5^th. The Shade Vanes, Screens, or darkening Glasses, which are pieces of colored glass, set in frames, & fixed so that any one or more of them may be interposed between the index glass, & that Horizon Glass which is used in making the observation.

6^th. The Sight Vanes; which are two pieces of brass, fixed perpendicular to the plane of the quadrant.

Each of them has two small holes through the upper part of it to one of which the Eye is to be applied in observing: That which is placed a little below the center of the quadrant on the radius which the divisions of the arch begin at, is used in making the fore observation: & that which stands immediately behind the back horizon glass is used for the back obsr.

The Instead of Sight Vanes those quadrants which are proper for observing the moons distance from the sun or a Star, for the purpose of finding the Longitude have a triangular stem which slides in a socket of the same shape fixed perpendicular to the plane of the Instrument.

This stem supports a double ring one of which is let into the other & turns in it on two points which are in a line parallel to the plane of the quadrant & the two parts are confined to each other by two screws which are in a line perpendicular to the plane of the quadrant.

A small telescope magnifying from four to five, or six-teen or twenty times screws into the interior ring; & to this telescope the eye of the observer is to be applied instead of the Sight Vane in ordinary quadrants.

The triangular stem is moveable in the socket by means of a screw which is at the back of the quadrant so that the telescope may be brought to point to any part of the horizon glass which the observer think most proper.

Two wires are fixed in the focus of the telescope parallel to each other; & when the Telescope is screwed into its place these wires must be placed parallel to the plane of the quadrant by turning the eye tube of the telescope in its socket.

8th These instruments have also a screw attached to the extreme end of the index which works in the clamp by which the index is fastened to the arch of the quadrant.

When the objects have been brought nearly into contact by observing the index by the hand, it must be fastened to the arch; & then by turning this screw the objects may be brought into contact with greater ease & certainty than by the hand alone. This screw on account of its position is generally called the Tangent Screw.

9th The Vernier which subdivides each of the smaller spaces of the arch into minutes. The twenty spaces which are formed by the 21 short lines cut on the index being exactly equal to nineteen of the spaces on the arch of the Instrument it follows that each space on the index is less by 1/20th part, than a space on the arch; that two spaces on the index are less by 2/20th parts than two spaces on the Arch, & so on; so that the whole twenty spaces on the index are less

by $\frac{20}{20}$ths parts or a whole space, than twenty spaces on the Arch, & as one whole space on the arch has been shewn to contain 20 minutes each of these 20th parts must correspond to one minute. Hence, as each space on the arch is greater than a space on the index by $\frac{1}{20}$th part of a space on the arch, or one minute; when the line, marked 0 on the index coincides with any line on the arch it is obvious that the second line on the index, or that marked 1, will not be so far advanced by $\frac{1}{20}$th part of a space as the next line on the arch, the third line on the index will not be so far advanced as the third line on the arch by $\frac{2}{20}$ths of a space & so on. It therefore follows that to make the second line on the Index coincide with the second line on the arch the index must be removed forward the $\frac{1}{20}$th part of one of the spaces on the arch or one minute & to make the third line on the index coincide with the third line on the arch the index must be moved forwards $\frac{2}{20}$ths of a space or two minutes & so on that is to have any one of the twenty other lines of the index to coincide with a line on the arch, the index must be moved as many 20th parts of a space of the arch or minutes as that line of the arch is numbered with:

And, on the contrary, whatever line on the index may be found to coincide with a line on the arch, it is plain that the index must, to join such a coincidence, have been pushed forward as many 20ths parts of a space on the arch from the line which 0 on the index stands beyond as the coinciding line is numbered with.

Problem 3rd

To read off the Degrees, Minutes &c which the index of the quadrant stands at.

Rule.

Note, first how many of the large spaces, or degrees the line marked 0, on the index stands beyond.

Secondly, whether it stands also beyond one, or both the short lines which are between the long ones; because if it has not passed neither of them; the minutes are under 20; if it has passed one of them, the minutes are above 20 but under 40; & lastly, which of the twenty lines on the index coincides with a line on the arch; & which ever does so many minutes as that line denotes must be added to the even degrees in the first case; to the degrees & 20 minutes in the second case; or to the degrees & 40 minutes in the 3rd; & the same will be the degrees & minutes which the quadrant stands at, or the result of the Observation.

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If none of the lines of the index coincide with a line on the arch, look for those two which are nearest to a coincidence and the number which the first of them is marked with will be the number of minutes which must be added to the whole degrees, & 20 minutes, or whole degrees & 40 minutes, & if the lines are equally near a coincidence half a minute more must be added to them, but if the first be evidently nearer to a coincidence than the latter, a quarter of a minute must be added, or if the latter be nearest to a coincidence, add three quarters of a minute, & the obs^{n} will be had to degrees minutes, & quarters of a minute. The halves & quarters cannot be distinguished without a magnifying glass

Problem IVth

To place the axis of the telescope parallel to the plane of the Instrument.

Rule.

Fix the telescope in its place, & turn the wires, which are in its focus, parallel to the plane of the quadrant.

Choose two well defined objects, the angular distance between which is nearly as great as the instrument will measure.

Look through the telescope at one end of them: & by

moving the index of the quadrant bring the image of the other to touch it very exactly & let this contact be made on that wire which is nearest to the plane of the quadrant, by altering the position of the quadrant a little, & not the index, bring that point of the two objects, where they touch into the other wire, & if they still remain in contact, the axis of the telescope is parallel to the plane of the quadrant.

But if the objects now appear to over-lap, the object end of the telescope is inclined from the quadrant; & therefore the lower of those two screws that confine the two rings together which the telescope is screwed into, must be eased, & the upper one tightened, until on trial, the two objects will appear in contact on both wires.

If, instead of over-lapping, the objects appear open on the latter wire, the object end of the telescope is inclined to the plane of the quadrant & must be rectified by easing the upper screw, & screwing up the lower one till the objects will appear in contact on both wires.

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Problem 5th

To set the Index Glass perpendicular to the plane of the Quadrant.

Rule.

Put the Index to the middle of the arch, hold the quadrant with its plane parallel to the Horizon its center to your eye, & the arch from you.

Look obliquely across the index glass toward that part of the arch which is on the left hand side of the index & you will see the image of the arch in the index glass exactly of the same height with the arch itself if the glass be upright.

If the glass be not perpendicular to the plane of the quadrant the image will appear higher or lower than the arch itself which will appear broken at the farther edge of the index glass, & in this case the glass must be altered by means of the three screws which attach the frame of the Glass to the index until the image & arch are exactly in the same plane.

Problem 6th

To set the Horizon Glass for the 5 obs.ns perpendicular to the plane of the quadrant.

Rule.

Set the index to 0, & hold the plane of the quadrant parallel to the plane of the horizon, look through the lower hole in the sight vane & observe whether the Horizon or line which separates the sea & sky, seen on each side of the Horizon glass, be in a straight line with the reflected image of it, seen in the silvered part of the glass; if it be, the glass is right: but if it be not the two screws which are in the front & at the back of that glass must be altered until the two parts of the Horizon are in a line; & the glass will be perpendicular to the plane of the Instrument

Problem 7th

To place the Horizon Glass parallel to the Index Glass when the index stands at 0, or the beginning of the divisions on the arch.

Rule.

Set the index very exactly to 0, hold the plane of the quadrant perpendicular to the plane of the

Horizon; look through the hole in the sight vane which is near-
est to the plane of the quadrant & observe whether that part
of the Horizon which is seen through the unsilvered
part of the Horizon glass, form one continual straight
line with the image of it, which is seen in the quicksilvered
part: if it does the glass is right; but if it does not un-
screw the fingle screw which is in the middle of the
lever at the back of the quadrant, & by means of the
handle which governs the lever turn the glass on
its axis until the two parts of the Horizon are in a
straight line, screw the screw tight, which is in the mid-
dle of the lever, & the quadrant will be adjusted
in every respect.

But instead of making this last adjustment
it is better to find the Error of the Quadrant that is
the number of minutes &c which 0 on the index
stands at when the surfaces of the two glasses are
parallel.

Problem 8th

To find the Index Error of an Hadley's Quadrant
or the angle under which the surface of the index glass
is inclined to that of the Horizon glass when the
index stands at 0 or the beginning of the divisions

of the arch.

Rule.

Hold the quadrant in a vertical position; look through the sight vane at the Horizon of the sea & move the index until that part of the Horizon which is seen in the transparent part of the horizon glass forms a straight line with the image of it seen by reflection in the quicksilvered part: read off the numbers which the index then stands at, write them down & repeat this operation five or six times setting the several results one under another.

Add these numbers together & divide the sum by the number of trials & the quotient will be the error of the quadrant.

This error must be added to the result of all observations taken with that Instrument, if the index stood off the arch or before 0, but it must be subtracted from the result of the observations if the index stood on the arch or beyond 0. If the index stood sometimes before 0, & sometimes after it take the sum of those ..... which fell before 0 & the sum of those which fell after it & divide the difference of these two sums by the number of trials & the quotient will be the error of the quadrant to be added if the sum of those in which the index stood before 0 be greater than the sum of those when the index stood after it, but

subtracted if it be otherwise. Any distant well defined straight line, as the mast ... ... of a ship, the flag-staff of a fort, the ridge or corner of a building, or the corner of a wall may be used for these purposes aswell as the horizon of the sun the plane of the quadrant being held at right angles to such a line. In finding the error of an Hadleys Quadrant by this method the sight vane must be used; in the following one which is much better, use the telescope:

Look directly at the sun & by moving the index of the quadrant from you separate the two images of the sun until the two edges touch very exactly; read off the numbers which the index of the quadrant stands at, write them down & repeat the operation 5 or 6 times setting the several results under another.

Add these numbers together & divide the sum by the number of trials & the quotient will be the suns diameter, as measured on the arch. Set the other edges of the two suns be now made to touch each other by moving the index of the quadrant towards you, read off the numbers it stands at which will now be on the other side of 0, or off the arch; & on this account the numbers

of the vernier must be read the contrary way calling the 20 stroke 0, the 15 stroke 5, the tenth as it was before, the 5 stroke 15, the 0, twenty & the intermediate ones accordingly.

Having made as many obs^ns this way as you did with the index on the other side of 0, or on the arch, take the mean of those also & half the diff^ce between this & the former mean will be the error of the quadrant, to be added to all obs^ns made with that quadrant, if the latter mean, that is if the measure of the sun's diameter before 0 on the arch, be greater than the former mean, or its diameter measured on the arch be beyond 0; but the error must be subtracted from the observations if the former diameter be greater than the latter. As the contact of the limbs is formed the first time in each set of obs^ns by separating the two images of the sun after the first obs^n has been read off, set the limbs open from each other, & form the contact the second time by bringing them together, the third time by separating them, & so on alternately.

Problem 9th.

To observe the sun's Altitude.

Rule.

Turn down one of the dark glasses, hold the quadrant in a vertical position, with the arch downwards, look through that hole in the sight vane which is farthest from the plane of the quadrant, & through the unsilvered part of the horizon glass toward that part of the horizon which is under the sun, & move the index along the arch until you see the image of the sun come on the horizon glass. If you find the Sun too bright for your eye turn down a darker glass than that which was turned down before, if the sun's image appear too faint put down one which is not so dark.

Give the quadrant a gentle motion from right to left, & from left to right partly by turning on your heel, & partly by a motion in your hips, so as to make the image of the sun to describe an arch of a circle, & yet remain constantly on the middle of the unsilvered part of the horizon glass; & at the time you do this, move the

index gently along the arch of the quadrant until the lower edge of the sun touches the horizon, as it describes the arch above mentioned, without dipping any where below it, & your quadrant will then be set to the apparent altitude of the sums lower limb.

Problem 10th

To observe the Altitude of a Star.

Rule.

Set the index of the quadrant to 0, & direct your sight through the sight vane & the horizon Glass to the star which you wish to observe; & having found the image of the star on the silvered part of the glass, move the index gently along the arch, which will cause the image of the star to descend toward the horizon. Follow the image with your eye moving the quadrant at the same time, so as to keep the image in the middle of the silvered part of the glass until you have brought it to touch the horizon, when you give the quadrant a motion from right to left which you were directed to give it in observing the sums Altitude, & the index of the quadrant will point out the apparent altitude of the star.

The Moon's altitude is observed in the night as the sun is in the day time, & by the rule which has been given for observing the altitude of a star.

When the meridional altitude is wanted begin about a quarter of an hour before the time that the object will be on the meridian & having set the index of the quadrant to the true altitude of the object at that time by the methods given above, fix it in that position by means of the screw which is at the back of the arch of the quadrant. Let it rest for a short time & then look whether the object or its limb has risen from the horizon if it has bring it again to touch the horizon by moving the index gently along the arch & causing the image of the object at the same time to vibrate on the horizon glass by the motions which have been already described.

Continue to repeat this operation at short intervals as long as the object continues to rise & when it ceases to rise the degrees &c which the index of the quadrant stands at must be read off and taken for the meridional altitude of the object.

Problem 11th.

To observe the moon's distance from the sun.

Rule.

Look through the telescope or sight vane of your quadrant at the moon, turning the face of the instrument upward if the sun be on your right, but downward if it be on your left, & bring the plane of it into the plane of that great circle which passes through the centres of the objects, by noting when the line joining the two cusps of the moon in the first & last quarters, or its longest diameter in the second & third is perpendicular to the plane of the Instrument.

Bring their nearest limbs together by moving the index slowly along the arch & when they approach very near to each other, screw the index fast & give the quadrant a gentle vibratory motion round the ray which passes from your eye to the moon moving the index at the same time, by means of the tangent screw until the limbs touch very exactly as they pass each other & when this happens the numbers which are shewn by the index on the arch of the Instrument will be the apparent distance of the nearest limbs of the sun & moon.

Problem 12th

To observe the moon's distance from any of the stars which are made use of in the Nautical Almanac.

Rule.

Turn y^e long^e of the ship by account into time & if it be west add it to the time at the ship when you intend to observe; but if it be east, subtract it from that time, & you will have the time at Greenwich. To this time take y^e distance of the moon & star which you mean to observe, out of the Naut. Alm. & set the index of the quadrant to this angle. Look to the eastward or westward of the moon, according as the star is situated, in a line which is nearly at right angles to her longest diameter, or to the line joining her cusps, & as near as you can judge about as many degrees from her as the index of your quadrant is set to, & choose out the brightest star you find in that part of the heavens. Look through (pretty) the sight vane (not the telescope) turning the face of the instrument upward or downward according as the moon is to the right or left of the star & give it a slow motion round the ray which passes from your eye to the star, & if that star be the right one the

images of the moon will pass over or very near the horizon glass. If the moon does not pass over or near the horizon glass you have made choice of a wrong star, you will never the less see whether the moon passes beyond or short of it & which ever way it does pass another bright star must be sought for, that way in the direction above mentioned.

But it will seldom happen that you will fail of finding the right star at the first trial, as it is always much the brightest of any in its neighbourhood.

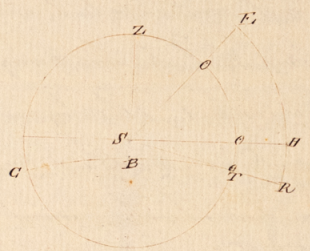
The proper star being thus found, fix the telescope in its place, look directly at ☿ star & turn your quadrant slowly round the ray, passing to it until the image of ☾ moon comes into the telescope. By a gentle motion of the instrument round the ray just mentioned, make the moon vibrate by the star moving the index at the same time by means of the tangent screw until her round edge touches the center of the star for it will appear of some breadth as it passes by it, & the index will point out the deg. &c which express the distance of the star from the moons enlightened limb. When the distance of the moons from ☉ sun or ☆ star is observed for the purpose of finding ☾ longitude it is convenient to have the altitudes of ☾ two

objects observed also at ye same instant.

Therefore, while one observer is employed in observing the dists two other must observe ye altitudes of the two objects, by ye directions which have been already given, holding themselves in continual readiness to leave off as soon as the first observer gives notice that he has brought the limbs of ye sun & moon into contact. The degrees & minutes which the index of each quadrant stands at, being then read off & written down in one line, the obsns must be repeated, six, eight, or ten times & the several results written one under another.

Move the index of the quadrant a little betwn every obsn that you may not be influenced in the succeeding obsn by what you had done in the foregoing one & it is necessary to move it different ways setting the objects open & to overlap alternately. The observed altitude of every object requires several corrns 1st. Because in observing, ye object is brought to the visible horizon or edge of the sea & the visible horizon is necessarily below ye true one, drawn through the eye of ye observer on account of the convexity of the Earth's surface, & the height of the observers eye above it, by carrying ye image of ye object down to the former instead of the latter. The altitude is made too great & the more so the

higher the observer is raised above the surface of the sea.
For let TDG represent a part of y^e earth B the place of the observer, BS the height of his eye above the surface of the sea, & O the sun in the heavens. Through S draw SH perpendicular to SB, & SR to touch y^e surface of y^e Earth in T, y^e point where the sky and water will appear to meet. From S at the distance of ST, describe the circle TDG, which will represent y^e visible horizon, & SH will represent y^e true one to which the image of the sun ought to be brought in observing, because the angle HSE is the true altitude of the sun's limb, but as this line is only imaginary & not visible the sun's image is to touch y^e visible horizon at T; & by that means y^e altitude ..... is made too great by y^e angle HSR, which is called y^e dip of y^e horizon. This error is contained in Tab II of the Requisite Tables out of which it must be taken with the height of the observers eye, & subtracted from the obs^d alt^d D^o. The air & vapours which surround y^e earth & form its atmosphere, cause all celestial bodies to appear higher than they really are except when they are in the Zenith; & the more so the nearer they are to the Horizon. The error arising from this cause is called

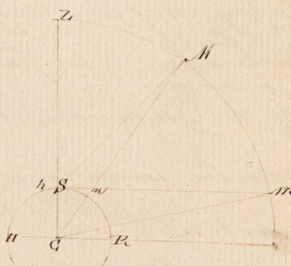


Refraction, & is cont^d in Tab 1 of the Requisite Tables out of which it must be taken with y^e alt^e of the object corrected for y^e dip of the horizon & subtracted from it.

3^dly These are all the corrections which the alt^e of a star requires, but the alt^es of y^e sun & moon must be further corrected for parallax & y^e semidiameter of y^e object. The semidiameter of the sun is found for every 8^th day in y^e year at y^e top of page III in y^e Nautical Alm^c & the moon's semidiameter will be met with on page VII where it is set down to every last of the hour.

The semidiameter must be subtracted from y^e apparent altitude of y^e limb when the alt^e of the upper limb is obs^d & added to it when y^e alt^e of the lower limb is obs^d

The parallax of celestial objects make them appear lower than they really are. Thus if a circle SRH represents the earth, C its center, M the moon or any other celestial object, HR the rational, & hr the visible horizon, & S the place of the observer. Then if C M & S M be drawn, hr produced out to meet y^e circle Z. M m, in m, & C m be joined, it is plain that the angle M S r is the app alt^e as it appears to a spectator at S, & that y^e angle M C R = M R m. is the true alt^e, which is greater than the app alt^e M S r, by the angle S M C, the par^x in alt^e.



It is farther evident that the par^{x} is greatest, when the object is in the spectators visible horizon at m, & nothing when it is in the Zenith at Z, because the lines drawn from C & S to the object then coincide.

The parallax of any object is also less, the greater its distance is from the observer; & on this account the parallax of the fixed stars is not discernable with the best instruments that have yet been made; & the parallax of the sun never exceeds nine seconds.

It is found in Tab III of the Requisite Tables, out of which it must be taken with the sun's app alt^{e}, & added to it. The moon's horizontal, or greatest par^{x}, is given on page VII of the Naut. Alm^{c}, where it is set down to every twelfth hour, & her par^{x} in alt^{e} will be met with in Tab III of the Requisite Tables, out of which it must be taken with the moon's horizontal par^{x}, & her alt^{e} corr^{d} for the dip of the horizon.

But the moon's par^{x} in alt^{e} & her refraction may be taken together, out of Tab VIII & added to the alt^{e} of ☽ moon's l^{mb}, after it has been corrected for the dip of the horizon.

Problem 13.th

Having given the long.e of any place, & the apparent time at that place, to find the Greenwich time, that is the appr. time at Greenwich.

Rule.

Find the long. of the place in time by Problem 1st in Astronomy, & if it be west, add it to, but if it be east subtract it from the appr. time at the place, the sum or rem.r will be the appr. time at Greenwich.

Examples.

On the 17th Dec.r 1802 at 2h 44m appt time at Yarmouth, what was the appr. time at Greenwich.

Appr. time at Yarmouth on the 17th Dec.r was 2h 44m 00s
Long.e in time ..... − 6. 40 E
Appr time at Greenwich 2. 37. 20

On the 25th July 1802 at 4h 44m 24s appr time at Canton? What was the appr. time at Greenwich

Appr. time at Canton on the 25th July 1802 was 24h 00m 00s
4h 44m 24s
28h 44m 24s
Longitude in time ..... − 7. 32. 28 E
Appr time at Greenwich on the 24th 21. 11. 36

On the 24th of March 1802 at 19h 16m appr time at Lima? What was the appr time at Greenwich

Appr time at Lima on the 24th March 1802 was 19h 04m 16s
Longitude in time ..... + 5. 06. 56 W
24h 11. 12
Appr time at Greenwich on the 24h 11. 12 25th

Problem 11th

To find the sun's decl.n for any given time & place from the Naut. Alm.

Rule.

Take the sun's decl.n from page II of the Nautical Alm. for noon at Greenwich on the given day, if the time be less than 12h, but for noon on the day following, if it be more. If the time at Greenwich be given, enter Tab VI of the Requisite Tables with the time from the nearest noon at the top & the day of the month in one of the side columns opposite to which & under the former stands the correction of the sun's decl.n.

If the time be noon under a meridian which is not that of Greenwich, find the long.e of the place at the top of the table, & the day of the month in one of the side columns, opposite to which & under the long.e stands the correction of the decl.n on that account.

If the decl.n be wanted for any other time than noon, & under a meridian which is not the meridian of Greenwich, both these corrections must be applied; & they must be added to, or subtracted from the sun's decl.n for noon at Greenwich, according to the directions which stand at the top of the column in which the day of the month is found.

To find the sun's decl^{n} on the 30th May 1802 at 18^{H}.23^{+} app time at Greenwich

Sun's decl^{n} on the 31st	21^{H}.51^{+}.41$^{''}$	N
Corr^{n} for 5^{H}.37^{+} AM 31st	-2.03	
True decl^{n}	21.48.36	N

To find the sun's declination on the 27th February 1802 at noon in Long^{e} 140° E

Sun's decl^{n}	= 8°.29^{+}.21$^{''}$	S
Corr^{n} 140° E	= +8.28	
True decl^{n}	= 8.37.49	S

To find the Sun's decl^{n} on the 24th Nov^{r} 1802 at 4^{H}.48^{+} app time in long^{e} 127° E

Sun's decl^{n}	20°.29^{+}.37$^{''}$	S
Corr^{n} for 4^{H}.48^{+} PM	+3.17	
	20.32.54	
Corr^{n} for 127° E	-4.00	
True Decl^{n}	28.28.54	S

To find the Sun's decl^{n} on the 4th April 1802 at 7^{H}.17^{+} app time at Greenwich

Sun's decl^{n}	5°.31^{+}.50$^{''}$	N
Corr^{n} for 7^{H}.17^{+} PM	+6.00	
True decl^{n}	5.37.50	N

Problem 15th

To find the time when any known star will be on the meridian of a given place.

Rule.

Take the sun's right ascension for noon at Greenwich on the given day, from page II of the Nautical Almanac, & the difference between the sun's right ascension on the given day, & the day following. Take the star's right ascension for the given time from Tab: VII of the Requisite Tables, & subtract the sun's right ascension for noon at Greenwich from it; the remainder will be the time of the star's culminating nearly.

Enter Tab: X & XII of the Requisite Tables with the daily variation of the sun's right ascension at the top, & the longitude of the given place in the right hand column, & take out the minutes & seconds that stand under the former, & opposite the latter, which must be added to, or subtracted from, the time of culminating nearly, according as the longitude of the place is east or west, & the sum or remainder will be the stars culminating corrected. Enter Tab: X & XII again, with the daily var: of the sun's right ascension at the top and the time of culminating corrected as above in the left hand column, & take out the correction corresponding to them, which being subtracted from the time of culminating corrected as above, will give the true

time of the Star's culminating.

In all operations where time is concerned, addl 24h to the time from 2d you subtract from, of the time which is to be taken, be the greater & the rem.r will be reckoned from noon of the preceding day.

When one time is to be added to another time, if the sum be greater than 24h subtract 24h from it & the rem.r must be reckoned from noon of the day following.

At what time will the Star Fomalhaut be on the meridian of St. Helena on the 2nd. Nov.r 1802.

	h m s
Star's R 1780	22 53 08
	+ 1 05
Star's R 1802	22 54 13
Sun's R	14 27 53
Time of culm.n nearly	8 26 20
Corr.t Sub Z XII for 5° 40′	− 04
Time of culm.n corr.d	8 26 16
Corr.n Sub Z XII	− 1 23
True time of culm.n	8 24 53

	h m s
Sun's R 2nd	14 27 53
Sun's R 3rd	14 31 49
Diff.ce	3 56
Years elapsed	22.84
Ann. Var.n Star's R	288
	18272
	18272.
	4568..
	63792

At what time will the Star Regel be on the meridian of Gibraltar on the 18th. August 1802.

	h m s
Star's R 1780	5 03 58
	+ 1 05
Stars R 1802	5 05 03
	24 00 00
	29 05 03
Suns. R	9 44 27
Time culm.n nearly	19 20 36
Corr.n Sub Z XII for 5° 40′	− 4
Time of culm.n corr.d	19 20 32
Corr.n Sub Z XII	− 3 00
True time of culm.n	19 17 32

	h m s
Sun's R 17th	9 44 27
Suns. R 18th	9 48 11
Diff.ce	3 44
Years elapsed	22.63
Ann. Var.n Star's R	287
	15841
	18104.
	4526..
	64,9481

Problem 16th

To find the Latitude of a Ship at sea, from the observed mer.c Alt.e of the Sun's Limb.

Rule.

Take the Sun's Decl.n from page II of the Nautical Almanac by Problem 14th & note whether it be north or south.

Correct the observed alt.e of the Sun's Limb by subtracting the dip. of the horizon (Tab II) & the Refraction (Tab I) from it, & by adding the parallax in Altitude (Tab III) to it.

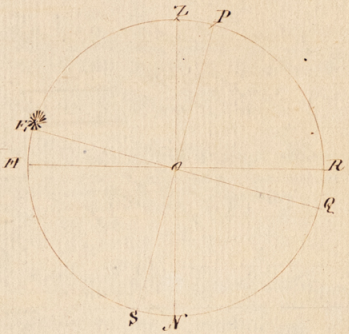
Also by adding to it, or subtracting from it the semidiameter page III of the Nautical Almanac according as the lower or upper Limb was observed, & you will have the true altitude of the sun's center.

Take the true altitude from 90° & it will leave the true distance from the Zenith which is north if the Zenith was North of the Sun at the time of observation, but south if it was south of it.

If the suns Zenith distance & declination, be both north or both south add them together, but if one be north & the other South subtract the less from the greater & the sum or difference will be the Latitude of the same name with the greater.

Example 4th

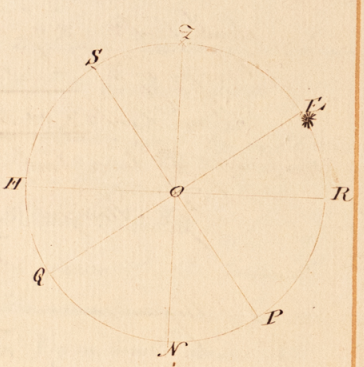
Sept. 17th 1802 longitude 67° W the meridianal altitude of the sun's Lower Limb was observed to be 15° 17' the Zenith being North of the sun & the height of the observers eye 18 Feet. What was the Latitude.



Sun's Decl.n	2° 27' 01" N
Corr.n for 67° W	− 4.22
True Decl.n	2.22.39 N
Mer.l Alt.e ☉ Lower Limb	15.17.00
Dep.t Horizon	− 4.03
	15.12.57
Refraction	− 3.26
	15.09.31
Par.x in Alt.e	+ 8
	15.09.39
Semi diam.r	+ 15.59
True Alt.e	15 25.38
	90.00.00
Zenith Dist.ce	74.34.22 N
Sun's Decl.n	2.22.39 N
Latitude	76.57.01 N

Example 2nd

March 27th 1802 longitude 5h E the meridional altitude of the sun's upper limb was observed to be 34° 30' the Zenith being South of the Sun & the height of the observers eye 22 Feet. What was the Latitude.



Sun's Decln		2° 22' 58" N
Corr for 5h E		− 3' 50"
True Decln		2° 22' 08" N
Mer. Alt. ☉ upper limb		34° 30' 00"
Dip Horizon	− 4' 28"	
		34° 25' 32"
Refraction	− 1' 23"	
		34° 24' 09"
Par in Alt.	+ 8"	
		34° 24' 17"
Semi diam.	− 16' 04"	
True Altitude		34° 08' 13"
		90° 00' 00"
Zenith Dist.		55° 51' 47" S
Sun's Decln		− 2° 22' 08" N
Latitude		53° 29' 39" S

Problem 17th

To find the Latitude of a Ship at sea, from the obs.d meridional Altitude of a Fixed Star.

Rule.

Take the Stars Decl.n out of Tab.r II of the Requisite Tables for the given year.

Correct the obs.d Altitude of the Star by subtracting the dip of the Horizon Tab.r III & the Refraction Tab.r I from it, which will leave the true altitude.

Take the true altitude from 90.° & it will leave you the true distance from the Zenith, which is north or south according as the Zenith was north or south of the star at the time of obs.n Then, if the Zenith Distance & Decl.n be both north or both south, add them together; but if one be north & the other South, subtract the less from the greater, & the sum or diff.ce will be the Latitude of the same name with the greater.

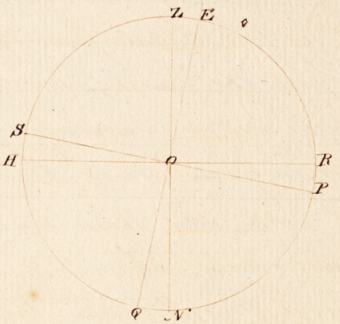
Example 4th

September 7th 1802 the meridional altitude of Aldebaran was observed to be 67°30'45" the Zenith being south of the Star & the height of the observers eye 22 Feet. What was the Latitude.

Stars Decl.n 1780 = 16°03'02" N
13.07
Stars Decl.n 1802 = 16.06.09 N

Stars Mar.l Alt.e 67.30.45
Dip Horizon − 4.28
67.26.17
Refraction − 23
True Alt.e 67.25.54
90.00.00
Zen. Distance 22.34.06 S
Stars Decl.n 16.06.09 N
Latitude 6.27.57 S

Years elapsed 22.67
Stars Var.n of Decl.n = 8.26
13602
4534.
18136..
3.07 = 187.2542



Example 9th

April 17th 1802 the meridional altitude of Sirius was observed to be 7° 05′ 47″ the Zenith being north of the Star & the height of the observers eye 14 Feet.

What was the Latitude

Stars Decl^{n} 1790 - 16° 25′ 39″ S

+ 1.34

Stars Decl^{n} 1802 = 16 27 13 S

Stars Mer. Alt^{e} 7° 05′ 47″

Dip Season -3.34

7.02.13

Refraction -7.15

True Alt^{e} 6.54.58

90.00.00

Zen Dist^{ce} 83.05.02 N

Stars Decl^{n} 16.27.13 S

Latitude 66.37.49 N

Years elapsed 22.29

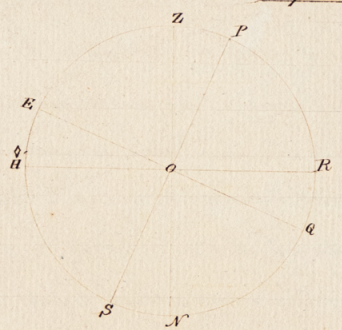
Ann Var^{n} & Dec 4.25

11145

4458.

8916.

+ 1.34″ = 97325



Problem 18th

To find the Latitude of a Ship at Sea, from the obsd meridional Alte of the Moon's Limb.

Rule.

To the Longitude of the given place, in time, add the number from Tab. XXI corresponding to that Longitude, & the daily variation of the moon's passage over the meridian & take the sum from the time of the moon's passage over the meridian on the given day (Naut. Alm. p. VII); if the Longitude be east, gradded to the time of her passage, if the long.e be west, & the sum or diffce will be the time at Greenwich when the moon was on the meridian of the given place.

To this time take the moon's horizontal parallax & semidiameter from p. VII of the Naut. Alm. & her decl.n from page 17 noting whether it be north or south. Correct the obsd Alte of the moon's limb by subtracting the dip of the horizon (Tab. II) from it, & adding the correction of her alt.e (taken out of Tab. VIII) with the moon's horizontal parallax, & the alt.e of her limb, corrected for dip, & also her semidiameter, if the lower limb was observed, or by subtracting her semi-diameter if the upper limb was obsd, which will give the true altitude of her center.

Take the true altitude from 90, & it will leave the distance from the Zenith, which is north or south according as the Zenith was north or south of the moon at the time of observation. Then, if the Zenith Distance & Decln be both north or both south, add them together; but if one be north, & the other south, subtract the less from the greater, & the sum or diffce will be the Latitude of the same name with the greater.

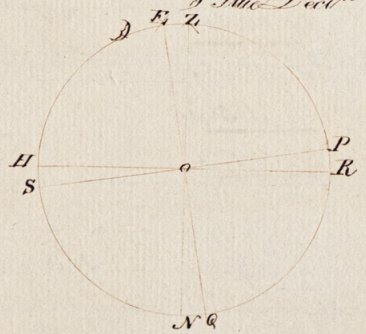
August 13th 1802 longitude 16° E. the mer. alte of the ☽s Upr. Limb was 65° 10′ 0″ the Zenith being north of the ☽ & the height of the observers eye 23 Feet. What was the Latitude.

☽ semidiam — 16′ 45″
☽ horz. par. 61 28

Mer. Alte ☽ W.L 67° 42′ 30″
Dip. Horizon — 4.34
App. Alte ☽ S 67.37.56
Corr. Tab VIII + 23.31
68.01.27
☽ semi diam — 16.45
True Alte 67.44.42
90.00.00
Zen Dist 22.15.18 N
☽ Declⁿ — 16.05.00 S
Latitude 6.10.18 N

Long in time H. 1′ 08
Corr. Tab XII + 16
1.24
☽ Southing + 2.24
Greenwich time 3.00

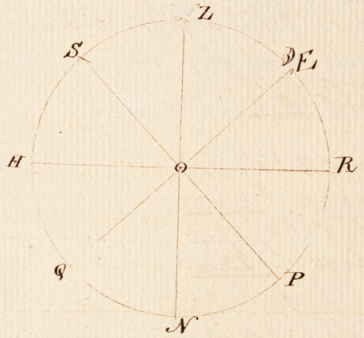
☽ Declⁿ for given — 17° 22′ 5
Corr. do 3h 0′ 0 — 1.47
☽ True Declⁿ — 16.05.5



Oct.r 8th 1802 Long.e 133° W.t the mer.l alt.e of the ☉ ☽ Limb was observed to be 48° 16′ the Zenith being South of the ☉ & the height of the observers eye 24 Feet: What was the Latitude.

☉ Decl.n Hor.y mid.t	=	6° 47′ 00″ S	
Corr.n for 7h 07′	=	2° 05′ 01	
☉ True Decl.n	=	4° 41′ 59 S	
☉ semi diam.r	=	16′ 27″	
☽ horiz.l par.x	=	60′ 57″	
Mer.n Alt.e ☽ ☾	=	48° 16′ 00″	
Dip.t Hor.on	=	− 4′ 40	
App.t Alt.e ☽ ☾	=	48° 11′ 20	
Corr.n Tab.r VIII	=	+ 30′ 45	
		48° 54′ 05	
☽ semi diam.r		+ 16′ 27	
☽ True Alt.e		49° 10′ 32	
		90° 00′ 00	
☽ Zen.l Dist.e		40° 49′ 18 S	
☽ Decl.n		+ 4° 41′ 59 S	
Latitude		45° 31′ 17 S	

Long.de in time	=	8h 53′	
Corr.n Tab.r VII	=	+ 18	
Sum	=	9h 11′	
☽ Southing	=	2h 56′	
Greenw.h time	=	19h 07′	



Remark.

If the meridional alt^{e} of a circumpolar Star be observed when it is below the pole, or the meridional altitude of the sun or moon at midnight, in any place where they do not set; then, if to such altitude, properly corrected, there be added, the polar distance of the sun, moon or star; that is, the complement of its Declination, the sum will be the Latitude of the place, of the same name with the declin^{n}.

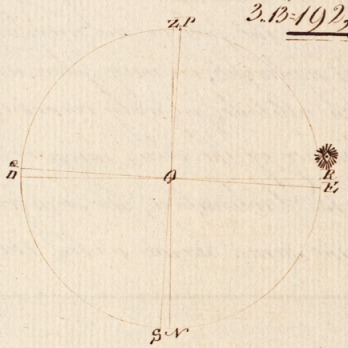
Example 1st

Nov.r 4th 1802 the meridional altitude of ♃ Stair at mid-night below the pole was observed to be 7° 10' the height of the observers 12 Feet. What was the Latitude

Stars Decl.n 1780 8° 17' 57" N
+ 3.13
Stars Decl.n 1802 8.21.10 N
90.00.00
Polar Distance 81.38.50

Years elapsed 22.84
Sun Var.n ☉ dec.r 8.44
9136
9136.
18272
3.13 = 192,7696

☉ Mer. Alt.e 7° 10' 00"
Dip. Horizon -3.18
App. Alt.e 7.06.42
Refraction -7.11
True Alt.e 6.59.31
Polar Dist. 81.38.50
Latitude 88.38.21 N



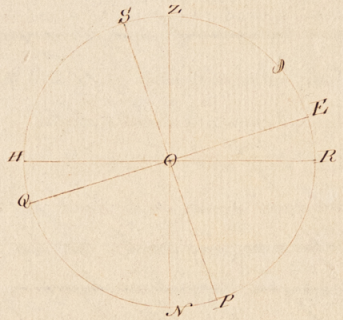
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Example 2nd

January 4th 1802 long.e 57° W. the meredional altitude of the Sun's Ll. limb at midnight below the pole was obs.d to be 4. 15 & the height of the observers eye 11 F.t What was the Latitude.

☉'s Decl.n 22..46..48 S
Corr.s for 57° W -58
☉'s Decl.n 22..45..50
90..00..00
P.D Distance 67..14..10

Altr ☉'s Ll. = 4..15..00
Dip Horizon -3.10
App.t alt.e 4.11.50
Refraction -11.20
4..00.30
Par.x in alt.e +9
4..00..39
semi diam -16.19
A. Alt.e ☉'s center 3.44.11
P.D Distance 67..14..10
Latitude 70..58..21 S



Problem 10th.

To find the Latitude of a ship at Sea, having the latitude by account, two observed altitudes of the Sun, the time elapsd between the observation, measured by a common watch, and the suns declination.

Rule.

To the log secant of the latitude by account, add the log secant of the suns declination; their sum, rejecting 20 from the index is the log ratio.

From the natural sine of the greater altitude taken out of Table VII subtract the natural sine of the least altitude, find the log^m of the remainder, & write it under log ratio.

With half the elapsed time enter Table VI & from the column of half elapsed time, take out the logarithm answering to it which is also to be put under the log ratio.

Add these three logarithms together, & look for their sum in Table VI in the column of middle time & having found the logarithm nearest to it, take out the time corresponding to it & put it under half the elapsed time, & subtract the less from the greater: their diffce will be the time from noon when the greater altitude was taken. With this time enter the Table again & from the column of log rising take out the log^m corresponding to it; from this logarithm subtract the log ratio, & the remainder will be the logarithm of a natural number which being found in Table XVIII & added to the natural sine of the greater altitude, will give the natural sine of the meridional alti-

tude of the sun.

From the meridional alt.ᵈ of the sun the latitude of the ship is to be found in the usual manner.

N.B. If the Latitude found in the preceding rule differ considerably from the latitude by account the operation must be repeated using the latitude last found instead of the lat.ᵉ by account until the result gives a latitude which agrees nearly with the latitude used in the computation.

July 20th 1802 Lat by acc. 38.03 N Long. 56 E at 1h 30. 15s by my watch the alt. of the ☼ L. Limb was observed to be 70. 58 & at 2h 26. 28s it was 63. 47. 15 What was the true Latitude of the ship the height of the obs. eye 22 Feet the Zenith north of the Sun at which the greater altitude was taken.

Alt. ☼ ☌ ☾ obs.	70. 58. 00	2nd obs. 63. 47. 15
Dip Horizn	−4.28	−4.28
	70. 53. 32	63. 42. 47
Refn	−10	−28
	70. 53. 13	63. 42. 19
☼ Par in alt	13	14
	70. 53. 1	63. 42. 33
☼ semi diam	+15. 47	+15. 47
☼ True Alt.	71. 09. 03	63. 58. 10

Times by W	T. Alt.	N. Sines
1h. 30. 15	71. 09. 03	94637
2. 26. 28	63. 58. 10	89856
56. 13 elapsed time		4781

Sun's decln. 20. 58. 13 acc. 10,010376
Sun's decln. 20. 48. 13 acc. 10,010928

Log ratio 0,13304
Log m 3,67952

28. 06. 30 ½ elapsed time ..... Log m 0,91244
1. 01. 34. 00 middle time ..... Log m 4,72500
33. 27. 30 time from noon Log rising .. 3,02684

Log ratio 0,13304
782 Nat. No. 2,89380
94637 N.S. greater alt 70. 66. 60
92420 N.S. Mer. alt 72. 35. 33
Zen. Dist 17. 24. 27 N
Decln 20. 49. 12 N
Latitude 38. 13. 39 N

Repeated

Lat^e last found	38° 13′ 39″	sec	10, 10481
Sun's declination	20° 48′ 13″	sec	10, 02928
	log ratio		0, 13409
Diff^ce N.S 4781	Cts log^m		3, 67952
½ elapsed time 28° 06′ 30″	Cts log^m		0, 91244
Mid. time 1° 01′ 13′ 00″	Cts log^m		4, 72605
Time from noon 33° 36′ 30″	log rising		3, 03350
	log ratio		0, 13409
789 . Nat. N^o			3, 89759
94637 N.S. greater	Alt^e		90° 00′ 00″
95426 N.S. Mer. Alt^e			72 36 13
	Lon. Dist		17° 23′ 47″ N
	Declination		20° 49′ 12″ N
	Latitude		38° 12′ 59″ N

Nov.r 21st 1802 Lat by acct 50.46 N, Long.e 48° W, at 22h 17′ 30″ by a watch the Alt.e of the Sun's L. Limb obs.d to be 17° 04′ 15″ and at 23h 17′ 30″ the Alt.e of y.e same Limb was 19° 31′ 45″: What was the true latitude the height of the observers eye above the sea the Zenith being north of the sun when the greater alt.e was taken?

Alt.e ☉ L. 1st Obs.n	17° 04′ 15″	2nd Obs.n 19° 31′ 45″
Dep.n Alt.e	−4.22	−4.22
	16.59.53	19.27.23
Refraction	−3.04	−2.40
	16.56.49	19.24.43
☉'s Par.x in alt.e	+8	+8
	16.56.57	19.24.51
☉'s semidiam.r	+16.15	+16.15
☉'s True Alt.e	17.13.12	19.41.06

☉'s decl.n 22nd		20° 04′ 43″
Corr.n for 48° W		+1.36
		20.05.50
Corr.n 1h 12.30 Mm		−36
True decl.n		20.05.14 S

T. by W	Tr. Alt.e	N. Sines
22h 17′ 30″	17° 13′ 12″	29604
23 17.30	19.41.06	33685
1.00.00	Elapsed time	4081
30.00	½ elapsed time	
00.34	middle time	
30.34	time from noon	

Sal by acct 50° 46′ } sec 10,19863
☉'s decl.n 20.05.14 } sec 10,02725
log ratio 0,22528
As log.m 3,61077
As log.m 0,88430
As log.m 4,72035
log rising 2,95786
log ratio 0,22528
540 Nat. N.o 2,73258
33685 N. Sy.a or alt.e 90.
34225 N.S.r Mer. Alt.e 20.00.51
Zen. Dist 69.59.09 N
Declination 20.05.50 S
Latitude 49.53.19 N

Repeated.

Lat^e last found 49°53'19" } sec. { 10,19093
Sum^s Decl^n 20°05'14" } sec. { 10,02725

Log ratio 0,21818

Diff. Alt 4081 ..... ats Log^m 3,61077

½ elapsed time 30°00 ..... ats Log^m 0,88430

Mid. time 59°54 ..... ats Log^m 4,71325

time from noon 29°54 .. log rising 2,92033

Log ratio 0,21818

51½ Nat. N° 2,71115

33685. N.S greater Alt^e 90.

24199. N.S. Mer. Alt^e 19°59'53

Lon. Dist 70°00'07" W

Declination 20°05'50" S

Latitude 49°54'17" N.

Remark.

In the two preceding examples it has been supposed that both altitudes were taken at the same place; but as that can seldom happen at sea, it is necessary to shew how to correct one of the altitudes so as to make it what it would have been if observed at the same place where the other was; & this may be done as follows:

Let the bearing of the sun be observed by the compass at the instant of the first observation: take the number of points between it & the Ships course, corrected for leeway, if she makes any; with which, if less than 8, or with what it wants of 16 points, if more than 8, & the dist.ce run between the observations, enter the Traverse Table, & take out the difference of lat.e corresponding to them. Add this difference of latitude to the first alt.e if the number of points between the sun's bearing & Ships course were less than eight; but subtract it from the first altitude, if the number of points were more than 8, & it will be reduced to what it would have been if obs.d at the same place where the second was.

Note. The result of the operation will be the Latitude of the ship at the time when the second altitude was taken, & must be reduced to noon by means of the log.

Nov^{r} 19th 1862 being at sea, in long^{e} 36° E & lat^{e} 47° 24' N by acc^{t} at 11^{h} 55^{m} 20^{s} by my watch the alt^{e} of the sans ☌ ☌ was obs^{d} to be 17° 24' & the bearing of its center by compass Aby E. ¼ E. & on the 20th at 3^{h} 4^{m} 10^{s} the alt^{e} of the same ☌ ☌ was 21° 55' 30" the height of the observers eye being 20 Feet the Ship's course between the obs^{ns} by the compass was E & S & S at the rate of 7 knots & she made no leeway. What was the true Lat^{e} when the latter obs^{n} was made the zenith bearing N. of the Sun.

Ships course	S 7½ E	Suns decl^{n} 20th 4^{h}	19° 37' 25" S
Sun bearing	S 4 E	Corr^{n} for 30° E	- 1. 01
Between	6¼ points dist run 21 m		19. 36. 19
Diff lat^{e} added	7. 00	Corr^{n} for 3^{h} 10^{m} 7" Alw	- 19
Alt^{e} ☌ ☌ obs^{d}	17. 24. 00	True Decl^{n}	19. 36. 00 S
Alt^{e} reduced	17. 31. 00	Alt^{e} 2nd obs^{n}	21. 45. 30
Dip & Parrac	- 7. 16		- 6. 29
	17. 23. 44		21. 38. 51
Par^{x} & semidia	16. 23		+ 16. 23
True Alt^{e}	17. 40. 07		21. 55. 14

Times by W^{k}	T. Alt^{e}	N. Times
21. 55. 30	17. 40. 07	30351
21. 54. 10	21. 55. 14	37332
1. 25. 38. 40	chap^{s} time	16981
1. 29. 20	½ chap^{s} time	
33. 14	Mid. time	
56. 06	Time from noon	

Lat^{e} by acc^{t} 47. 24	acc^{t}	46, 17^{m} 87
Suns Decl^{n} 19. 36	acc^{t}	10, 0 23 92
Log ratio		6 19 6 79
ds Log^{m}		3, 8 43 92
ds Log^{m}		5, 4 26 22
ds Log^{m}		4, 6 6 92

Log rising 3, 47 436
Log ratio 6, 19 6 76
Nat^{l} N^{o} 3, 28 3 57
Log cater alt^{e} 90. 00. 00
N. Mer alt^{e} 23. 06. 43
Lon Dist 66. 53. 17 N
Decl^{n} 19. 36. 19 S
Latitude 47. 16. 58 N

1921
37332
59253

Repeated

Lat.e last found 47.16.38 } sec.t { 10,16853
Sun's Decl.n 19.36.00 } { 10,02592
Log ratio 0,19445
Diff.ce N.S 6981 ℔s Log.m 3,84392
2 last sed time 1'29,20 ℔s Log.m 0,42022
Mid time 33.03 ℔s Log.m 4,45859
Time from noon 56.17 Log rising 3,47718
Log ratio 0,19445
1917 . Nat. N.o 3,28273
37332 N.S greater Alt.e 90.00.00
29249 N.S Mer.l Alt.e 23.06.30
Len Dist.ce 66.53.30 N
Declination 19.36.19 S
Latitude 47.17.11 N

WILLIAMS & 3 1/2 PATENT L.B

July 9th 1802 being at sea, in lat.e 49° 25′ N Long.e 23° W by acc.t at 22h 02m 20s by my watch the alt.e of the sun's L.b was obs.d to be 60° and the azimuth of its center S 86° E by the compass & on the 10th at 11h 10m the obs.d alt.e was 62° 38′ 15″. The Ships course was E.S.E ½ E without leeway at the rate of 6½ knots between the obs.ns & the height of the observers eye 18 Feet. What was the true Latitude when the latter obs.n was taken the Zenith being then North of the sun.

Ships course S.E ½ E
Suns bearg S. 0
between 6½ dist.e run 6,6
D.e Lat.e adm.d 0.4.54
Alt.e & L.t 62.38.00
Alt.e reduced 60.01.54 . Alt.e 1st obs.n 62.38.15
Dip & Hf. . -4.36 . -4.32
59.57.18 . 62.34.13
Par & Semi. +15.31 . +15.31
True Alt.e 60.13.09 . 62.50.04

Suns decl.n 10h 22° 19′ 46″ N
Corr for 23h . . -31
22.19.15
Corr for 22h 15m . +18
True Decl.n 22.19.23 N

Semesby W.	S. Alt.e	N. Tenas
23h 02m 20s	60.12.09	86793
24.11.10	62.50.04	88968
1.08.50	elapsed time	2175.
34.25	½ elapsed time	
38.51	Mid time	
4.26	Time fam noon	

Lat.e by acc.t 49° 24′ } sec.t 10,18657
Suns Decl.n 22.19.23 } 10,03284
Log ratio . . . 0,22041
its Log.m . . . 3,23746
its Log.m . . . 0,97038
its Log.m . . . 4,32825

Log rising 4,27090
Log ratio 0,22041
Nat. N.os 4,05049

88968. N.t greater alt.e 90° 00′ 00″
88079. N.t lesser alt.e 62.50.42
Lon.g Dist 27.09.18 N
Decl.n 22.19.15 N
Latitude 49.28.33 N

Repeated

Latitude Last found	49°.28'.33"	sec.ª	16, 18721
Sun's Declⁿ	22..19..23		16, 03384
Log. ratio			0.22105
Diff. N.S. 2175	its Logᵐ		3,33746
½ Elapsed time 3'.25"	its Logᵐ		0,97038
Mid time 385"	its Logᵐ		4,52889
Time sam. morn 4.20	Log. rising	4,28174	
	Log. ratio	0,22105	
11	Nat. Nᵒ	1,06069	
88968	N.S greater Altᵉ	90..00..00	
88979	N.S. Mer. Altᵉ	62..30..42	
	Lon Dist	27..09..18 W	
	Declⁿ	22..19..15 N	
	Latitude	49..28..33 N	

WILLIAMS' 83 1/2 PATENT

Problem 20th.

The Lat.e of a place, the sun's decl.n & its alt.e being given to find the app.t time at that place.

Rule.

If the sun's decl.n & the colatitude of the ship be one north & the other south, take their diff.ce but if they be both north or both south take their sum, & if this sum exceeds 90°, take it from 180 for the sun's mer.n altitude. From the obs.d alt.e take the dep.t of the horizon, & the refraction; to the rem.r add the sun's semi-diameter; & par.x in alt.e the sum will be the true alt.e of the sun's center. Subtract the nat. sine of the alt.e thus corrected, from the nat sine of the mer.n alt.e & to the log.m of the remainder add the log. secant of the ship's latitude, & the log secant of the sun's decl.n; their sum, rejecting 20 from the index, must be sought for in Tab.e XI under log rising & the time corresponding to it will be the app.t time from the nearest noon when the sun's alt.e was taken. Consequently if the obs.n was made in the forenoon the time thus found must be taken from 24h, & the rem.r will be the app.t time from noon on the preceding day. Or, having found the sun's true alt.e & decl.n as above the operation must be performed by rule 5th for the solution of oblique angled ..... spherical triangles.

February 2nd 1802 about $2^h\ 0'$ in latitude $47°\ 19'\ 30''$ N long^e $14°\ 7'$ W, the alt^e of the sun's L.L. was obs^d to be $17°\ 13'\ 30''$ the height of the observers eye being 21 Feet: What was the app^t time.

Alt^e O^s L.L.	$17°\ 13'\ 30''$
Dip & Ref^n	$-\ 7\ 26$
	$17°\ 06'\ 04$
Par^x & semidiam	$16'\ 24$
True Alt^e	$17°\ 22'\ 28$

Sun's decl^n	$16°\ 55'\ 36''$ S
Corr for $47°$ W	$-\ 6.47$
	$16°\ 48'\ 49$
Corr for $2^h\ 0'$ P.M	$-\ 1.51$
True Decl^n	$16°\ 46'\ 58$ S ..... sec^t 10,01890
Colatitude	$42°\ 40'\ 30$ N ..... cosec^t 10,16887
Mer^t Alt^e	$25°\ 53'\ 32$. N S = 43668
True Alt^e	$17°\ 22'\ 28$. N S = 29861
	13807 Log^m ... 4,14009
App time	$2^h\ 32'\ 16''$ Log. rising ... 4,32786

By another method

☉ Decl^{n} 16.46.38 S	Tran. Alt^{e} 17.22.28	Latitude = 47.19.30 N
90.00.00	90.00.00	90.00.00
Codecl^{n} = 73.13.02	Zen. dist 72.37.32	Co-Latitude 42.40.30



To find ∠P

ZP = 42.40.30	} versch	10,16887	
P☉ = 16.46.38		10,01850	
Z☉ = 72.37.32			
sum 2)131.25.00			
½ sum = 41.02.30	sine	9,97003	
Z☉ = 72.37.32			
diff^{ce} 30.34.58	sine	9,79325	
		2)19,95115	
½∠P 19.02.00	cosi	9,97557	
2			
∠P 38.04.00	= 2.32.16.	App. time	

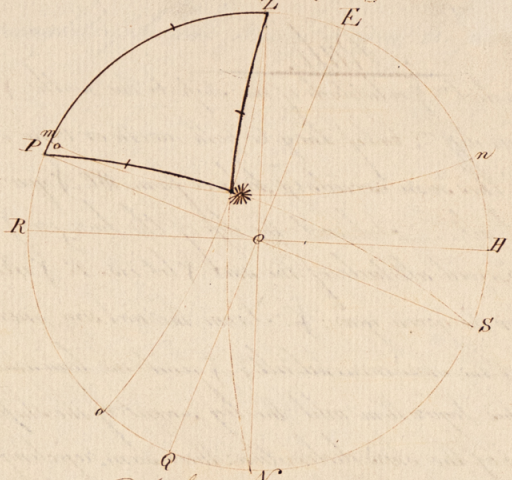
May 17th 1802 about 19h 15' at Bombay the altitude of the sun's L.L. was observed to be 23° 15' 30" the observers eye being 21 Feet.

What was the apparent time when the observation was made.

Alt. ⊙ L.L.	23° 15' 30"	
Dip & ½ ⌀	−6. 36	
	23. 08. 54	
Par. & semid.	+15. 39	
True Alt.	23. 24. 33	
Sun's decl. 18th	19° 27' 20" N	
Corr for 72° 38' E	−2. 46	
	19. 24. 34	
Corr for 4h 45' Mn	−2. 42	
True Decl.	19. 21. 52 N	sec. 10,025 29
Colatitude	71. 02. 36 N	cosec. 10,024 19
	90. 25. 11	
	180. 00. 00	
Mer. Alt.	89. 34. 49 . N.S =	99997
True Alt.	23. 24. 53 . N.S =	39738
	24° 00' 00"	60259 Log. 1,78002
	4. 44. 13	Log rising 1,82050
Appt time	19. 15. 47	

By another method

☉ Decl = 19. 21. 57 N	☉ Alt = 23. 24. 53	☉ Alt = 18. 56. 40 N
90. 00. 00	90. 00. 00	90. 00. 00
P Dist = 70. 38. 09	Zen Dist 66. 35. 07	Z Alt = 71. 03. 20



To find ∠ P

P ☉ = 70. 38. 09	co sec	10, 02531
PZ = 71. 02. 20		10, 02419
Z ☉ = 66. 35. 07		
sum 208. 16. 36		
½ sum 104. 18. 18	sine	9, 98662
Z ☉ = 66. 35. 07		
diff = 37. 33. 11	sine	9, 78494
		2
		19, 82207
½ ∠ P 35. 27	cosi	9, 91053
		12
∠ P 71. 04 =		4. 44. 12
		24. 00. 00
		19. 15. 48 = Appt time

Problem 21st

The Lat.e & Long.e of a place, the ☉ R, Decl.n, & alt.e of a known fixed star being given, to find the appr. time at that place.

Rule.

If the stars decl.n & colatitude of the ship be one north & the other south take their diff.ce; but if they be both north or both south take their sum, & if this sum exceeds 90°, take it from 180° & you will have the star's mer.l alt.e. Subtract the dip of the horizon & refraction from the observed altitude of the star & let it's ☉ R & decl.n be taken out of (Tab. IV) for the given year: &c. From the nat. sine mer. alt. take the nat. sine of the star's corrected alt.e & find the logarithm of the rem.r. To this Logarithm add the log secant of the ship's lat.e & the log secant of the star's declination: their sum, rejecting 20 from the index, must be sought for in (Tab. XVI) under log rising, & the time corresponding to it will be the distance of the star from the meridian; which being added to the star's ☉ R in time, if the star was west of the mer.n at the time of observation, or subtracted from it, if the star was then east of the mer.n will give the right ascension of the mid. heaven.

Find the sun's right ascension in time, by help of (Tab. XXII) for noon at the given place, & subtract it from the ☉ R of the mid. heaven; the remainder is the estimated time.

Enter XXIII a second time, with the estimated time, & daily var.n of the sun's R.A & subtract the minutes & seconds thus found, from the estimated time, the remr is the appt. time when the altde of the star was obsd; or the hour angle representing the star's dist. from the meridian may be found by rule 5th. for the solution of obliqueangled Spherical triangles as in the preceding rule.

August 27th 1802, about 46m past 9 Clock, Lat. 27° 48′ 20″ N. Long. 99° 41′ W. the obs. altitude of Antares west of the meridian was 12° 24′ 30″ height of the observers eye being 24 Feet above the surface of the sea.

What was the App. time.

☉ Alt. = 12° 24′ 30″ ☉ R 1780 = 16h 15′ 57″ ☉ Decl. 1780 = 25° 55′ 28″ S
Dip & Ref. = − 8 28 + 1 dy + 3.20
☉ Alt. 12 15 52 ☉ R 1802 = 16 16 57 ☉ Decl. 1802 = 25 58 48 S

Ann. Var. ☉ R = 3″,65 Ann. Var. ☉ Decl. = 8″,84
Years Elapsed 2265 Years elapsed 2265 Suns R = 16h 21′ 27″
1325 4420 Corr for 99° 41′ 00
1590. 5304. Suns true R 16 22 27
530. 1768.
530. 1768.
1 = 60,0225 3.20 100,2260

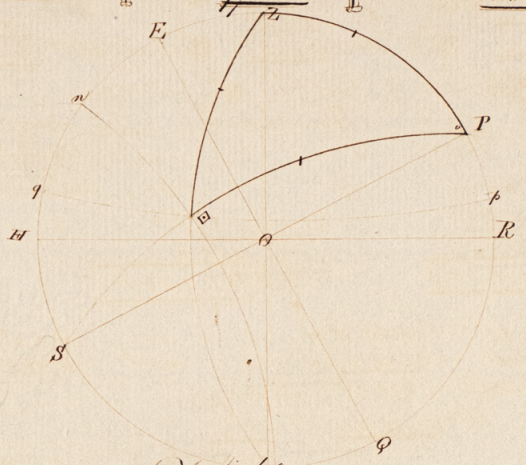
☉ decl. = 25° 58′ 48″ S ..... sec. 10,04626
Colat. = 62 11 30 N ..... cosec. 10,05330
Mer. Alt. = 36 12 42 NS = 59′77
S. Alt. = 12 15 52 NS = 21′42
37835 Log. 4,57789
Stars Dist. from ☉ mer. = 3h 52′ 33″ ..... Log rising 4,67745

Stars R in time 16 16 57
R of ☉ mid heaven 20 10 30
Suns R 10 22 27
Estimated time 9 48 03
Corr. Tab XXII ..... − 1 22
App. time 9 46 41

WILLIAMS'S 33 1/2 REFLECTOR

By another method.

D^{a} Decl^{n} 25° 58′ 48″ S.	D^{a} Alt^{e} 12° 15′ 52″	Latitude 27° 48′ 30″ N
90° 00′ 00″	90° 00′ 00″	90° 00′ 00″
P. Dist 115° 58′ 48″	Zen. Dist 77° 44′ 08″	Co. Lat^{e} 62° 11′ 30″



To find ∠P

PZ = 62° 11′ 30″	co sec^{t}	10,053330
PB = 115° 58′ 48″		10,061626
Z B = 77° 44′ 08″		
sum = 255° 54′ 26″		
½ sum 127° 57′ 13″	sine	9,89681
Z B = 77° 44′ 08″		
diff = 50° 13′ 05″	sine	9,88362
		2
		19,88200
29° 11′ 37″	cosi	9,94100
		2
∠P		58° 23′ 14″
D^{a} dist from 1st mer^{n}		19° 27′ 44″
D^{a} R		9° 33′ 33″
R mid heaven		16° 16′ 37″
Sun's R.		20° 10′ 30″
Estimated time		10° 22′ 27″
Corr^{n} Tab. 7 III		9° 46′ 03″
App time		− 1′ 32″
		9° 46′ 31″

April 14th 1802, at ¾ past 7. Latitude 48° 56' N. Long.t 66° W. the obs.d altt.e of ☉ Aldebaran, west of ☽ meridian, was 22° 24' 30" the height of the observers eye being 21 Feet.

What was the appt. time.

D.s Alt.e	22° 24' 30"	D.s R 1780	4h 22' 19"	D.s Decl.n 1780	16° 02' 02" N
Dip & Ref.n	− 6 12		+ 1 16		+ 2 04
☉ Alt.e	22 17 18	D.s R 1802	4 24 35	D.s Decl.n 1802	16 06 06 N

Years elapsed = 22.29
Ann.l Var.n D.s R = 3.42

4 4 58
8 9 16
6 6 8 7
1.16 = 76.23 18

Years elapsed 22.29
Ann.l Var.n D.l Decl.n = 8.26

13371
4458

17832
30" = 184.11 54

Sun's D R = 1h 28' 09"
Corr.n for 600 + 41
Sun's true R 1 28 50

D.s Decl.n 16° 06' 06" N ..... sec.t 10,01738
Colat.e 41 04 00 N ..... cosec.t 10,18248
Mer.n altt.e 57 10 06 N ☉ 8,026
☉ altt.e 22 17 18 N ☉ 3,929

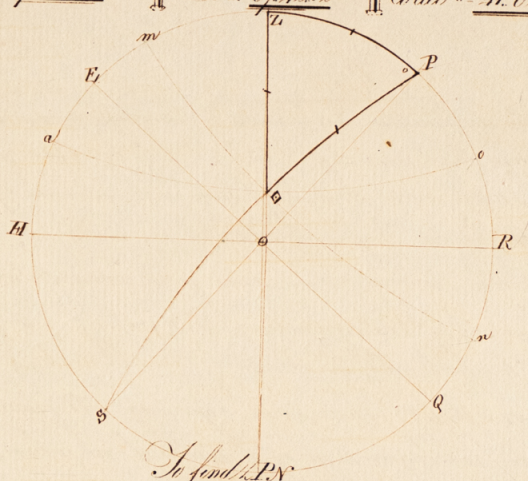
4,6087 Log.m 1,66358
D.s Distance from mer.n 1h 57' 23" ..... Log. rising 4,86244

D.s R in time 1 24 35
R mid heaven 9 21 58
Sun's R − 1 28 50
Estimated time 7 53 18
Corr.n Tab. XXIII − 1 12
Appt. time 7 52 56

By another method

$\odot$ Decl^n 16.6.66 N | $\odot$ Alt^e 22.17.48 | Latitude = 48.56 N

P Dist $\frac{90.00.00}{73.53.54}$ | Z Dist $\frac{90.00.00}{67.42.12}$ | Colat^e = $\frac{90.00}{41.04}$



To find $\angle$ P N

$\angle$ P	= 41.04.00	cosec	10,18248
P S	= 73.53.54		10,01738
Z S	67.42.12		
Sum	182.40.06		
$\frac{1}{2}$ Sum	91.20.03	sine	9,99988
Z S	67.42.12		
diff	23.37.51	sine	9,60297
			19,80171
	37.15.50	cosi^e	9,90085
	12		
$\angle$ P	3.4.21.20		
	24.50.26		
D^e dist from mer^n	4.38.03		
D^e R on time	4.24.35		
R mid heaven	9.22.40		
Sun's R	−4.28.50		
Estimated time	7.53.50		
Corr^n Tab XXIII	−10		
Appt time	7.52.38		

Problem 22nd

The appr. time, the ships lat.e & long.e & the suns declr. being given, to find its appr. altde.

Rule.

If the suns declination & colatitude of the ship be one north & the other south, take their diffce; but if they be both north or both south, take their sum, & if this sum exceeds 90 degrees, take it from 180 degrees & you will have the suns meridional altitude. With the apparent time from noon enter (XXI), & take the logarithm corresponding to it out of the column of log rising, to which add the cosine of the latitude, & the cosine of the suns declre: their sum, rejecting 20 from the index, will be the logarithm of a natural number, which being subtracted from the natural sine of the meridional altitude, will give the natural sine of the suns true altitude, at the given time.

Add the difference between the suns refraction & its parallax in altitude to the true altde, & it will give the suns apparent altitude. Note. The suns altitude may likewise be found with the colatitude, Polar Distance, & hour angle by rule 4th for the solution of oblique angled spherical Triangles.

What was the appt. altde of the sun's center at St. Helena on the 20th October 1802 at 19h 42m 16s.

Sun's decln 21st 10.33.04 S
Corr for 4h 0m 44s 111 − 3.43
10.29.21
Corr for 5.43.30 + 29
True Declne 10.29.50 S

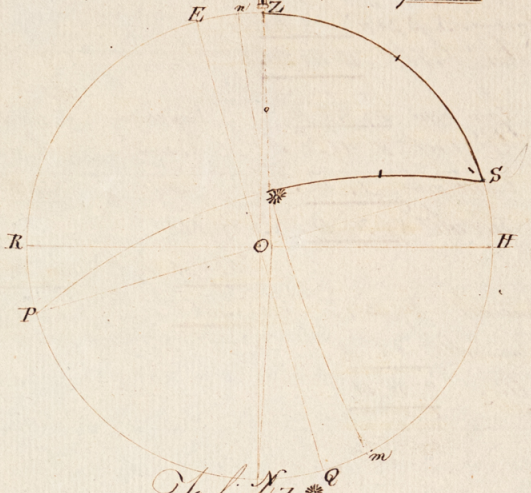
Appt time 19h 42m 16s ..... Log rising 4,75468
T. Declne 10.29.50 S .... Cosine 9,99267
Colatitude 74.05.00 S ..... sine 9,98362
Mer altde 84.34.50 NS 99555
53748 log m 4,73037

Sun's true altde 27.15.14 NS 45807
Corr of Sun's altde + 1.42
Appt altde 27.16.56

By another method

Suns decl^n 10.29.50 S Latitude 15.55 S

P Dist 79.30.10 Colatitude 74.05



To find Z ✱

Z S 74.05 ha... 10,54480
ZS 64.26 cosi... 9 63504
Arc 156.33.36 ha... 10,57993
cosec 10,67868
ZS 74.05 ... cosi. 9 43813
diff 22.57.34 ... cosi. 9 96416
Z ✱ 62.44.22 ... cosi. 9 66090

90.00.00
Suns hour. Alt^e 27.15.38
Corr^n Suns. Alt^e 1.1.42
Appr. Alt^e 27.17.20

WILLIAMS 83 1-2 GREENLD

What was the Appt altitude of the suns center in Lat 49° 57′ N & Long 54° W. on the 1st Sept 1862 at 18h 56′ 10″.

Suns Decln 2nd = 8° 06′ 44″ N
Corr for 54° 0′ = −2′ 22″
= 8° 03′ 22″
Corr for 3h 03′ 41″ = +4′ 44″
True Declⁿ = 8° 08′ 06″ N

App time = 18h 56′ 10″ ..... log rising 4,87894
Suns declⁿ = 8° 08′ 06″ N ..... cosine 9,99561
Colatitude = 40° 03′ 00″ N ..... sine 9,80852
Mer alt = 48° 11′ 06″ NS = 74531

48202 Log 4,68307

Suns true alt 15° 16′ 00″ NS 26329

Corr Suns alt +3′ 10″

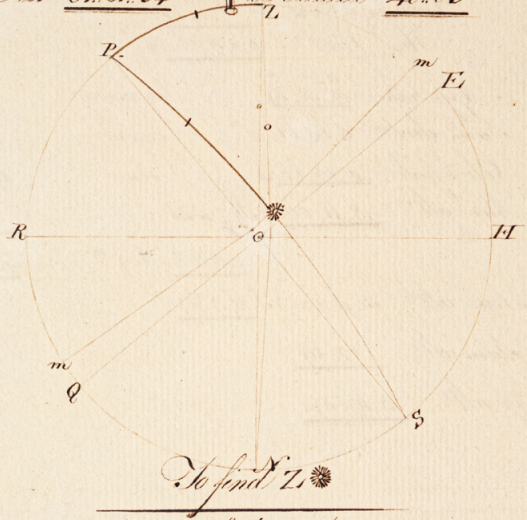
App alt 15° 19′ 10″

By another method.

Sun's decl^{n} 8°.08'.06" N — Latitude 49°.57' N

90°.00'.00 — 90°.00

P Dist 81°.51'.54 — Co Latitude 40°.03



To find Z

PZ = 40°.03'.00 sa ... 9,92458

∠P = 75°.55'.15 cosi ... 9,38608

Ac^{a} 4 11.23.24 { sa ... 9,31066 / cosec! 10,69824

PZ = 40.02.00 cosi ... 9,88394

Diff 70.18.30 cosi ... 9,52758

Z 74.54.06 cosi ... 9,42042

90.00.00

Sun's true alt^{e} 15.15.54

Corr^{d} Sun's alt^{e} + 3.19

Appr^{d} alt^{e} 15.19.13

Problem 23rd

The app.t time, the Lat.e & Long.e of the Ship being given, to find the app.t altitude of any known fixed star.

Rule.

If the stars decl.n & co-latitude of the ship be one north, & the other south take their difference, but if they be both north or both south take their sum, & if this sum exceeds 90° take it from 180°, & you will have the star's meridional altitude.

Turn the Long.e of the Ship into time, & if it be West add it to, but if it be East subtract it from the app.t time at Ship, & it will give the time at Greenwich.

Take the sun's R.A for that time (out of the Naut. Alm.) by help of (Tab XXII) & add it to the app.t time at the ship, which will give the R.A of the mid. heaven.

Take the star's Decl.n & R.A out of (Tab XII) & take the difference between the R.A & the R.A of the mid. heaven which will give the distance of the star from the meridian.

With the distance of the star from the meridian take the log. rising out of (Tab XVI) to which add the cosine of the ship's latitude, & the cosine of the star's decl.n, their sum, rejecting 20 from the index, will be the log. of a natural number which being subtracted from the nat. sine of the meridional altitude of the Star, will give the nat. sine of the star's altitude at the given time.

Add the Refraction to the true alt.e & the sum will be the app.t alt.e of the Star. Or, having found the distance of the star from

the meridian as directed above together with the colat.e & decl.n the operation may be performed by rule 4.th for resolving obliquangled spherical triangles.

What was the appl.d alt.e of Aldebaran on 11.th April 1802 at ^{H} 5. 56. 20 in Latitude 51° 32′ N. & Long.e 145° 30′ W.

App.e time = ^{H} 5. 56. 20 Sun's AR = ^{H} 1. 17. 08 . Ans.r Cor.r ☉ AR 3,42 Ans.r Cor.r ☉ D.n 8,26

Long.e in time = 9. 42. 00 W Corr.d ☉ AM = 12,24 Years elaps.d 22,28 Years elaps.d 22,28

Gr.n time = 15. 38. 20 Sun's True AR 1. 19. 32 4456 13368

App.e time 5. 56. 20 8912. 4456

AR Mid.n heav.n 7. 15. 52 6684. 17824

1. 16 = 76. 19. 76 2.0 = 18. 40. 32. 8

☉ AR 1780 = 1. 23. 19 ☉ Decl.n 1780 16. 02. 02 N.

☉ AR 1802 = 1. 21. 35 ☉ Decl.n 1802 16. 06. 06 N.

AR. Mid.n heav.n 7. 15. 52

☉ Dist.e from mer.r 2. 51. 17 Log rising ... 4,425772

☉ Decl.n 16. 06. 06 N. .... Co.si 9,98262

Colat.e of place 38. 28. 00 N ..... sine 9,79283

Mer.n alt.e 57. 31. 06 N.S. 84181

15928 Log.m 4,20217

☉ True Alt.e 40. 57. 30 N.S. 65553

☉ Refraction 1. 04

☉ App.e Alt.e 40. 58. 34

By another method.

☉ Decl^n 16. 06. 45 N Latitude = 51. 32 N ∠ P 42. 49. 15

P Dist 90 / 73. 53. 54 Colatitude 90. 00 / 38. 28



To find Z ☉

ZP = 38. 28. 00	tan	9,90009
½P = 42. 49. 15	cosi	9,86528
Arc A = 30. 13. 32	tan	9,76537
Arc A = 30. 13. 32	cosec	10,29801
ZP = 38. 28. 00	cosi	9,89374
Diff = 43. 41. 02	cosi	9,85936
½ ☉ = 49. 02. 28.	cosi	9,81658
	90. 00. 00	

☉ True Alt^e 40. 57. 32

☉ Ref^n + 1. 04

☉ App. Alt^e 40. 58. 36

What was the appt. altitude of Pomalhaut at Rio Janeiro Aug^t 29th 1802 at ^{H}11.19.^{M}36.

App^t time ^{H}11.19.^{M}26 Sun's AR ^{H}10.28.^{M}40. Am. Var^n R 339 Am. Var^n D^n 1898
Long^e in time 2.32.43 Corr^n S22 +2.09 Years elap^d 2266 Years elap^d 22,66
Green^h time 11.12.09 Sun's True R 10.30.55 4532 11388
App^t time 11.19.26 6798. 11388.
MR Mid heav 21.30.21 6798. 3796.
Decl^n 1780 30.46.58 +115 15,2312 3796.
−7.10 R for 1780 ^{H}22.15.27 7.10 130,0868
Decl^n 1802 = 30.39.48 S R for 1802 22.16.52
MR Mid heav 21.30.21
Dist from mer^n 56.21 Log rising 3,47820
Decl^n 30.39.^{M}48 S ..... cos 9,92439
Colat^e 67.05.30 S ..... sine 9,96433
97.45.38
180.00.00
Mer. Alt^e 82.14.22 N S 99084
2383 Log^m 3,27712
True Alt^e 75.14.30 N S 96701
Refraction +15
App. Alt^e 75.14.45

By Another Method

☉ Decl^{n} 30° 39' 48" S Latitude = 22° 54' 10" S ∠ ^{p} = 14° 05' 15"

P Distan 59° 20' 12" Co-latitude = 67° 05' 50"



To find Z ☉

S ☉	= 59° 20' 12"	tan	= 10,22692
∠ S	= 14° 05' 15"	cosi	= 9,98674
Spc^{l} A	= 58° 33' 27"	tan	= 10,21366
Arc A	= 58° 33' 27"	cosec^{t}	= 10,06807
S ☉	= 59° 20' 12"	cosi	= 9,70756
Diff	= 8° 32' 23"	cosi	= 9,99516
Z ☉	= 14° 47' 00"	cosi	= 9,98535
	90° 00' 00"		

☉ True Alt^{e} 75° 13' 00"

☉ Ref^{n} + 15

☉ Appr. Alt^{e} 75° 13' 15"

Problem 24th

The app. time, the lat.e & long.e of the ship being given to find the app. alt.e of the moon's center.

Rule.

If the moon's decl.n & colat. of the ship be one north & the other south take their diff.; but if they be both north or both south take their sum & if this sum exceeds 90° take it from 180° & you will have the moon's mer.l alt.e Turn the long.e of the ship into time & if it be west add it to bet if it be east subtract it from the app. time at ship & it will give the time at Greenwich.

By the help of (LXXII) take the sun's RA from page II of the Nautic. Almt. for this time & add it to the app.t time at the ship which will give the RA of the mid. heaven.

To the time at Greenwich take also the moon's hor. par.x (page VII) & her RA & Decl.n (page VI) Naut. Almt. by help of (LXXI).

Turn the RA into time & take the diff. bet.n it & the RA of the mid. heaven which will be the distance of the moon from the meridian. With the dist. of the moon from the mer.n take the log. rising out of (LXXI) to which add ..... the cosine of the ships lat.e & the cosine of the ☽'s decl.n the sum rejecting 20 from the index must be sought for in (LXXIII) which being subtracted from the nat. sine of the moon's mer.l alt. will give the nat. sine of the moon's true altitude at the given time. With the moon's hor. par.x & the true alt. diminished by the number which stands against it in (LXIII) take the corr.n of her alt.e out of that table & subtract it from the true altitude which

will give the appt alt^e of the moon's center.

The moon's true altitude may likewise be found by the second method of the preceding problem.

What was the appt altitude of the ☽ center on Nov^r 30^th 1802 at 8^h 27′ 42″ at New York.

App^t time = 8^h 27′ 42″ Suns R.A. = 16^h 23′ 20″ ☽ hor par^x = 59′ 10″
Long^e in time = 4 56 26 W Cor^n to ☉ = + 2 24
Green^h time 13 24 18 Suns T.R.A. = 16 25 44 ☽ Decl^n for mid = 10° 09′ S
App^t time = 8 27 42 Corr^n ☉ = -20
☽ R.A. for mid 323° 04′ R.A. Mid heav^n = 53 26 ☽ True Decl^n = 15 49
Corr^n ☉ + 46 ☽ R.A. = 21 35 20
☽ True R.A. 323 50 20 41 54
5 21 00 00
107 56 40
Rise time 7^h 35′ 10″ ☽ Dist from ☉ 3° 18′ 06″ mer^s Log rising 4,54516
☽ Decl^n 15° 49′ 00″ S ..... cosi 9,98224
Co Latitude 49 17 00 N ..... sine 9,87964
Mer^n Alt^e 33 28 00 N.S 53145
25589 Log 4,40804
☽ True Alt^e 17° 11′ 28″ N.S 29556
Corr^n ☉ - 53 29
☽ App^t Alt^e 16 17 57

By another method

3^{d} Dist^{e} = 15.49 S Lat^{e} = 40.43 N Z P 49.31.30

P Dist 90.00 / 105.49 Co lat 90.00 / 49.17



To find Z D

Z P	= 49.17.00	la 10,06518
P	= 49.31.30	cosi 9,81227
Arc 1	= 37.01.16	la 9,87745
Arc 1	= 37.01.16	cos^{a} 10,22033
Z P	= 49.17.00	cosi 9,81446
Diff	= 68.47.44	cosi 9,55834
Z D	= 72.48.41	cosi 9,47058

	90.00.00
3^{d} Sine Alt^{e}	17.11.19
Corr^{n} 18	-53.29
3^{d} App. Alt^{e}	16.17.50

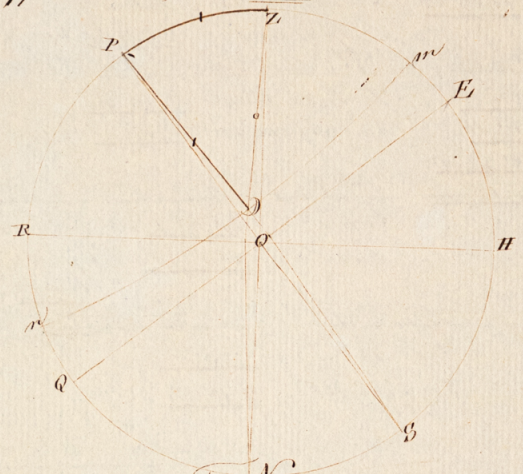
What was the appt alt^e of the ☉ center on 3^rd July 1802 at 20h 59m 38s at London.

Appt time	= 20h 59m 38s	San's R for mer.	6h 46m 47s	☉'s Hor. Par^x	54.14
Long^e in time	+ 23 56	Corr^n (22)	+ 3 27		
Green^h time	21 00 01	San's True R	6 50 24	☉'s Decl^n for mid	4.28 N
		Appt time	20 59 38	Corr^n (21)	1 33 34
☉'s R for mid	148.03	☽ Mid. heav^n	3 50 02	☉'s True Decl^n	12 44 06
Corr^n (20)	+ 4.08	☉'s R in time	10 08 44		
☉'s True R	152 11	☉'s Dist from mer^n	6 18 42	Log rising	5,03421
	50 43 40				
☉'s R in time	10 08 44	☉'s Decl^n	= 12.44.06 N	cosc	998918
		Colat^e	= 38 28 00 N	sine	9,79383
		Mer. Alt^e	= 51 12 06 N	S	77926
					6617 Log 4,81702
		☉'s True Alt^e	7.04.35	N S	12319
		Corr^n (58)	- 46.32		
		☉'s App. Alt^e	6 18 03		

By another method

S^{s} Decl^{n} 12.44.56 N | Sa Alt^{e} 51.32 N | L P 94.40.20

P Dist 90.00.00 / 77.15.54 | Sa Alt^{e} 90.00 / 38.28



To find Z D

Z P	= 38.28.00	tan	9.90009
L P	= 94.40.20	cos	8.91117
Arc A	= 2.42.48	tan	8.81126
Arc A	= 3.42.48	cos^{l}	11.18064
Z P	= 38.28.00	cos	9.89274
Diff	= 80.58.12	cos	9.10576
Z D	= 82.55.36	cos	9.09040

90.00.00

S^{s} True Alt^{e} 7.04.24

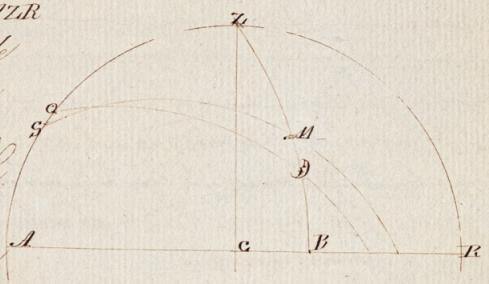
Corr^{n} 58 − 46.32

S^{s} App. Alt^{e} 6.17.52

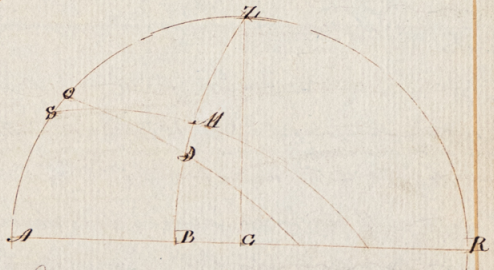
Problem 25th

Having the app or observed distance of the moon from the sun or a fixed star together with the observed altitude of each & the moons hor par^{x} to find the true distance.

Rule.



Let the primitive circle AZR represent the vertical circle which the sun or star is on at the time of obs^{n} Z the zenith S the app place of the sun, ZDB the vertical circle which the moon was on, & D its app place. Then will AS be the suns app alt^{e}, BD that of the moon & the arc SD of a great circle described through these two points will be their app distance.



It has been already shewn that all celestial objects appear higher than they really are on account of par^{x} & consequently the diff^{ce} betw^{n} their true & app alt^{s} or zenith dist^{s} will always be equal to the diff^{ce} betw^{n} the refraction & the suns par^{x} in altitude. As the refraction of the sun is always greater than its par^{x} in altitude it is plain that the app alt^{e} of that object will always

be greater than its true altitude & consequently its app zenith distance will be less than the true.

On the contrary the moons par^{x} in alt^{e} being always greater than its refraction its app alt^{e} will always be less than the true & its app zenith distance greater than the true.

If then OS be taken downward from O, & equal to the diff^{ce} betw^{n} the $\odot$ refraction & its par^{x} in altitude & DM, be taken upward from D & equal to the diff^{ce} between the moons par^{x} in altitude & its refraction. AS & BM will be the true altitudes of the sun & moon ZS & ZM will be their true zenith distances & the arc SM will be the true distance of their centers which is req^{d}.

Hence in the spherical triangle OZD the three sides OD, ZO & ZD namely, the app dist of the objects, & their app zenith distances are given to find the angle OZD which is done by the 5th case of oblique angled spherical triangles.

Then adding to ZO the diff^{ce} bet^{n} the refraction of the sun (II) & its par^{x} in alt^{e} (III) if the objects be the OD & $\odot$ or the refraction of the star (II) if the objects be the D & Star we obtain ZS the sun & stars true zenith distance, & subtracting from ZD the diff^{ce} between the D^{s} par^{x} in alt^{e} & refraction (IV) we obtain ZM its true zenith distance consequently as the $\angle$ SZM remains unaltered there will be two sides ZS, ZM & the contained $\angle$ SZM given, in the triangle ZSM to find SM the true dist which may be done thus.

To the cosine of the diff^{ce} of the azimuth SZM add the log^{m} of ZM the D^{s} true zenith distance & their sum rejecting 10 in the index will be the log^{m} of arc ZD.

when the ∠ SZ. M is greater than 90° the sum of ZD & ZS the Sun's true Zenith dist.ce will be SD but when the angle SZ. M is less than 90° take their diff.ce for SD. Then add together the tangent of ZD the cosec.t of ZD the versine of Z. M the D.s true zenith dist. & the cosine of SD the same rejecting 20 in the index will be the cosine of the true distance.

This method is very plain & strictly true is recommended by those who have the logarithmic sine & tangents to every second of the quadrant, but those who have not may use following which seems well adapted to the disposition & capacities of seamen in general.

From the sums refraction (51) take its par.x in alt.e (3m) & the rem.r will be the correction of the sun's altitude.

The correction of the star's alt.e is its Refraction (51)

Take the proportional log.m of the D.s hor. par.x (out of 57) put 10 for its index & subtract the log cosine of the D.s app.t alt.e from it, the rem.r will be the prop.l log of her par.x in altitude from which take the moon's ref.x (51) & there will remain the corr.n of the D.s altitude.

Add the sun's app.t altitude to the D.s & take ½ the sum, subtract the less from the greater & take ½ the diff.ce. Add together the cotangent of the ½ sum the tang.t of the half diff.ce & the cotangent of half the app.t distance the sum rejecting 20 from the index is the log tangent of an arc A.

When the sun's altitude is greater than the moon's take the diff.ce bet.n arc A & ½ the app.t dist. but if the moon's altitude be greater than the sun's take their sum & ½ to the cotang.t of this sum or diff.ce add the cotang.t of the sun's app.t altitude & the prop.l log.m of the corr.n of

the sun's altitude their sum rejecting 20 from the index is the prop log of the 1st correction of the app dist. If the diff of arc 1 & half the app dist was taken in the preceding article take now their sum but if their sum was then taken take now their diff & to the log co-ar^{ts} of this sum or diff add the log co-sin^{e} of the 3d app altitude & the prop log of the correction of the 3d alt^{e} the sum rejecting 20 from the index is the prop log of the second corr^{n}. If the arc 1 be less than ½ the app dist the 1st corr^{n} must always be added & the second subtracted from the app distance, but when the arc 1 be greater than half the app distance both the first & second corr^{ns} must be added & the sun's altitude be less than the moons but if the moon's be least both must be subtracted to give the corrected distance of the sun & moon.

Take the sum & diff of the corr^{n} of the moons alt^{e} & the second corr^{n} of the app dist & to the prop log of this sum or diff add the log sin^{e} of the corrected dist & the constant log 4.3820 the sum of these four logs will be the prop log of the 3d corr^{n} to be added to the corr^{d} distance if it be less than 90° or subtracted from the corr^{d} dist if it be greater than 90° the sum or diff we be the true distance.

To work a True Distance by Rios's Method.

Add together the two appr. altitudes. Take out of (16) the complementary argt answering to the appr. altr of the sun or star & out of (19) the corrn of the Dappr altn, according to the minutes of Hor parx & the propr parts for seconds which add together with the sum of appr altns & you will have the corrd sum of altns

At the time of taking the corrn out of (19) take likewise the Auxiliary argument out of (10) which will be found in the opposite page. Take out of (11) Nº 1 answering to the sum of appr altitudes & to the minutes of Auxr angle & also the parts for seconds; Nº 2 answering to the sum of corrd altitudes & the parts for seconds & Nº 3 answering to the degs & of appr dist. (reserving the seconds) & the minutes of auxr argument & also the parts for seconds.

Add together the three numbers & parts for seconds & the corrd distance answering to that sum in the same Table with the addition of the reserved seconds will be the true dist from the moon to the sun or star.

October 15th 1802 about 10h 30m Sat 34° 0' S Long.e by acc.t 102° E the following observations were made for finding the True Distance. Note −1'.30" from the distance, −1'.30" from the ☉ Alt.e +1'.30" to the Sun's for Errors. Height of the Eye 21 Feet

☾ Alt.e ☉ I	☾ Alt.e ☉ I	Dist.e ☾ ☉
23° 04' 00"	18° 42' 00"	118° 33' 00"
23. 18. 00	18. 31. 00	118. 33. 00
23. 39. 00	18. 17. 00	118. 32. 00
23. 56. 00	18. 00. 00	118. 31. 00
24. 30. 00	17. 37. 00	118. 29. 00
5) 118. 27. 00	5) 91. 06. 00	5) 592. 38. 00
23. 41. 24	18. 13. 12	118. 31. 36
+ 1. 30	− 1. 30	− 1. 30
23. 42. 54	18. 11. 42	118. 30. 06
− 4. 22	− 4. 22	+ 31. 44
23. 38. 32	18. 07. 20	119. 01. 50
+ 16. 06	− 15. 28	
23. 54. 38	17. 51. 42	

☾ Appt time = 10h 30m ☽ Semidiam = 15'.32"
Long.e in time = 6. 48 E Aug.to = + 6
Green.h time = 12. 42 ☽ Aug.to Semid.m 15. 38

Sun's Refraction 2'.08"
Sun's Par.x − 8
Sun's Cor.x 2. 00
☽ Hor. Par.x 57'.00" PL. 10,4094
☽ App. Alt.e 17. 51. 42 cosi 9,0785
☽ Paren. Alt.e 54. 15" PL 0,5309
☽ Refraction − 2. 56
☽ Cor.x 51. 19

☾ App. Alt.e = 23° 54' 38"
☽ App. Alt.e = 17. 51. 42
Sum = 41. 46. 20
Diff = 6. 02. 56
App. Dist = 119. 01. 50
1st Cor.n + 1. 16
119. 03. 06
2nd Cor.n − 34. 14
Cor.d Dist 118. 28. 52
3rd Cor.n − 7
True Dist 118. 28. 45

half is
20° 52' 10" colan 10,418 46
3. 01. 28 tan 8,7292
59. 30. 55 colan 0,3698 0
Area 4. 39. 27 tan 8,9117
Diff 54. 51. 18 colan 9,8475
☾ App. Alt.e 23. 54. 38 colan 10,3532
☾ Cor.n 2. 00 PL 1,9542
1st Cor.n 1. 10 PL 2,1549
Sum = 54. 46. 32 col 9,6840
☽ App. Alt.e = 17. 51. 42 col 10,4918
☽ Cor.n = 51. 19 PL 1,5750
2nd Cor.n = 34. 14 PL 1,7208
Sum = 1. 25. 33 PL 0,3221
Diff = 17. 05 PL 1,0227
Cor.d Dist 118. 28. 52 tan 10,2656
Constant Log 1,5820
3rd Cor.n 7 PL 2,1924

By Rios's Method.

	δ^{s} App. Alt^{e}	23.55	
	δ^{s} App. Alt^{e}	17.52	
δ^{s} Hor. Par^{x}	57.04	Sum. Alt^{e}	41.47. N^{o}1	125 7714
		Comp. Cor^{n} δ^{s} Alt^{e}	57.59	101
Aux^{y} A. Sag^{ta}	9 14	δ^{s} Corr^{n}.	51.20	
	12	Parts for Sec^{ds}	0	
Aux^{y} A. Angle	0.16	Corr^{d} Sum of Alt^{e}	43.36.10. N^{o}2	1735900
				134
		App. Dist	110. 01. N^{o}3	2482618
		Red^{n} Sec^{ds}	50	182
		Cor^{d} Dist	118. 20. 00	5176649
		S. Dist.	118. 28. 50	

July 5th 1802 about 3h Latitude 45.25 S Long.e by acc.t 79.15 E
The following Obs.ns were made for finding the True Dist. N B 17°
to ☽ Dist 1.05 to ☉ Alt.e 2.00 from ☽ & ☉ for errors Dep 4.20.

☉ Alt.e ☽ Dist	☽ Alt.e ☉ Dist	Dist ☽ ☉
32° 18.00	65° 57.00	51° 41.30
32.03.00	66.11.00	38.44.45
31.56.00	66.17.00	38.42.00
31.23.00	66.38.00	38.43.00
☽ 127.40.00	☉ 265.03.00	☽ 234.48.15
31.55.00	66.15.45	38.42.03
+1.05	-2.00	+17
31.56.05	66.13.45	38.42.20
-4.20	-4.20	+30.40
31.51.45	66.09.25	39.13.00
+15.47	-15.02	
32.07.32	65.54.23	

App.t time = 11h 27.00
Long.e in time = 5.17 E
Green time = 21.43

☽ semidia = 14.47 ☽ Ref.n 1.31
Aug.n = +13 ☽ Par.x -8
Aug. semidia = 15.02 ☽ Corr.n 1.23

☉ Mer. Par.x = 54.17 PL 1.5206
☉ App. Alt.e = 65.54.23 cosec 9.6100
☉ Par.x Alt.e = 22.10 PL 0.9097
☉ Ref.n = -25
☉ Corr.n = 21.45

☽ App. Alt.e = 32.07.32
☉ App. Alt.e = 65.54.23
Sum = 98.01.55
Diff.ce = 33.46.51
App. Dist = 59.12.00
1st Corr.n = +1.13
59.14.22
2nd Corr.n = -4.01
Corr.d Dist = 59.10.21
3rd Corr.n +2
T. Dist = 59.10.23

half is
40.00.57 ... cot ... 9,93889
16.53.23 ... tan ... 9,48235
20.36.34 ... cot ... 10,24543
Arc 24.53.56 ... tan ... 9,66667
Sum 51.30.30 ... cot ... 9,8331
☽ App. Alt 32.07.32 cot ... 10,2020
☽ Corr.n = 1.23 PL ... 2,1143
1st Corr.n = 1.13 PL ... 2,1694
Diff = 4.42.38 cot = 11,0840
☉ App. Alt. 65.54.23 cot = 9,6502
☉ Corr.n = 21.45 PL = 0,9178
2nd Corr.n = 4.01 PL 1,6520
Sum = 25.46 ... PL = 6,8442
Diff = 17.44 ... PL = 1,0065
Corr.d Dist = 59.10.21 tan = 10,2242
Constant Log ..... = 1,5820
3rd Corr.n 2" PL = 2,6369

By Rios's Method.

	☉ App. Alt. = 32° 18′		
	☽ App. Alt. = 65. 55		
D° Hor. Par. = 57′. 17″	Sum. Alt. = 98. 13	N°1	2 1 3 8 0 5 7
Aux. A. Arg. b = 27. 21	Comp. Corr. Alt. = 58. 37		3 4
9	D° Corr. = 21. 37		
3	Pals for Sec. = 1		
Aux. A. Arg. c 27. 33	Corr. Sum of Alt. = 99. 23. 21	N°	0 8 5 3 9 1 7
			1 8 5
	App. Dist. 30°. 12′.	N°3	1 4 9 5 1 8 5
	Red. Sec. 9		1 4 1
	Corr. Dist. 30°. 10′. 15″		1 4 8 7 5 1 9
	T. Dist. 30. 10. 24		

Of Finding the Longitude.

It has been shewn that the Long^e of a place is an arc of the Equator contained between the meridian of that place, & the meridian of some other given place, from which the Long^e is reckoned to begin, & it is measured by the deg^s, minutes, & sec^s contained in that arc. Now as the Earth performs its revolution eastward round its axis, uniformly in 24^h; it is manifest that the meridian of every place on the Earth will in that period, successively come opposite to the Sun, & make noon to that point of the Equator which the meridian passes through, as well as to all places on that mer^n.

Hence, it is evident that the times of noon at different places, differ in proportion to their difference of Long^e & consequently the difference between the times at any two places will always be proportional to their difference of longitude; that is as 24^h are to the difference between the times at any two places, so are 360 to the difference of longitude between those places; therefore if the difference bet^n the times at any two places be known, the diff^e of long^e bet^n those places will be known also.

Now as we can always find the time at the place where we are, by observing the sun's Altitude in the day time, or the altitude of a bright star in the night, if we can by any means know what time it is at Greenwich; it is manifest that the difference of Long^e between those places will be known by turning the difference of the two times into degrees, minutes, & seconds, at the rate of 360 degrees to twenty four Hours, or fifteen degrees to one hour of time.

And it is farther evident that if the time at the place of Observation .....

be greater than the times at Greenwich the longitude of ye place is east; but if the time at the place of observation be less than the time at Greenwich, the Longitude is west; because the sun coming to the meridians of such places as lie to the eastward before it comes to the meridians of those which lie to the westward, makes the times at the former meridians greater than the times of the latter.

Of the many methods which have been thought of for finding the time at the first meridian, two have been found on trial, to perform it with tolerable accuracy.

1st, If the time when any particular phenomenon will happen at Greenwich could be predicted, & the time when it does happen under any other meridian be observed, it is manifest, from what has been said, that the difference between those two times, turned into degrees, minutes, &c. would be the longitude of the place of Observation. Experience has shewn that the Lunar Tables are not capable of assigning the place of the moon in the heavens, & consequently its dist from the sun or a fixed star with sufficient exactness, for this purpose its distance, therefore, from the Sun or from certain fixed stars, is now computed at stated times, & inserted in the Naut. Alm. Consequently, if ye moon's distance from the Sun, or one of these star, be observed, & compared with those computed distances, the time at Greenwich when

such distances was obs.d may be found, & the difference between that time, & the time when it was observed, be turned into degrees, minutes, &c will be the Long.e of that place from Greenwich.

The second method is by means of a Watch or Time Keeper, regulated to keep equal time, & set to the time at the 1st meridian.

If it be regulated also to keep true mean time, it is manifest that it will always shew the mean time at the 1st meridian, but if on trial, it be found to gain, or loose, uniformly any number of Seconds a day on mean time then that number of Seconds being multiplied by the number of days which have elapsed since the watch was set to the time at Greenwich, or the 1st meridian, & the product subtracted from, or added to the time shown by the watch, according as the watch gains or looses, will give the true mean time at Green.h

Problem 26th

To find the Longitude of a Ship at sea by observations of the moon's distance from the sun, & their altitudes, taken at the same time; the latitude of the ship, & its longitude by account being known also.

Rule.

Turn the longitude by account into time, & if it be west add it to, but if it be east, subtract it from the estimated time at the ship, when the observation was made, & it will give the time at Greenwich, nearly. To this time take the moon's semidiameter, & hor. par: from p. VII of the Naut. Alm: also the Sun's semidiameter for the day, from p. III & augment the moon's semidiameter by adding to it the number of seconds found in Tab. IV with the moon's observed altitude.

Correct the obs.d Distance by adding to it the semidiameter of the Sun, & the augmented semidiameter of the moon: correct also the observed altitudes by subtracting the dip of the Horizon & adding, or subtracting the semidiameter's of the objects according as the altitudes of the lower or upper limb was observed: & the appr. distance & altitudes of the sun, & moon, will be obtained. With the appr. distance & the two appr. altitudes find the true distance by Prob. 25th

Amongst the distances of the moon's center from the Sun, & Fixed Star's put down in Pages VIII,

II, I, II, of the Naut. Alm, find those two distances of the sun & moon which are next less, & next greater, than y^e true distance by observation, take the difference between them, also betw^n that which stands first in the Naut. Alm. & the true distance by observation & subtract the prop. log of the former difference from the proportional logarithm of the latter, the remainder will be the proportional logarithm of a portion of time, to be added to the time which the distance standing first in the Ephemeris was computed for, & the sum will be the apparent time at Greenwich.

To this time take the sun's declination out of (p^r II) of the Naut. Alm, & correct the app^t. altitude of the sun's center by subtracting from it the difference between the Refraction of the sun & its parallax in altitude, Taken out of Tables I & III with these & the ship's Latitude find the app^t time by Problem 20.

Take the difference between the app^t time at Greenwich & the app^t time at y^e Ship, & convert it into degrees, minutes, &c & it will be the true Longitude of the ship at the time of observation, East if the time at ship be greater than the time at Greenwich, but west, if the time at Ship be less than the time at Greenwich.

Feb.y 23d 1802 about 21h½ Sid. t. 9h 03m S. long by acc.t 66.30 W. of Edin.r
obs.ns were made for finding the True Long.e Nov.r 14th to the East. - from
y.e ☉ Alt.e & - 2.40 from ☽ Alt.e for errors dip 4.16

☉ Alt.e ☽ L.	☉ Alt.e ☽ L.	Dist.e ☉ ☽
53.58.00	23.02.00	102.17.40
54.04 〃	22.32 〃	102.17.20
54.15 〃	22.40 〃	102.17 〃
54.26 〃	22.29 〃	102.16.50
216.43 〃	91.03 〃	409.08.50
54.10.45	22.43.45	102.17.10
-1.00	-2.20	+40
54.09.45	22.43.25	102.17.50
-4.16	-4.16	+31.33
54.05.29	22.39.09	102.49.47
+16.12	+15.43	
54.21.41	22.54.52	

App.t time 21h 30m
Long.e in time 4.26 40
Green.h time 1.50 24 nearly

D.o Semidiameter = 15.37
Augmentation = 16
D.o Aug.d Semidia. = 15.43

D.o Hor. Par.x = 57.14 PL = 10,4976
D.o App.t Alt.e = 22.54.52 cos = 9,9643
D.o Parin. Alt.e = 52.43 PL = 6,5333
D.o Refraction = -2.16
D.o Correction = 50.28

☉ Ref.n = 41
☉ Par.x = -5
☉ Corr.n = 36

☉ App.t Alt.e = 54.21.41
D.o App.t Alt.e = 22.54.52
Sum = 77.16.33
Diff = 31.26.49
App.t Dist 102.49.47
1st Corr.n + 37
102.50.24
2nd Corr.n -50.33
Cor.d dist 101.59.51
3rd Corr.n -0
True dist = 101.59.51

halfs

38.38.10	cot	10,0072	5
15.43.24	tan	9,4490	52
51.24.32	cot	9,9010	93
Arc 45.41.42	ta	9,4487	70
Diff 35.43.11	cot	10,1432	
☉ App.t Alt.e 54.21.41	cot	9,8355	
☉ Corr.n = 36	PL	2,4771	
1st Corr.n 37	PL	2,4758	
Sum = 67.06.35	cot	9,0255	
D.o App.t Alt.e 22.54.52	cot	10,3729	
D.o Corr.n = 50.28	PL	0,5522	
2nd Corr.n 50.33	PL	0,5516	
Sum ... 1.41.01	PL	0,2300	
diff 5	PL	3,3344	
Cor.d dist 101.59.51	tan	10,6726	
Constant Log.m		1,5820	
3rd Corr.n 0	PL	3,8309	

By Rios's Method

☉ App. Alt. = 54.22
☽ App. Alt. = 22.55

☽ Hor. Par. = 57.14
Aux. 1. Arg. II. 55
3
4
Aux. 1. = 12.02

Sum. Alt. = 77.17 No. 1 1781202
Comp. Corr. 59.24 4
☽ Corr. 30.16
Par. by Sec. 13
Corr. Sum. 11. 29.06.32 No. 2 .... 1205920 23

App. dist 102.40 No. 3 2220378
Rios Sec. 47 106
Corr. dist 101.59.02 5207642
☽ Dist 101.59.50

Dist at noon = 102.11.44
Dist at 3h = 101.40.50
1st diff 1.30.48 Pl. 0,2074
2nd diff 1.11.54 Pl. 0,2985
Aportion of time 2.22.37 Pl. 0,1011
1st dist 0.00.00
App. time at Grn. 2.22.37

Sun's decl. = 9.36.18 S
Corr for 2.22.37 R. II = 2.07
Sun's True decl. 9.34.11 S

Sun's Decl. = 9.34.11 S ... sec. 10,00608
Co-latitude 80.37.00 S ... cosec. 10,00544
90.31.11
180.00.00

☉ App. Alt. = 54.21.33
☉ Corr. = -36
☉ ☽. Alt. = 54.21.05

Hor. Alt. 89.28.49 No. 99996
True Alt. 54.21.05 No. 81260
18736 log. 4,27268
2.24.33 log rising ☽. 4,28470

App. time at Shg. 21.35.27
App. time at Grn. 26.22.37
4.47.10
×3
23.55.50
×3
☽ Longitude 71.47.30 W

March 12th 1802 about 3½ latitude 50.16 S longitude by account 15 W^est the following observations were made for finding the true Long^e N 13 + 22" to the Sun's Alt^e 4.35 hd the dist 8 + 2.00" to ☿ S^r for Errors dip + 1.22"

Alt^e ☉ LL	Alt^e ☉ UL	Dist^e ☉☿
31.25.00	11.35.00	105.14
31.15	11.34.00	105.15
31.06	11.30	105.15
30.54.30	12.03	105.15.15
30.46.30	12.04	105.15.15
5	5	5
155.7.00	59.13	520.15
31.05.24	11.50.30	105.15.00
+ 22	+ 2.00	+ 1.35
31.05.46	11.52.30	105.19.35
+ 22	+ 22	+ 31.07
31.01.24	11.48.14	105.50.42
+ 16.08	- 14.30	
31.17.32	11.33.15	

App^e time	4	3.30
Long^e in time		+ 1.00
G^r^k time nearly		3.31
S^d's Semidiameter		14.36
Augmentation		+ 2
S^d's Aug^d semidiam		14.50
D^o Hor. Par	54.47	PL 16.5166
D^o App. Alt^e	11.35.15	cosi 9.9911
D^o Par^x Alt^e	53.40	PL 0.5255
D^o Ref^n	4.33	
D^o Corr^n	49.07	
☉ Ref^n	1.34	
☉ Par^x	- 8	
☉ Corr^n	1.26	

☉ App. Alt^e	31.17.32		
D^o App. Alt^e	11.33.15		
Sum	42.50.47		
diff	19.44.17	half is	
App^d Dist	105.50.42		
1st Corr^n	+ 36		
	105.51.18		
2nd Corr^n	- 29.55		
Corr^d Dist	105.21.23		
3rd Corr^n	- 4		
T. Dist	105.21.19		

	21.25.23	cot	10,40632
	9.52.08	tan	9,24047
	32.55.21	sin	9,87834
Arc	18.21.27	tan	9,52513
diff	24.23.54	cot	10,16.45
☉ App. Alt^e	31.17.32	cot	10,2162
☉ Corr^n	1.26	PL	2,0980
1st Corr^n	36	PL	2,4766
Sum	1.20.48	cot	9,3238
D^o App. Alt^e	11.33.15	cot	10,6891
D^o Corr^n	49.07	PL	0,3640
2nd corr^n	29.55	PL	0,7703
Sum	1.19.02	PL	0,3574
diff	19.42	PL	0,0720
Corr^d dist	105.21.23	tan	10,3612
Constant Log^m			1,5820
3rd Corr^n	4	PL	3,4720

By Rios's Method.

D^s hor par^x = 5.47
Aux^y Arg^t 5.21
6
3
Aux^y L 5.31

D^s App. Alt^e = 31.18
D^s App. Alt^e = 11.34
Sum^s Alt^e = 42.52 N^o 1 ... 1 2 6 8 9 0 8
Comp Corr^x = 38.35 1 8 6
D^s Corr^x = 48.21
Par^l for Dec^n = 46
Corr^d Sum^s Alt^e 44.39.42 N^o 2 ... 1 7 2 3 3 6 9
6 0

App dist 105.50 N^o 2 2 2 7 2 0 1 5
Ros Dec^n 42 6 8
Corr^d dist 105.20.37 5 2 6 4 6 0 6
True dist 105.21.10
Dist at 3^h 105.09.42
Dist at 6^h 106.32.16
1^st diff 1.22.34 Pl^t = 0,3285
2^nd diff 11.37 Pl^t = 1,1902
A Bericch & Sume 25.20 Pl^t = 0,8517
1^st Dist 3.00.00
App time at 3.25.20 Gr^k

Sun's decl^n = 3.28.34 S
Corr for 3.25.20 R.A. = 3.10
Sun's T. Decl^n 3.25.15 S

Sun's App. Alt^e 31.17.32
Sun's Corr^n = 1.26
Sun's T. Alt^e 31.16.06

Sun's decl^n 3.25.15 S sec^t 10,00077
Colatitude 40.44.10 S cosec^t 10,11745
Hor. Alt^e 53.09.15 N.S 80025
T. Alt^e 31.16.06 N.S 59004
28021 log^m 4,44903
App time at Ship 3.23.34 log rising 4,56725
App time at Gr^k 3.25.20
1.46
X2
5.18
5
T. Long^e 26.30.0

Problem 27th

To find the Longitude of a Ship at sea by observations of ye moon's distance from a known fixed star, & their altitudes, taken at the same time; the latitude of ye Ship, & its longitude by acct being known also.

Rule.

Turn the longitude of the Ship by accot into time & if it be west add it to; but if it be east subtract it from ye estimated time at Ship, when ye obsn was made, & it will give the time at Greenwich nearly.

To this time take ye moon's semidiameter & hor parx out of pp VII of the Nautk Alm. & augment ye moon's semidiameter by adding to it the number of Seconds which stands against its altitude in Tab IV.

Correct the obsd distance by adding to it the augmented semidiameter of ye moon, if ye enlightened limb be that which is nearest to ye Star; or by subtracting the augmented semidiameter of ye moon from it, if ye enlightened limb of ye moon be that which is farthest from ye Star, & the result will be the appt distance of ye star from ye moon's center. Correct also the two altitudes, by subtracting ye dip of ye horizon from each: & by adding the augmented semidiameter of ye moon to or subtracting it from ye moon's

obs^{d} altitude according as its lower or upper limb was obs^{d} & y^{e} app^{t} altitude of each will be obtained.

With y^{e} app^{t} distance & y^{e} two app^{t} altitudes find y^{e} True Distance by Problem 25th.

Amongst y^{e} distances of y^{e} moon's center from the Sun & fixed Stars, put down in pages VIII, IX, X & XI, of the Naut. Alm. find those two distances of y^{e} moon & star which are next less & next greater than the true distance by observation: take y^{e} difference between them, also betw^{n} that which stands first in y^{e} Ephemeris, & y^{e} true obs^{d} distance & subtract y^{e} proportional log of y^{e} former difference from y^{e} proportional logarithm of y^{e} latter, y^{e} remainder will be y^{e} proportional logarithm of a portion of time, to be added to y^{e} time which y^{e} distance standing first in y^{e} Ephemeris was computed for; & y^{e} sum will be y^{e} app^{t} time at Greenwich.

Take y^{e} Star's right ascension & declination out of Tab XII of the Requisite Tables, & correct its app^{t} altitude by subtracting its refraction, taken out of (Tab I) from it. With these & the latitude of the ship, find y^{e} app^{t} time by Prob. XXI.

Take y^{e} difference between y^{e} apparent time at Greenwich & the apparent time at Ship, & convert it into degs, mins, &c. & it will y^{e} True Long^{e} of the Ship at y^{e} time of obs^{n}, east if y^{e} time at Ship be greater than y^{e} time at Greenwich, but west if it be less.

September 17th 1802 about 13h Sat. 19′ 35¾″ I long.e by acc.t 139° 0′ the following distances of Arcet(s) from the D.s F. were taken for finding the F. Longitude. N.B. 1 2 to the dist.s 2 from the D.s Alt.e Edge 4′ 22″.

D.s Alt.e	Alt.e D.s Ll	dist D.s
42′ 36″ 00‴	65° 42′ ∽	45° 20′ ∽
43 06 ∽	65 25 ∽	45 19 ∽
43 14 ∽	65 13 ∽	45 19 ∽
43 22 ∽	65 05 ∽	45 18 30
43 30 ∽	64 30 ∽	45 18 15
43 38 ∽	64 46 ∽	45 18 ∽
259 46 ∽	391 04 ∽	271 52 45
43 17 40	65 10 10	45 18 47
−4 22	−2 00	+2 00
43 13 18	65 08 10	45 20 47
	−4 22	−15 38
	65 03 48	45 05 09
	−15 38	
	64 48 10	

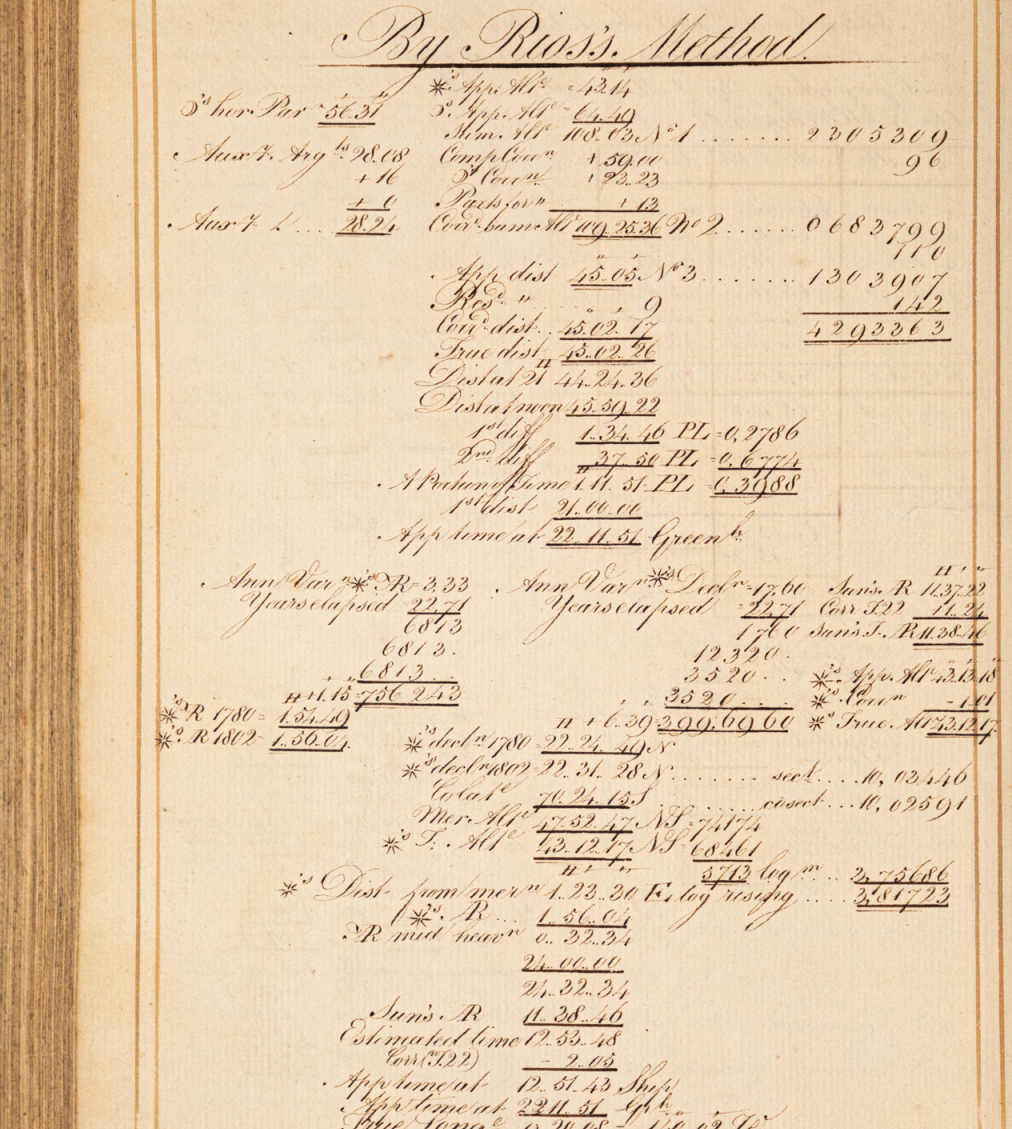
Appt time 13h 00′ 00″
Long.e in time 9 47 40
Green.h time 22 47
D.s Semidiam.r = 15′ 24″
Augmentation +14
D.s Aug.d semidi 15 38
D.s Hor. Par.x = 56′ 31″ Pl. = 10, 5031
D.s Appr. Alt.e = 64 48 10 cos. 9,6291
D.s Par. in Alt.e = 24 04 Pl. = 2, 8740
D.s Refraction = −27
D.s Correction − 23 37
Corr.n 1 01

Appr. Alt.e D.s = 43° 13′ 18″
D.s Appr. Alt.e = 64 48 10
Sum = 108 01 28
diff = 21 34 52
Appr. dist = 45 05 09
1st Corr.n +36
45 05 59
2nd Corr.n −3 36
Corr.d dist 45 02 23
3rd Corr.n +5
T. Dist 45 02 28

half is

54° 00′ 44″	cot	9,86107	
10 17 26	tan	9,28010	
22 32 34	cot	10,38186	
Arc 4 18 26 27	tan	9,52303	
Sum 10 59 01	cot	10,0611	
D.s Appr. Alt.e 43 13 18	cot	10,0270	
F.s Corr.n 1 01	Pl.	2,2481	
1st Corr.n 36	Pl.	2,3362	
Diff. 4 06 07	cot	11,0444	
D.s Appr. Alt.e 64 48 10	cot	9,6775	
F.s Corr.n 23 37	Pl.	1,8820	
2nd Corr.n 3 36	Pl.	1,6989	
Sum 27 13	Pl.	0, 8804	
Diff.ce 20 01	Pl.	0, 9539	
Corr.d Dist 45 02 23	tan	10,0006	
Constant Log.		1,5820	
3rd Corr.n 5	Pl.	3, 3569	

By Rios's Method.



May 6th 1802 about 6h 30' Sat 26° I long by acct 21° 1/4 E
the following distances of the ☽ & ☌ sam ☿ Star Regulus
were obs^d for finding the T. Long. Note 41' 45" to ☿ dist 3' to
the ☽. All ☽ - 1 push ☽ for Errors dip 1' 22".

☽ & ☿	All ☽ & ☿	Dist ☽ & ☿
49. 08. 30	31 27	46. 32
49. 10. 00	31. 46	46. 31. 15
49. 18	30. 52	46. 31
49. 23	30. 42	46. 30. 45
49. 33. 30	30. 28	46. 30. 45
49. 34	30. 26	46. 28
49. 39. 30	29. 55	46. 27. 30
345. 46. 30	215. 17	325. 30. 45
49. 23. 47	30. 45. 17	46. 30. 06
+ 3. 00	- 1 00	+ 1. 45
49. 26. 47	30. 44. 17	46. 31. 51
- 4. 22	- 4. 22	- 15. 12
49. 22. 25	30. 39. 55	46. 16. 39
	+ 15. 12	
	30. 55. 07	

App time = 6h 30'
Long in time = 1. 25 E
App time = 5. 05 nearly.
☽ Semidiam = 15' 04"
Aug^n = + 8
☽ Aug^d semidi = 15. 12
☽ Hor Par^x = 55. 18 Pl. 10,5125
☽ App Alt^e 30. 55. 07 cos 9,9334
☽ Par^x in Alt^e = 17. 26 Pl. 0,5791
☽ Ref^n - 1. 35
☽ Corr^n 15. 57
☽ Corr^n 49"

☽ App Alt^e 49° 22. 25
☽ App Alt^e 30. 55. 07
Sum 80. 17. 32
Diff 18. 27. 18
App Alt 46. 16. 39
1st Corr^n
2nd Corr^n - 29. 54
Obs^d Dist 45. 46. 45
3rd Corr^n + 10
T. Dist 45. 46. 55

halfs

	40. 08. 46	cot	10,07207
	9. 13. 30	tan	9, 21074
	22. 08. 19	cosa	10, 36623
App Alt ☽	49. 22. 25	sin	9,63391
	1. 07. 30	cot	11, 7069
☽ App Alt^e	49. 22. 25	cot	9, 9334
☽ Corr^n	49	Pl.	2, 3432
1st Corr^n	2	Pl.	3, 9835
Sum	17. 25. 18	cot	9, 9635
☽ App Alt	30. 55. 07	cot	10, 2226
1st Corr^n	15. 12	Pl.	0, 5939
2nd Corr^n	20. 32	Pl.	0, 7800
Sum	1. 14. 13	Pl.	0, 3761
Diff	15. 59	Pl.	1, 0516
Obs^d dist	45. 46. 45	sin	10, 0118
Constant log			1, 5820
3rd Corr^n	10	Pl.	3. 0215

By Rios's Method

Remark.

In the two preceding Problems & Examples: the apparent time at the Ship was found from the altitudes of the sun, or star, which was taken at the same time with the distances: but if it should so happen that the sun or star, from which the moon's distance is observed be very near the meridian; or if either through haziness of the atmosphere or badness of the horizon there be reason to suspect that such altitude is not exact enough for that purpose, which may be the case & yet the altitude be sufficiently accurate for the purpose of clearing the observed distance of the effects of parallax & refraction, or if from some local impediment, one or both of the altitudes cannot be observed at all, then the times when those distances, & altitudes were taken must be noted by a watch; & other altitudes either of the sun or a bright star must be taken at a greater distance from the meridian, or when the air or horizon is clearer; & the times noted by the same watch.

By means of these last mentioned altitudes the apparent time at the ship may be found by Prob.s 20, 1.mo or 21.st & of course, how much the watch is too fast or too slow.

Correct the mean of the times when the distances were taken by adding to it what y^e watch was too slow; or subtracting from it what the watch was too fast, & the sum or difference will be the apparent time at the Ship when the distances were observed, from which the longitude may be found as before & such altitude as could not be obtained may be computed by Prob^a 23^d. 24^th. or 25^th.

That the longitude thus found is the longitude of the ship at the instant when y^e altitudes were observed for finding the error of the watch, is obvious: for the time being found at the meridian which the ship was then under, the watch, if it goes right as it is supposed to do will shew the time at that meridian, let the ship be where it will.

Hence, therefore, it is the difference between the time by the meridian of Greenwich, & the time by that meridian which the ship was under when the altitudes were obs^d. which we take for the longitude of the ship, & consequently it must be the longitude of that meridian from the meridian of Greenwich, & not the longitude of the meridian which the Ship was under when the distances were obs^d.

May 14th 1802 about 4h 1/2 Lat 17° 58' S long by acct 18° 3/4 W the following altitudes of the sun's L Limb were observed for finding the app. time N.B. – 1'.5" from the suns altitude for the errors dip. 1'.22"

Times by W	Alt ☉ L.
4h 32m 47s	17° 15' 15"
4 33 09	17 09 –
4 33 35	17 04 15
4 33 55	16 59 15
4 34 14	16 55 15
☽ 22 17 37	☽ 85 23 00
1 33 31	17 04 36
	– 1 05
	17 03 31
	– 4 22
	16 59 09
	+ 15 52
	17 15 01
	– 2 54
	17 12 07

App time = 4h 15' ☉ Ho Sr = 3' 02"
Long in time = 1 13 W ☉ Par = – 8
Gr time nearly 5 30 ☉ Corr = 2 54

☉ decl = 18° 31' 26" N
Corr for 5 30 P.M. + 3 28
☉ True decl 18 34 49 N. sec 10,02325
Colat lude 72 02 00 S. cos 10,02171

Mer. Alt = 53 27 11 NS 80337
True Alt = 17 12 07 NS 29574
50763 log 4,70555
App time at Ship 4 16 21 log rising 4,75031
Time by W 4 33 31
Watch too 0 17 10 fast.

The same evening about $7\frac{11}{4}$ the follg dist^{s} of Regulus from the 3^{d} & Lyr obs^{d} for finding the True Long^{e} N.B. 1.11″ from the dist & 2 from the *alt^{e} for the Errors.

Times Obs^{n}	* Alt^{e}	3^{d} Alt^{e}	Dist^{s} *
7. 31. 56	57. 16. —	46. 45. —	52. 54. 45
7. 34. 07	57. 20. —	47. 10. —	52. 55. 45
7. 36. 11	57. 10. —	47. 48. —	52. 56. 30
7. 37. 52	57. 10. —	48. 05. —	52. 57. 15
7. 39. 31	57. 02. —	48. 34. —	52. 57. 15
7. 41. 16	56. 52. —	48. 56. —	52. 58. 00
45. 41. 03	342. 27. —	287. 27. —	317. 39. 30
7. 36. 50	57. 14. 30	47. 54. 30	52. 56. 35
−17. 10	− 2. 00	− 4. 22	− 1. 11
7. 19. 40	57. 12. 30	47. 50. 08	52. 55. 24
	− 4. 22	−15. 30	−15. 30
	57. 08. 08	47. 34. 38	52. 39. 54

App time	7. 15
Long^{n} in time	1. 13. 16
Gr^{t} Time mean^{n}	8. 30
3^{d} Semidi	− 15. 18″
Aug^{n}	− 1. 12
3^{d} Aug^{t} semidi	− 13. 30
3^{d} Hor Par^{x}	− 36. 00 Pl. 10. 3030
3^{d} App alt^{e}	47. 34. 38 cosi 9. 8209
3^{d} Par^{x} in alt^{e}	37. 52 Pl. 0. 7860
3^{d} Ref^{n}	− 52
3^{d} Colu^{n}	37. 00
* Corr^{n}	37″

* App alt^{e}	= 57. 08. 08		
3^{d} App alt^{e}	47. 34. 38		
Sum	104. 42. 46		
Diff	9. 33. 30	half is	
App dist	52. 39. 54		
1st Corr^{n}	+ 20		
	52. 40. 14		
2nd Corr^{n}	− 27. 03		
Corr^{d} dist	52. 13. 11		
3rd Corr^{n}	+ 4		
T. Dist	52. 13. 15		

	52. 21. 23	cot	0. 88723
	4. 46. 45	tan	8. 91160
	26. 19. 57	cot	10. 30345
Arc	17. 34. 15	tan	9. 11428
Diff	48. 54. 12	cot	10. 46654
* App alt^{e}	57. 08. 08	cot	9. 8103
Corr^{n}	37	Pl.	2. 4652
* 1st Corr^{n}	20	Pl.	2. 7409
Sum	33. 44. 42	cot	10. 1752
3^{d} App alt^{e}	47. 34. 38	cot	9. 9609
3^{d} Corr^{n}	37. 00	Pl.	0. 6871
2nd Corr^{n}	27. 03	Pl.	0. 8737
Sum	1. 04. 03	Pl.	0. 1488
Diff	9. 57	Pl.	1. 2574
Corr^{d} dist	52. 13. 11	tan	10. 1406
Constant Log			1. 8820
3rd Corr^{n}	4	Pl.	3. 3988

By Ross's Method.

S^r Hor Ver^r = 56.06	1^st Appr alt^e 57.09		
	2^d Appr alt^e 47.35		
	Sum alt^s 104.44	N^o 1	2251368
Aug 7. Arg 1 22.51	Comp. Corr^n + 59.23		11
4	2^d Corr^n + 36.55		
0	Parks for^n + 6		
Aug 7. Arg 22.52	Corr^d sum alt^e 96.20.1	N^o 2	07352851
			168
	Appr dist = 52.30	N^o 3	14000551
	Ross^s n^o = 54		282
	Corr^d dist = 52.12.10		13871691
	True dist = 52.13.13		
	Dist at 8^h = 51.10.21		
	Dist at 9^h 52.54.35		
	1^st diff 1.35.14	Pl^s = 0,2765	
	2^nd diff 53.52	Pl^s = 0,3239	
	At Ports for^n time 11.50	Pl^s = 0,2174	
	1^st dist 6.00.00		
	Appr time at 7.41.50	Gr^t	
	Appr time at 7.10.10	Shp	
	L Long^e 0.22.10 =	5.32.30	E^e

April 13th 1809 about 11½ Sat. 13.18 Long by acct 76½ M. Rec. fill'd distances of the D's P.L. from Virgilia's Spike were obs.d & dr. finding the True Long.e Dep.r 4.25

Times by W	* Alt.e	Alt.e P.L.	Dist.s *
8.30.44	33.27.00	68.23	44.21.00
8.52.43	33.57	68.38	44.20.10
8.54.27	34.18	69.13	44.19.00
8.57.38	35.04	69.49	44.18.00
9.00.19	35.44	70.14	44.17.00
44.30.01	172.27	346.47	221.35.10
8.55.12	34.29.24	69.21.24	44.19.02
-1.40.31	- 4.32	- 1.32	-15.07
7.14.21	34.25.02	69.17.02	44.03.55
		+ 15.07	
		69.32.09	

* Hyp alt.e 34.25.02
D.s Hyp alt.e 69.32.09
Sum 103.57.11
D.o 35.07.07
Hyp dist 14.03.55
1st Corr.n + 9.36
& 2nd Corr.n
Corr.d dist 44.13.31
3rd Corr.n + 0
T. dist 44.13.31

halfs

	51.58.35	cot	9,89318
	17.33.33	ta	9,50028
	22.01.57	cot	10,39288
Arc.	1.31.26.33	ta	0,74834
Sum	53.28.30	cot	9,86906
* Hyp. Alt.e	34.25.02	cot	10,16422
* Cor.n	1.23	Pts	2,1143
1st Corr.n	1.17	Pts	2,1481
Dif	9.24.36	cot	10,7806
D.s Hyp Alt.e	69.32.09	cot	9,5719
D.s Corr.n	18.43	Pts	0,6830
2nd Corr.n	8.19	Pts	1,3355
Sum	9.36	Pts	1,2730
Dif	7.07	Pts	1,4081
Corr.d dist	44.13.31	ta	9,9882
Constant log			1,5820
3rd Corr.n	0	Pts	4,2513

Hyp time = 7.30
Long in time = 5.06.0
G.t.e time near = 12.36.6
D.o semidi = 14.52
D.o Aug.n = 1.15
D.o Aug.n semi = 12.0
D.o Hyp Par.x = 54.55 Pts 10,3488
D.o Hyp alt.e 69.32.09 cos 9,5436
D.o Dec.n alt.e 19.05 Pts 10,0749
D.o Ref.n - 22
D.o Cor.n 18.43
* Cor.n 1.22

WILLIAMS 33 1 2 PATENT

By Rios's Method

*Appr Alt 34.26

D° Hor Par 24.33 D° Appr Alt 60.33

Aux ∠ Arg 28.05 Semiall 103.59 No 1 ... 2238100

+ 18 Comp Corr +58.27 73

Aux ∠ Angle 28.23 D° Corr +18.31

Parts for 1 +12

Corr Semiall 105.16.20 N°2 .. 0753283

188

Appr dist 44.03 N°3 1291430

No 1 55 138

Corr dist 44.12.36 1283212

1st Dist 44.13.31

Distat mid 44.31.43

Distat 15 43.01.52

1st diff 1.29.51 Pts = 0,3018

2nd diff 18.12 Pts = 0,0952

Alt Corrected 0.36.28 Pts = 0,6934

1st dist 12.00.00

Appr time at 12.36.28 Gr.

Appr time at 7.44.31 Ship

1st Long 5.22.57 = 80.34.15 W

The next morning about 19h½ the following altitudes of the Sun's L Limb were obsd for finding the appt time long e by acct 77° 3′ 0″

Times by W	Alt ☉ L.L.
21h 13m 00s	19° 10′ 00″
21. 14. 10	19. 54. 30
21. 15. 10	20. 09. 30
21. 16. 10	20. 24. 00
21. 17. 10	20. 38. 30
☽106. 15. 40	☽100. 16. 30
21. 15. 08	20. 00. 18
	−4. 29
	20. 04. 56
	+15. 59
	20. 20. 55
	−2. 24
	20. 18. 31

Appt time 19h 30′ ☉ Rfr. 2′ 32″
Long in time 5. 11 70 ☉ Par. −8
Gr. time 0 41 ☉ Corr. 2. 24
☽ Decl. − 9. 15. 55 N
Corr. for 4h PM +36
☽ Decl. 9. 15. 41 N sec. 10,00570
Colat 76. 42. 00 S cos. 10,01481
Mer. Alt. 67. 26. 19 NS 9,92346
L. Alt 20. 18. 31 NS 24707
57639 log 4,76071
4h 25m 43s log rising 4,77822
24. 00. 00
Appt time 19. 34. 12
Times by 21. 15. 08 W
Watch is 1. 40. 51 fast.

WILLIAMS'S 33 1/2 PATENT LD

Problem 28th

To determine how much a watch gains or looses on mean time in a day or 24 hours.

Rule.

There are many methods of resolving this Problem; but that which follows may be put in practice by every seamen. Sometime before the ship is ready to sail (a month may be sufficient for most voyages) observe several altitudes of the sun's limb, when it is at least 4 points from the meridian, & note the times by the watch when they were observed.

Take the mean of these altitudes & also the mean of the times. Correct the mean observed altde for the dip of the horizon, refraction, parallax, & semidiameter; & with the altitude so corrected, the latde of the place & the suns decln find the app time by Prob 20.

Take the equation of time from p. 2 of the naut. almanack for the time at Greenwich & apply it, according to its sign to the app. time & you will have the mean time at the place where you obs: the diff between which & the mean of the times shewn by the watch, when the observations were made will be what the watch was too fast or too slow, for mean time.

Immediately before the ship sails, repeat these observations, & find again, how much the

watch was too fast, or too slow for mean time.

Take the difference between the mean time of the first obsn, & the mean time of 2d second; also the difference between what the watch was too fast, or too slow, at these two times, & the latter difference will be the loss or gain of the watch, in the interval between the observations. Then as the interval between the observations, is to one day, so is the loss or gain of the watch, between the observations, to its loss or gain, in one day.

To obtain the rate which a watch goes at with greater certainty, observations may be made on more days than one, both at the end & beginning of the interval; or more than one set of observations may be taken on the same day, if opportunities offer, & the result of each set of observations, made at the beginning of the interval, must be compared with the result of each of those taken at the end of it; by which means several daily rates will be obtained which may differ a little one from another.

In this case the mean of them all must be taken for the true rate which the watch goes at.

April 6th 1802 at Gardners Island in Lat 41.5 N
Long.e 175.1 W the following Obs.ns were made for finding the Error
of the watch, height of the eye 21 Feet.

Times by W	Alt.s ☉ L.L.
19h 13m 10s	19° 50′ 00″
19. 14. 10	19. 57. 30
19. 15. 10	20. 00. 30
19. 16. 10	20. 24. 00
19. 17. 10	20. 38. 20
3 19. 15. 50	3 100. 46. 30
19. 15. 10	20. 09. 18
	−4. 22
	20. 04. 56
	+16. 01
	20. 20. 57
	−2. 25
	20. 18. 32

☉ Semidiameter 16′ 01″
☉ Ref.n = 2. 33 Estim.d time 19h 15m 10s
☉ Par.x = −8 Long.e in time 11. 40 W
☉ Corr.n = 2. 25 Gr.t time 7h 6. 36 P.M.
☉ decl.n for noon = 6° 40′ 06″
Corr.n for 6h 59m 30s P.M. + 6. 36. 32
☉ L. Decl.n 6. 46. 42. 32 N sec.t 10.00305
Colatitude 72° 30′ 10″ cos.t 10.02167
Mer.l Alt.e 65. 16. 17. 08 } 8.89829
L. Alt.e 20. 18. 32. 08 } 34702
36177 log 4.55917
4h 24m 13s 16th rising 4.77380
24. 00. 00. 00

Appt time 19 35 46 44
Equation of 12 14 40 time
Mean time 19. 38. 01. 24
Times by W 19. 15. 10. 00
Watch too 22. 51. 24 Slow

And May 14th 1802 the following observations were made Error of the quadrant -1.05

Sumas by W	☉'s Dist. ☽'s ☽
4h 32' 44" 30'''	17° 15' 15"
4. 33. 09. 00	17. 09. 00
4. 33. 35. 30	17. 04. 15
4. 33. 55. 00	16. 39. 15
4. 34. 14. 00	16. 55. 15
22. 47. 38. 00	85. 23. 00
4. 33. 31. 36	17. 04. 36
	-1. 05
	17. 03. 31
	-4. 22
	16. 59. 09
	+15. 52
	17. 15. 01
	-2. 54
	17. 12. 07

☉'s Semidiameter 15' 52"
☽'s Refraction = 3. 02
☽'s Parallax = -8
☽'s Correction 2. 54
Estim'd Time = 4h 33' 31" 36'''
Long'd by time = 11. 41. 08. 00 W
Gr'n time = 16. 14. 39. 36
☉'s Decl'n for noon = 18° 45' 50" N
Corr'n for 7h 45' 20" 7''' Am = -1. 46 38
☉'s True Decl'n 18. 46. 03. 32 N sect ... 16.02353
Colat 72. 03. 00. 00 S cosec 10.02167
Mer alt'e 53. 21. 36. 28 } N 2 80246
True alt'e 17. 12. 07. 00 } 29571
30672 log 4.50477
App'e time = 4h 16' 16" 23''' log rising 4.76907
Equation of time -3. 58. 50
Mean time 4. 12. 11. 33
Sumas by W 4. 33. 31. 36
Watch too fast 21. 20. 03 2nd obsn
Watch too slow 22. 51 24 1st obsn
Watch gains 44. 11. 27

Mean time 1st obsn D 19h 38' 01" 24'''
Mean time 2nd obsn 38. 04. 12. 11. 33
Interval betn the obsns 37. 04. 34. 10. 09

Theor. As 37d 04h 34' 10" 09''' ..... 1d ..... 44' 11" 27''' : 1' 11" 12'''

Ans. The watch gains 1' 11" 12''' P. Day

Problem 20th.

To find the Long.e at sea by a watch.

Rule. Obs.e the alt.e of the sun's lower limb, either in the morning, or evening, when y.e sun is at least 3 points of the compass from the meridian, & note the time when it was obs.d by the watch. Multiply the daily rate of the watch by the number of days which have elapsed since that on which the last obs.n was made for finding it, & add the product to the time shown by the watch when the sun's alt.e was obs.d; if the watch be losing, but subtract it from that time if the watch be gaining; & to the sum, or diff.ce add what the watch was too slow, or subtract from it what the watch was too fast for mean time at the place departed from, on the day when the last obs.n was made for finding its rate, & the result will be the mean time at that place when the sun's alt.e was obs.d.

To this time add the long.e of the place in time, if it be west, or subtract the long.e in time from it if it be east, & the sum, or rem.r will be the mean time at Greenwich. Correct the sun's alt.e for dep.e of the horizon, refraction, par.x, & semidiameter; with which, y.e lat.e of the ship & the sun's decl.n find the app.e time at the ship by Problem 20. Take y.e equation of time from (p.2) of the Naut.l Alm.c with y.e time at Gr.k & apply it to y.e app.e time according to its sign, & it will give the mean time at y.e ship; y.e diff.ce between which & y.e mean time at Gr.k will be y.e long.e of y.e ship in time if y.e time at ship be greater than y.e time at Gr.k, but W. if it be less

June 27th 1802 Latitude 26° 59' S the following Obsvns were made for finding the Longitude by a Watch that had been set 2h 55m 55s 21 too slow for mean time at Portsmouth Jan 14th 1802 & was then losing at the rate of 0s33 a day on mean time

Times by W	Alt ☉ L.L
19h 17' 06"	24° 23' 30"
19. 48. 02	24. 35. 45
19. 18. 18	24. 40. 15
19. 19. 38	25. 01. 00
19. 20. 12	26. 13. 15
5)96. 23. 46	5)123. 53. 45
19. 18. 45. 12	24. 46. 45
	− 4. 22
	24. 42. 23
	+ 15. 47
	24. 58. 10
	− 1. 34
	24. 56. 16

Jan^{y} = 17
Febr^{y} = 28
March = 31
April = 30
May = 31
June = 26
No^{o} of 163 Days
Daily 0.33 Fast
489
6
3270
Watch 0. 53. 47. 22 lost since Jan 7th 14h
Times by 19. 18. 46. 12. 00 Watch
19. 19. 38. 59. 24 Diff
Watch 2. 35. 55. 12. 36 Loses
Mean 22. 18. 34. 12. 00 Time at Portsmouth
Long^{e} in + 4. 25. 00. 00 Time
Mean 22. 19. 59. 12. 00 Time at Gr^{n}

☉ Semidi 15' 17"
☉ Ref^{n} 2. 02
☉ Par^{x} − 8
☉ Coun^{e} 4. 54

☉ Decl^{n} for noon = 23° 19' 51" S
Corr^{n} for 19h 18' 46" + 12
☉ True Decl^{n} 23. 20. 03 S sec^{t} 10. 03706
Latitude = 26. 11. 00 S cos^{t} 10. 04341
Mer^{t} Alt^{e} 39. 50. 57 N 64078
True Alt^{e} 24. 36. 70 42763
h ′ ″ 21015 log 4. 32407
2. 31. 35. 26 log rising 4. 42721
24. 00. 00. 00
App^{t} time at 21. 08. 24. 34 Ship
Equation of Time + 2. 35. 50
Mean time 21. 11. 00. 24 at Ship
Mean time 22. 19. 59. 12 at Gr^{n}
1. 08. 58. 48
×5
5. 44. 54. 00
×2
True Long^{e} 17. 14. 42. 00 W

Of the Azimuth Compass.

The Azimuth or Amplitude Compass differs from the common sea compass in the following particulars.

1st. The circumference of its card is divided into degrees & half degrees as well as into points.

2nd. A Box of brass which carries two sight vanes one at each end, fits on to the top of the circular brass box to which it is a diameter. The sight vanes are two upright pieces of brass, with a slit down the middle of each: the slit in that which the eye must be applied to, is very narrow; but that in the other is pretty wide & a thread is stretched up the middle of it parallel to its two perpendicular sides, which thread passing through a small hole at the top is carried across from the top of that vane to the top of the vane which is nearest to the eye parallel to a line, drawn on the index from the bottom of the slit, in the eye vane to the bottom of the thread which bisects the other.

3rd. A Piece of brass about half an inch broad, & in length somewhat less than the depth of the compass box is fixed in an upright position close to one side of it.

This piece turns on a joint at the bottom of the Box; & toward the top of it, is filed concave on the inside to fit the Edge of the Card when it is brought into

contact with it, seven parallel & equidistant lines are drawn down the concave side of this piece at such a distance from each other that the six spaces contained between them are exactly equal to five half degrees, on the Circumference of the Card. Those lines are numbered 0, 5, 10, 15, 20, 25, & 30 & serve as Vernier's Divisions; to subdivide the circumference of the Card into arcs of five minutes each: that line which is marked 0, is placed exactly at right angles to the line which joins the two sight vanes.

A Pin is fixed to the outside of the piece those lines are drawn on, which passing through a hole in the side of the box, turns upwards until it rises a little above the brim of the box, so that, by means of it the piece may be pushed by the finger of the observer, against the edge of the Card when he sees the thread in the remote sight vane exactly on the center of the object which he is observing & the pin being kept against the edge of the card will prevent its motion until the results of the obs^{n} is read off.

The use of the Azimuth Compass is to observe the bearing of objects from the ship & the Azimuth and amplitudes of the sun moon & stars for the purpose of determinating the var^{n} of the compass for it has been found by experience, that there are few places on the Earth where the north point of the compass points exactly to the true north point of the horizon neither is this variation the same in all places nor yet in the same place at all times but is continually changing.

Problem 30th

To observe the bearing of any object

Rule. Turn the circular box which carries the sight vanes round on the square one until you see the object through the narrow slit in one of the sight vanes exactly on the thread which bisects the broad slit in the other.

When this can be done push the piece which is in the side of the box against the edge of the Card & read off the deg.s & minutes that stand on the 1st or principal line on the stop & it will be the bearing required. care must be taken to annex the proper letters to denote the quarter of the Horizon it is in.

Problem 31st

To observe the amplitude of a celestial object

Rule When the center of the object is about a diameter of the sun above the visible horizon turn the circular box & sight vanes round in the square one until you see the center of the object through the two sight vanes exactly on the thread which bisects the slit in the more remote one. When this can be done push the stop in the side of the box against the edge of the card & read off the deg.s & minutes that stand against the 1st or principal line on the stop & it will be the amplitude req.d care must be taken to annex the proper letters to denote the quarter of the horizon it is in.

Problem 32nd

To observe the azimuth of a celestial object

Rule. Turn the circular box with the sight vanes until you see the center of the object through the narrow slit which is in one of the sight vanes exactly on the thread which bisects the slit on the other or until the shadow of that thread of the object be the sun falls exactly along the line which is drawn on the

index from the bottom of one sight vane to the bottom of the other. When either of those happens push the piece which is at the side of the box against the edge of the card & read off the deg.s & minutes that stand by the principal line on the ship. It will be the azimuth be req.d care being taken to annex the proper letters & denote the quarter of the Horizon it is in.

At the instant when the observer pushes in the stop another person must observe the altitude of the objects by the rules which have already been given for that purpose.

Of the Variation of the Compass.

The Variation of the compass is the distance in deg.s & bet.n the true north point of the horizon & that point which the north end of the compass, or needle points to.

The Variation is east, or west, according as the north end of the magnetical needle, falls east, or west, of the true north point of the Horizon: Or. The Variation of the compass is equal to the diff. between the true amplitude or azimuth of any celestial object & its amplitude or azimuth observed by the compass.

And Consequently. The variation may be found by comparing the true & magnetical amplitudes or azimuths of celestial objects.

Problem 33.rd

To find the var.n of the compass by an amplitude.

Rule. Observe the magnetical amplitude. Work the log from the preceding noon to the time when you made the obs.n & find by that means the lat.d of the ship.

Take the sun's decl.n from p.g 2 of the Naut. Alm.c allowance for the time of the sun's rising or setting & also for the longitude of the place. If the object be a star its declination must be taken from (p.g 57) of the requisite tables.

Shew. As the cosine of the latitude
To the radius
So is the sine of the decl.n
To the cosine of the true amplitude.

Or. Add the secant of the lat.e to the sine of the declination & the same will be the cosine of the amplitude from that point of the horizon which is of the same name with the decl.n

By the Gunter Scale.

Extend from the sine comp.t of the lat.e To radius
That extent will reach from the sine of the decl.n
to the sine comp.t of the true amplitude.

If the true amplitude be not reckoned from that point of the horizon which the magnetical amplitude was observed from, take the true from 180° & they will be reckoned from the same point.

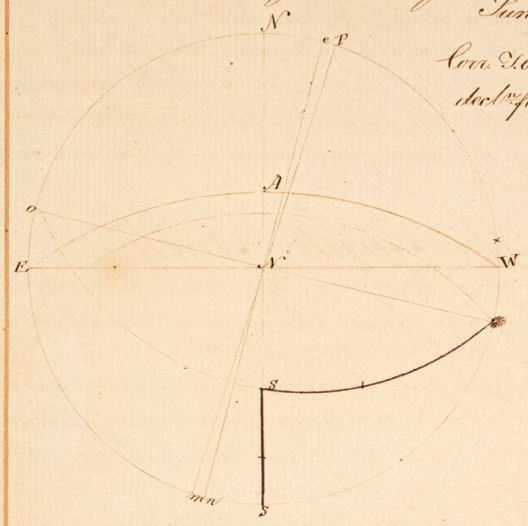
Take the diff.ce bet.n the true & magnetical amplitudes & it will be the variation of the compass east of the point which terminates the true amplitude if to the right of that which terminates the magnetical. but if west of it lie to the left of it.

October 14th 1792 in lat 37°.19' N. long 56°.10' the sun's setting amp.de was N86.15°W by the compass. What was the variation.

Sun's decl.n at noon at Q.h 8°.30'.46".8

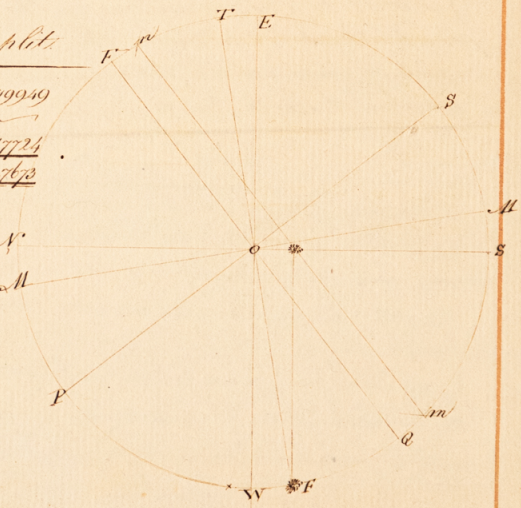
Corr.n to { for long 56°W. = + 2.. 2
for 6h.24' P. M. + 5.. 48

decl.n for 6h.24' P. M in long 56W 8.39..36.8



To find the True Ampl.e

As co-sine lat.e	= 37°.19'	9.99949
To to radius	= 90.00	10.
To sine of decl.n	= 8.39.36	9.17724
To co-sine true amp.e	S79.5.36W.	9.27673
	180.00.00	
	N 100.54.24 W.	N.
Magn.c amp.e N.	86.15.00 W.	
Var.n of the comp.s	14.39.24 W.	M



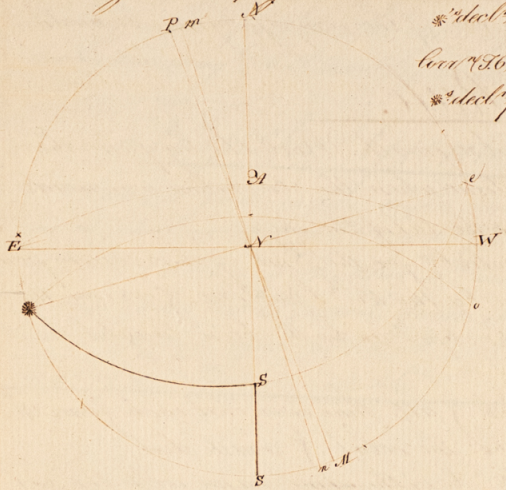
April 25th 1792 latit 28° 21 S long 174 E ☉ sunś rising amplitz was N 86. 15 E by the compass what was the variation.

☉ decl^{n} mean at Gr^{h} = 13° 49′ 39″ N

corr^{n} for 17½ long^{e} = − 9. 41

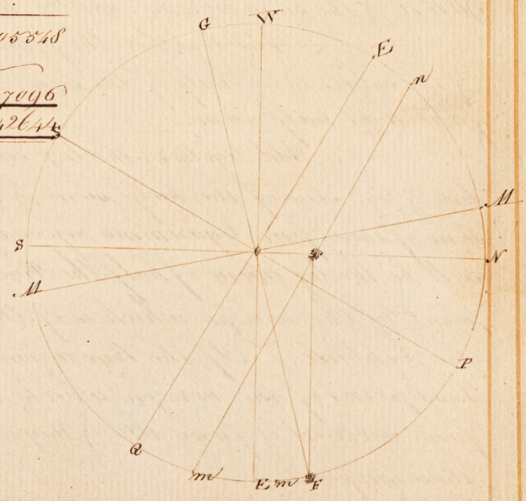
corr^{n} for 5 h. 36′ A. M = − 4. 41

☉ decl^{n} for 5 h. 36′ A.M long 174½ 13. 35. 17 N



To find the true amp.

Asces lat	= 28° 21 . . .	0,05548
Sec rad	= 90. 0 . . .	10,
Sec sine decl^{n}	= 13. 35. 17 . . .	9,37096
Sec sine true amp N 76. 21 E . .		9,42644
Mag^{l} amp N 86. 15 E		
Variation 11. 54 W		



Problem 34th

To find the varn of the compass by an Azimuth.

Rule.

Observe the magnetical azimuth. Work the log from the preceding noone to the time when the observation was made & find by that means the lat of the ship.

Take the suns declin for the time of observation from p 2d of the Naut. Alm. by the 11th Problem, & correct the observed altitude of the object as hath been taught in finding the latitude.

Take the lat. & alt. thus corrected each from 90°, & the two remrs will be the colat & zenith dist.

If the declin be of the same name with the lat take it also from 90° but if they be of different names add 90° to it & the sum or remr will be the Polar Distance.

Add together the polar distance zenith dist. & the colat, take half of the sum & subtract the polar dist from that half sum.

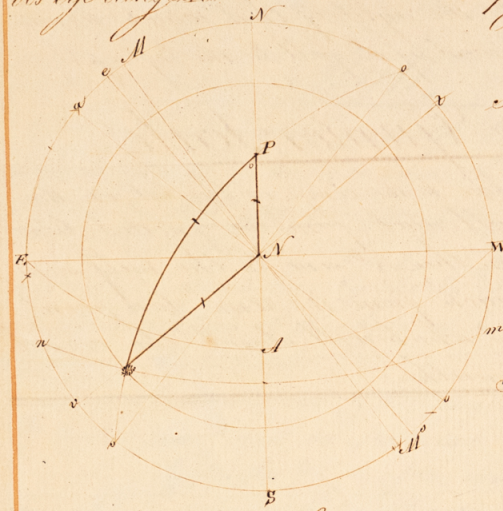
Add together the log secants of the colat & zenr dist. & log sine of the half sum & remainder; half the sum of these four logarithms rejecting 20 from the index will be the log cosine of half the true azimuth, from that point of the horizon which is of the same name with the latitude. If the true azim be not reckoned from that point of the horizon which the mag. azim. was obd from subtract it from 180° & they will be reckoned from the same point.

Take the diff between them & it will be the variation of the compass east if the point which terminates the true azimuth fall to the right of that which terminates the mag.ᵉ; but west if it fall to the left of it

By the Gunter Scale.

Extend from rad.ᵈˢ to the comp. of the lat. on the sines that extent will reach from the zen. dist. to a 4.ᵗʰ sine. Extend from this 4.ᵗʰ sine to the half sum & this extent will reach from the rem.ᵈᵉʳ to a point on the sines, against which will stand the versed sine of the true azimuth

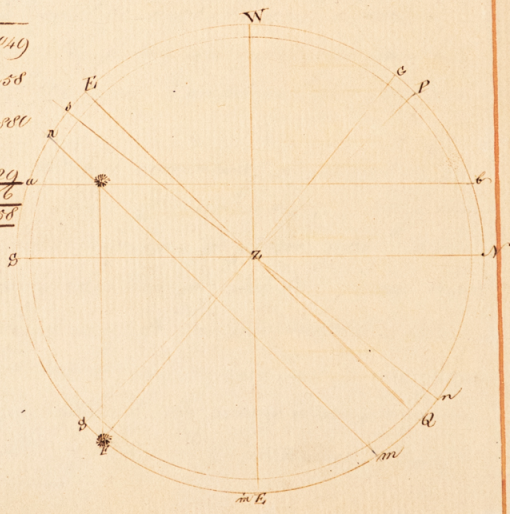
October 26th 1792 lat 42° 59 N long 12° 54 W the obs^d azim. of the sun's center was 88° 19 E when his obs^d alt & L L was 17° 19 the height of the obs^r eye being 22 Feet What was the var^n of the compass.



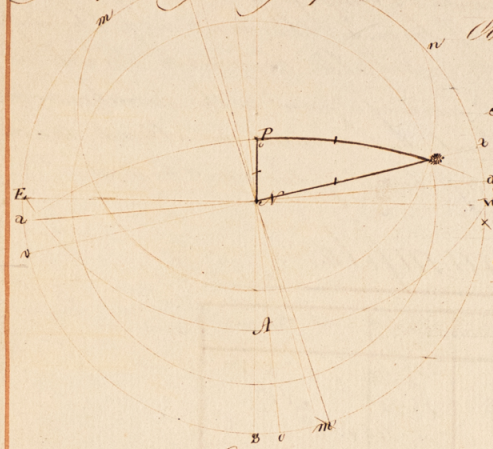
Obs^d alt ☉ L L = 17° 19' 00"
Dip . . . . . - 4. 22
Refraction . . . - 2. 02
Semidiam. . . . + 16. 09
Parallax + 8
True alt 17° 27' 47"
90. 00. 00
Zen. Dist 72. 32. 13
☉ decl. n. at 6½ = 13° 07' 46" S
Corr for long 12° W = + 6. 50
Corr for 2h 58'. 1. 11 - 2. 27
m. S the decl^n = 13. 12. 69 S
90. 00. 00
Polar Dist = 103. 12. 09

To find the true azimuth

Zen dist	= 72° 32' 13"	cosec^t	10, 020 49
Polar dist	103. 12. 09		
Colat	47. 11. 00	cosec^t	10, 134 58
Sum	222 55. 22		
½ Sum	111. 27. 41	sine	9. 9 6880
Polar dist	103. 12. 09		
diff	8. 15. 32	sine	9. 157 29
			19. 281 16
	64. 4. 51	cosine	9. 640 58
	2		
True az	N 128. 09. 42 E		
	180		
	S 51. 50. 18 E		
Mag^t S	87. 10. 00 E	Azim.	
Var^n	35. 28. 42 E		



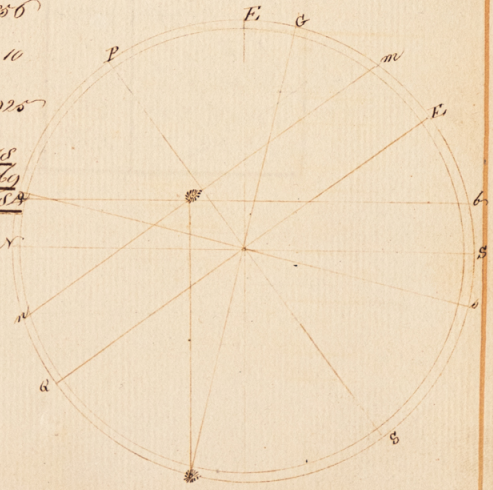
August 2nd 1792 lat 54° 19' N long 15° 52' E the obs. azim. of the sun's center was S 85° 06' W when the obs. alt. of his L.L was 11° 13' 15" height of the eye being 20 feet what was the var. of the compass



Obs. alt. ☉ L.L	= 11° 13' 15"
Dip	= −4. 16
Refn	= −4. 43
Semidiam	= +15. 49
Par.	= + 0
True Alt.	11. 20. 14
	90. 00. 00
Zen. dist	78. 39. 46
☉ decl. at Gr.	= 17° 23' 26" N
Corr. for long 15° E	= + 6. 54
Corr. for 6h 20' P.M	= −4. 18
True decl.	17. 35. 56
	90. 00. 00
Polar Dist	72. 24. 04

To find the true azim.

Zen. Dist	= 78° 39' 46"	cosec 10.006856
1st Dist	= 72. 24. 04	
Colat	= 35 41 00	cosec 10.234410
Sum	= 186. 44. 50	
½ sum	= 93. 22. 26	sine 9.999925
1st Dist	= 72. 24. 04	
Diff.	= 20. 58. 21	sine 9.553378
		19.794569
	37° 46' 42"	cose 9.897784
	42	
T. Azi	N 75. 33 24 W	
	180	
	S 104. 26. 36	
Mag	S 85. 06. 00 W azim	
Varn	19. 20. 36 W.	



Problem 35th

To correct the courses by compass for Variation

Rule. Suppose yourself looking the same way the ship sails then if the var.n be east, alter the course steered as many degs or points & parts of a point as the variation amounts to towards the right hand, but it must be altered the contrary way towards the left hand if the var.n be west.

Examples

Course by Com	Var.n	T. Course
N E ½ E	P/A 2½ W	N. N E
S S E ½ E	1½ W	S E
East	½ W	E b. N ½ E
E S E	1½ E	S E. ½ E
W. b. S.	2 E	N. b. N
S W ½ W	2½ W	S S W
N. b. W	1 E	North
E b. N	2 E	E. b. S
N E. b. E ½ E	¼ E	N E b. E ¾ E
N W b. N	2 W	N W. b. W

The Journal of a Voyage from England towards Madeira 1772.

Winds	Remarks.
N.W.b.W N.E N.W.by.N	4 July 2nd. At 2 P.M the Pilot came on board; ordered the ship to be cast loose from the shear hulk at Deptford & made sail down the river. At 5 P.M made fast to the shear hulk at Woolwich, the wind not permitting us to proceed farther. At 4 A.M cast loose from the hulk & at 10 anchored in Long Reach; moored with a cable each way.
N.W	4 July 3rd. Employed taking the gunner's stores on board. Read the articles of war to the ship's company & punished Jonas Sturdy with 12 lashes for drunkenness & disobedience. Fine weather & moderate wind.
W.b.S	5 July 4th. Light airs & fine weather. At 4 A.M unmoored & sailed down the river to Gravesend. At noon anchored with the small bower in 6 fathoms.
N.N.E	6 July 5th. At 6 A.M weighed & turned to windward down the reach & at noon anchored in the Hope in 7 Fathoms. Fine weather & moderate wind.
Westerly	7 July 6th. Brisk wind & cloudy weather. At 6 P.M weighed & sailed & at 8. At 10 anchored with the best bower at the Nore in 9 Fathoms. At 7 weighed & sailed.
	8 July 7th. At 3 P.M came to in Margate

road with the best bower. At 8. AM weighed & made sail Brisk wind & hazy weather

Variable Westerly	3 July 8th. At 3. PM came to an Anchor in ye Downs in 7 Fathoms & moored with a cable each way. The town of Deal bearing N.W. N. & ye S. Foreland S.W.b.S. sent the pilot on shore. Moderate wind & cloudy with showers.
N.W. N.W.	4 July 9th. The former part of these 24 hours brisk wind & clobady weather the middle strong gales with rain. At 2 PM got down the top gallant yards. At 3 struck ye yards top masts & top gallant masts. At 12 AM. exceeding strong wind with rain let go ye sheet anchor under fore.
N.W.	2 July 10th. More moderate. PM hove up ye Sheet anchor & got up ye Yards & top masts AM overhauled ye rolying & set it up. Cloudy weather with some rain.
North.	1 July 11th. Brisk wind & flying clouds. PM Stowed the Boats cleared ye Decks & made all ready for going to sea. At 6. AM unmoored. At 8 weighed & sailed & at noon the S. Foreland bore N.W. dist 5 miles.

H.	H.	F.	Courses	Winds	Remarks on Thursday July 12th 1770
1	1	2	S S W ¾ W	N ½ W	Little wind & fine weather
2	1	2			Stowed the Anchors. Set top gallant
3	2				sails & steering sails.
4	3				The South Foreland. North.
5	3	2	W ½ N		
6	3	2			
7	3				Took in Steerg sails & top gallant sails
8	3			North	Moderate wind & fine weather.
9	2	6			
10	2	6			
11	3	2			
12	3		West	N.E.	Ditto weather.
1	3				
2	3	2	W W W ½ W		Fairlight point N.E. 1 about 4 miles
3	3	2			
4	3	4			Moderate wind & clear weather.
5	3	6			
6	4	2			Beachy head N.W. about 6 miles
7	5	4	W ½ N		
8	6	4			Brisk winds & cloudy
9	7				Took in a reef of each Topsail
10	7	4			
11	8				
12	8	4			Bembrige Point N W 5 leagues.

Hs	K	F	Courses	Winds	Remarks on Thursday July 13th 1772.
1	7	6	W N W	N.E.	Fresh gales & flying clouds
2	7	6			Dunnose W.N.W. about 7 miles
3	7	5	West		
4	7	4			Ditto Weather
5	7	4		North	
6	7	4			
7	7	6			The bill of Portland N.W. 6 L.
8	7	4	W by W W½ W	N.E.	Shortened Sail.
9	6	2			
10	6				
11	6				
12	6				Weather as above
1	5	4			
2	5	4			
3	5	2			
4	4				Moderate wind & clear
5	4				
6	4	2			The Start W.N.W. 4 leagues.
7	6				Let out all reefs
8	6	4			Brisk wind & fine weather.
9	6	4	W by W W½ W	N.N.E.	
10	6	4			
11	6	5			
12	6	5			Mer. Alt. ☉ ☽ L.L. 61° 28′ 30″
					Ramhead N.E. ¾ E.

Course	D	Dlat	Dep	Lat in	Long made	Bear & dist at noon
S 31° W	13	12½	7½	50° 5½ N	0. 15′ W	Ram ¾ N.N.E. ¾ E. 5 leag.

H.	R.	F.	Courses	Winds	Remarks on Sunday July 14th 1772.
1	5	6	S W.	W N W	Brisk wind & clear
2	6				
3	6				
4	6				Wind as above: cloudy.
5	5	2			
6	3	4	S W ½ S	W N W ½ W	
7	3	4			
8	3	4	S W b S	W b N	Moderate wind & cloudy
9	3				
10	2	2			
11	2	2			
12	1	6	S W b W.	N W b W.	Foggy, & drizzling rain.
1	2				
2	1	6			
3	2				
4	2				Ditto weather
5	2				
6	2				
7	1	6			
8	1	6			Sucked Ship. Weather as above.
9	2		N W. ½ N	W S W ½ W	
10	2				
11	1	6			
12	1	2			Light breezes & foggy.

Course	Dist	Dlat	Dep	Latin	Long made	Bearg & dist at noon
S. W	59	58	11½	49 7 N	W. 29½ W	Rum N 75 E dist 24

Hs	Kt	F	Courses	Winds	Remarks on Tuesday July 15th 1772.
1	2	4	N W b W	S W b W	Settle wind & foggy weather
2	2				
3	2		N W	W S W	
4	2				Weather ditto.
5	1	6	N N W	West	
6	2				
7	1	4	N W b W	S W b W	
8	2				Tucked Ships. Thick fog
9	2	4	S S W	West	
10	2	4			
11	2	4	S W b S	W b N	
12	2	4			Weather as above
1	1	6			
2	1	4			
3	1	4			
4	1	6	W S W	N W	Ditto weather
5	1	2	W b S	N W b N	
6	2				
7	1				
8	1	4			Very hazy little wind
9	2	2			
10	1	7	W S W		
11	3		West	North	
12	4				Wea. D. mer. alt ☉ 2 LL. 62. 25 S.

Course	Dist	Dlat	Dep	Latin	Long made	Bear & dist at noon
S 53 W	29	17.	23	48. 50 N	7 5½ W	Rum N 34 E dis 68 L

H	K	F	Courses	Winds	Remarks on Thursday July 16th 1772.
1	3	6	W. 6. S	North	Moderate wind & hazy
2	4	2			
3	6		W S W		A great swell from y^e N N E.
4	5				Ditto weather
5	5				Shortened sail
6	5				
7	4				
8	3	4			The weather as above
9	3	4			
10	4				
11	5				
12	5			N E	Ditto weather
1	5				
2	4	4			
3	4				
4	4				Moderate wind & cloudy
5	4				Sun rose by compass N 79° 30' E
6	4				Variation 22° 16' W
7	3	5			Set y^e top gallant sails.
8	3	5			Moderate wind & fine weather
9	3				Set y^e shearing sails.
10	2	4			
11	3	3			The swell continues from y^e N N E.
12	3	3			Dit. weather meridl. Alt. ☉ 62° 37' S

Course	Dis	DLat	Dep	Latin	Long. made	Bear^gs dist at noon
S 41° W	108	81	71	47° 28' N	2° 51' ½ W	Ramh N 3° E d. 681

Hs	Hs	F	Courses	Winds	Remarks on 2 July 17th 1772.
1	4		W. S.W.	N.N.W.	Moderate wind & clear. Hauled down
2	4	2			under steering sails
3	4	2			
4	4	2			Ditto weather
5	4				
6	3	2	S.W.b.W.	N.W.b.W.	Hauled down fore top mast steer sails
7	4	3			Clued up & handed top gallant sails
8	4	6			Weather as above.
9	4				
10	4				
11	3	4			
12	3				Moderate wind & cloudy
1	3	4	S.W.b.S.	W.b.N.	
2	3	4			
3	4				
4	4				Squally weather. Tacked ship
5	4		N.N.W.	West	
6	3	2			
7	3				
8	3				Brisk wind & cloudy. Tacked
9	3	4	S.by W.	W.by S.	
10	4	4			
11	4				
12	4				Very hazy. Mer alt ☉ LL 64. 30 S

Course	Dis	DL.	Dep	Lat in	Long made	Bearg & dist at noon
S.19°W.	67	58	20	46.25 N	3.23.30 W	Ramsh N 30 E dist 96 l.

Hs	Ks	F.	Courses	Winds	Remarks on y 9th July 1772
1	4	5	South	W S W	Moderate wind & hazy
2	4		S b E	S W b W	
3	3	2			
4	3		South	W S W	Ditto weather
5	3	2			
6	4		S S E	S W	Az. ☉ center by comp. N 53° 56' W
7	3		N W b N	W b S	Alt ☉ his ☾ 15° 56' var. obs. 22° 12' W
8	3				Weather as above
9	2		N W	W S W	
10	2				
11	1	5			
12	1	4			Rainy, unsettled weather
1	2	2	N W b N	W b S	
2	2	2			
3	2				
4	2				Squally, with rain
5	2	4			
6	3				
7	3				
8	2	6			Weather as above
9	2	1			
10	2	3	N N W	West	
11	2				
12	1	6			Foggy, with rain

Course	Dis	DL	Dep	Lat in	Long made	Bear & dist at noon
N 32° W	19	16 1/2	10	46° 41 1/2 N	3° 38 1/2 W	Ram-h N 33 1/2 E dist 87

Ho	R	F	Courses	Winds	Remarks on July 19th 1772.
1	5	7	W S W.	N W.	Brisk wind & cloudy
2	3	4	S W. ½ W.		
3	2				
4	1	2			Moderate wind & cloudy weath.
5	1	2			
6	3	3	S W. b. S		
7	4	5			
8	4				The weather as above
9	4				
10	4				
11	4				
12	4				Ditto weather
1	3	4			
2	3	6			
3	3	4			
4	3	6			Ditto weather
5	3	6			
6	4				
7	3	5			
8	3	3			
9	3	6	S W. ½ W.		Brisk wind & cloudy
10	3	3			
11	3	3			
12	4	2			Very hazy. Meridl. ☉ L.L 65. 15 S

Course	Dis	DL	Dep	Lat in	Long made	Bear & dist at noon
		7¼	43¼	45. 20 N	4. 44½ W	Rumb N 33 E dist 118 L

Hs	K.	F	Courses	Winds	Remarks on Friday July 26th 1772.
1	4	4	SW.b.W.	NW.b.W	Moderate wind & clear
2	4	4			
3	3	4			
4	1	4			Light airs intermixed with calms.
5	1	2	SW	WNW	Tried the current, which set W.b.N. ½ N. ¾ of a K. an hour.
6	2	4			
7	3	4			Brisk wind & cloudy: took in a reef of each top sail.
8	3	5			
9	3	4	SW.b.W.	NW.b.W	
10	3	6			
11	2				Took in 2nd reef of the top sails.
12	3		SW	WNW	Squally, with heavy rain
1	2	5	SW.b.S	W.b.N	Handed the mizen top-sail.
2	2	5			Close reefed the fore top-sail.
3	2	3			
4	2	4			Squally, with showers
5	3				
6	3	4	SSW	West	Close reefed the main top-sail
7	4				
8	4				Heavy squalls of wind & foggy
9	3		South	WSW	Saw Cape Orlaybal S.b.E. about 8 leag.
10	2	6			
11	2				Handed the top sails
12	2	2			Strong wind in squalls.

Course	Dis	DL	Dep	Lat in	Long made	Bear & dist at noon
S.b.W	68	67.2	11.3	44. 13 N	5. 1 W	Ram-h N 39 E d. 13 L

Hs	Rs	Fs	Courses	Winds	Remarks on 8 July 21st 1772
1	2		S.W.	W.S.W.	Strong wind & foggy
2	2				
3	2				Wore ship C. Ortegal S.W. 5 or 6 m
4	2		N.N.W.	West	Weather as above
5	2				
6	2				Hauled up the mainsail
7	2				Brought to under y mizs stay sail
8	Up. N.W. off N.N.E.			Variable	
9	Up N.W. off N.E.				Wore Ship. Balanced the mizs & set it
10	Up N.N.E. E. off W.S.W.				
11	Up N.N.E. off S.E. & E.				Set the forsail
12	3		W.W.S.W.	East	More & moderate
1	5				Set the mainsail
2	5				
3	5	4			
4	5	3			Moderate wind with raine
5	4				Set the top sails
6	3		W.S.W.		
7	3		West		Set a reef out of the top sails
8	3	4			Moderate wind with raine
9	3				
10	2				Set out all reefs
11	2				
12	2				The Graygn lights S.E. & S. 4 or 5 L.

Course	Dis	DL	Dep	Lat in	Long made	Bear & dist at noon
S 52 W	40	24.7	31.5	43. 48 N	5. 45 W	Ramh N 37 E dist 15 L

Hrs	Knots	Courses	Winds	Remarks on Friday 22nd 1772
1				Calm & cloudy
2				
3				
4				
5		Ships head round 3 points.		
6				
7				
8				Calm & foggy
9	1	S.E.	S.E.b.S.	
10	2			
11	2			
12	1			Little wind & foggy
1	2	N.W.b.N.	Variable	
2	2 6			
3	3			
4	3 2			Ditto & hazy
5	2	N.N.W.	West	
6	2			
7	2	S.S.W.		
8	2			Ditto & fine weather
9	2			
10	2			
11	2			
12	2			Flying Clouds. Meridl. ☉ D 66° 23′

Course	D	DL	Dep	Lat in	Long made	Bear & dist. at noon
		4	4.6	43.35 N	5.56 W	Run in. N 31 E dist 13 Ls

Ho	Mk	P.	Courses	Winds	Remarks on 4, July 23rd 1772.
1	3		W S W	N. N W	Moderate wind & hazy
2	3	6			
3	3				
4	2	6		North	Ditto weather
5	2	4			
6	2	4			
7	2	4			
8	4			N b E	Cape Finisterre S S W. 6 or 7 Leag.
9	3	2	S W		
10	3	6			
11	3	6			
12	3	6			Weather as above
1	3	6			
2	3	4			
3	3	4			
4	3	5			Moderate wind & flying clouds
5	3	6			
6	4				About ½ past 6 A Obs the dist of the
7	4	4			nearest Ly of the ☉ 84. 34. 2⅝.
8	5				Upp. alt ☉ L 16. 13½. Hence the
9	5				Lon. of the ship by the obs[n] is 16. 8 W
10	5				the Lupp alt of the 2[d] ☽ L 39. 60. ½
11	5				Error of the Sext in long 18½ too much
12	5				Mor alt ☉ L 6. 28 ½

Course	Dis	DL	Dep	Lat in	Long[e] in	Bear[g] & dist at noon
S 28° ½ W	89	78	42	42. 18. N	6. 50 W	Ram-h N 30 E dist 185 L

Hs	Ks	Fs	Courses	Winds	Remarks on 24th July 1772
1	6		S W.		Brisk wind & clear weather
2	6				
3	6				
4	6				Weather as above
5	6				
6	6				
7	7				Obs. Azim. of Center N 30° 16' W
8	7				& the alt. of his L.L. 7° 56', var. 20° 09' W
9	6	6			Weather as above
10	6	4			
11	6	3			
12	7				Ditto weather
1	7				
2	6	4			
3	6				
4	6				Ditto weather
5	6				Meas'd ½ past 6. 4. Mob. dist. Comb's sun
6	6				& ☽ 75° 13' 54" alt. ☽ L.L. 20° 28' ⅔ alt. of
7	6				☽ U.L. 69° 41' ½. Long. dist. 11° 25' ¼ W.
8	6				Brish wind & clear weather
9	6				
10	6				
11	6				
12	6				Merid. alt. sun's L.L. 69° 31' S

Course	Dis	D Lat	Dep	Lat in	Long made	Bearg & dist at noon
S 25° W	130	136	63½	40° 02' N	8° 14' W	Ram h. N 29° E dis 23 L

Ho	H	F	Courses	Winds	Remarks on b July 25th 1772.
1	7		S W.	N E	Brisk wind & clear weather
2	6	4			
3	6	4			
4	7				Ditto weather
5	7				
6	7	2			
7	7	4			
8	7				Brisk wind & hazy
9	7				
10	7				
11	6	4			
12	6	3			Ditto weather
1	6	4			
2	6	5			
3	6	4			
4	6	4			
5	6	2			
6	6				
7	6				
8	6				Brisk wind & cloudy
9	6		S W b W		
10	6				
11	6		S W b S		
12	6				Ditto weather: meridl Alt. 71° 40' S

Course	Dis	DL	Dp	Lat in	Long made	Bearg & dist at noon
S 25 W	158	142	66	37° 40' N	9° 39' W	Ramsh 28½ E dis 288 L.

Hs	Hs	F.	Courses	Winds	Remarks on Ds July 26th 1772
1	6				
2	6	4	SW	NE	Brisk wind & cloudy
3	6	4			
4	6	4			Brisk wind & cloudy
5	6	4			
6	6	4			Variation 06° 3' by azim. 17° 11' W
7	6	4			
8	6	6			Brisk wind & cloudy
9	6	4			
10	6	3			
11	6	3			
12	6	4			Brisk wind & flying clouds
1	5	4			
2	5	4			
3	5	4			Ditto weather
4	5	4			At 2 h 10' after 4 AM by wa. ob. dis
5	6				far I of ♃ & sun a Attick. d 33. 15, alt.
6	6				of a Ar. 65. 56, & that of ♃ DL 29. 43 3/4
7	6				At 6. 32. 45' by wa. alt ☉ DL was 16. 52
8	5	4			Weather as above
9	5	2			Saw a sail in the N Equarter
10	5	2			
11	5	4			
12	5	6			Ditto weath. mer alt ☉ DL 73. 36. S

Course	Dis	DL	Dep	Lat in	Long made	Bear & dist at noon
S 28° W	145	128	68	35. 21 S	W. 12. 45 W	Rame Hd N 37° E d. 332 L Punch S. 36 W d. 79 L

Hs	Ks	Ls	Courses	Winds	Remarks on July 27th 1772.
1	5	4	SE.	NE.	Moderate wind & cloudy
2	5	4			
3	5		SbE.½E		
4	4	6			Ditto weather
5	4	3			
6	4	3			
7	4	6			
8	4	6			Weather as above.
9	4	5			
10	4	3			
11	5				
12	4	3			Moderate wind & hazy
1	5				
2	5				
3	4				
4	4				Ditto weather.
5	4				
6	4	4			
7	4	4			
8	4	2			Moderate wind & hazy
9	4	5			At 9h 30' 4" by watch the alt of his Ll was 18° 53¼ & y azim. of his center S63E. At 11h 5' 15" the alt of his Ll was 7° 31'. Cloudy weather.
10	5				
11	4	3			
12	4	6			

Course	Dis	DL	Dep	Lat in	Long made	Bear & dist at noon
S 15 W	116	110	34.7	33.14 N	10.56½ W	Ramah N 25½ E d. 26 Lgs

Ha	K	P	Courses	Winds	Remarks on 8 July 28th 1772
1	4	2	S W.	N E.	Moderate wind & hazy
2	3	2	W b. S		
3	3	4			
4	3	6			Moderate wind & clear weath.
5	3	6			
6	4	2			
7	4	2			Sand seen from y mast head
8	4	4	West		which we take to be Porto Sancho
9	3	4			Weather as above
10	4				
11	3		W ½ S		
12	3				Weather the same
1	2				
2	1				
3	3				
4	3	2			Moderate wind & hazy
5	3	4			
6	3	4			
7	3	4			
8	3	4			Porto Sancho N. 76 d. E
9	3				Weather as above
10	3	4	W b S	N N W	Porto Sancho North
11	4	3			At noon Porto Sancho N 73 E Funchiera
12	4	2			Madeira S W b W. Meridl Alt of L.L. 75° 56 S

Course	Dis	DL	Dep	Latin	Long made	Bearg & distat noon
S 54 W	89	51½	73	32° 48½ S	12° 36¾ W	Funch. S 39 W. d. 51.

Again, Funchal, which is in lat.t 32° 37′ N. bore N.W.b.W. by compass; that is, 13° W. true: &; the lat.t of the ship being 32° 48′ ½ N. the diff.e of lat.t was 11.5, & the merid.l diff.e of lat.t 13.7; the ship was therefore 15 m. dist.t from thence; & east of it in long.e 11′ ¼; & consequently the longit.e of Funchal, by this acc.t is 16° 56′ of Greenwich

Finis.





WILLIAMS & S 1-2 PATERNOSTER L



WILLIAMS'S PATENT.



